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Birzeit University
Mathematics Department
Math 1411 Calculus I

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First Exam First Semester 2018/2019 .... Time: 90 Minutes

بجوز الشبهة 9

Question 1 (66%). **Choose the most correct answer:**

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(1) $\int \frac{x^2}{\sqrt{x^3+1}}dx$ $=\int \frac{x^2}{\sqrt{u}} \cdot \frac{du}{3x^2}$

$u = x^3+1$

$du = 3x^2 \cdot dx$

$dx = \frac{du}{3x^2}$

$= \frac{1}{3}\int u^{-\frac{1}{2}} \cdot du$

$= \frac{1}{3}\left[\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right] + c$

$= \frac{2}{3}\sqrt{x^3+1} + c$

(a) $\frac{2}{3}x\sqrt{x^3+1}+C$

(b) $\frac{2}{3}\sqrt{x^3+1}+C$

(c) $\frac{2x^2}{3}\sqrt{x^3+1}+C$

(d) $\frac{2}{3\sqrt{x^3+1}}+C$

(2) The graph of the function $y=\cos(x-\frac{\pi}{2})$ is symmetric about

$= \cos(x)$

(a) origin

(b) x-axis

(c) y-axis

(d) none

(3) If $F(x)=\int\limits_{\cos x^2}^{0}\frac{-1}{1-t^2}dt$. Then $F'(x)=$

$F(x) = -\int_0^{\cos x^2} \frac{-1}{1-t^2}\, dt$

$F'(x) = +\left(\frac{-1}{1-(\cos x^2)^2}\right) + \sin(x^2)\cdot 2x$

$= \frac{-2x \sin(x^2)}{\sin^2(x^2)} = \frac{-2x}{\sin(x^2)} = -2x \csc(x^2)$

(a) $2x\csc x^2$

(b) $-2x\csc^2(x^2)$

(c) $-2x\csc(x^2)$

(d) $-2x\cos(x^2)$

(4) Let $f(x)=\begin{cases}\frac{x^2+x-2}{1-x}, & x\neq 1\\ A, & x=1\end{cases}$ Find the constant A so that $f(x)$ is continuous at $x=1$.

$\lim_{x\to1}\frac{x^2+x-2}{1-x} = \lim_{x\to1}\frac{(x+2)(x-1)}{-(x-1)} = \lim_{x\to1} -x-2$

$= -1-2 = -3 = A.$

(a) $A=3$

(b) $A=1$

(c) $A=-3$

(d) $A=-1$

(5) The domain of the function $f(x) = \frac{1}{\sin^2 x}$ is $\sin^2(x) \neq 0$

(a) $x \neq n\pi$, n integer $\cancel{\sin^2(x) \neq 0}$

(b) $0 < x < \pi$ $\sin(x) = 0$

(c) $x > 0$ $x = 0, \pm\pi, \pm 2\pi$

(d) $x \neq (2n+1)\frac{\pi}{2}$, n integer $\cancel{\sin^2(x) \neq 0}$

(6) If $z = a + ia$ is a complex number, where $a \neq 0$ is a real number, , then $\frac{z}{\overline{z}} =$

$a \neq 0$

(a) i

(b) $\overline{z}$

(c) a^2

(d) $2a$

$\frac{z}{\overline{z}} = \frac{a+ia}{a-ia} = \frac{a(1+i)}{a(1-i)} \cdot \frac{(1+i)}{(1+i)} = \frac{(1+i)^2}{1+1} = \frac{(1+i)^2}{2}$

$= \frac{1+2i-1}{2}$

(7) $\lim_{x \to 0} \frac{x - x\cos x}{\sin^2(x)} =$ $\lim_{x\to 0} \frac{x(1-\cos x)}{1-\cos^2(x)} = \cancel{\lim_{x\to 0} \frac{1-\cos(x)}{\frac{\sin(x)}{x}\cdot\sin(x)}} = \lim_{x\to 0} \frac{1}{\sin(x)} - \frac{\cos(x)}{\sin(x)}$

(a) 2

(b) 0 $= \lim_{x\to 0} \frac{x}{1+\cos(x)} = \frac{0}{2} = 0$

(c) ∞

(d) $\frac{1}{2}$

(8) $\lim_{x \to -2^-} \frac{2-x}{2-\sqrt{2-x}} =$ $\frac{2--2}{2-\sqrt{2+2}} = \frac{4}{0}$

(a) 4 $\frac{-4.1}{2-\sqrt{4.1}} = \frac{+}{-}$ -2.1

(b) 2

(c) ∞

(d) $-\infty$

(9) The graph of $\frac{\sin x}{x}$ has $\lim_{x\to\infty} \frac{\sin x}{x} =$

(a) a vertical asymptote $x = 0$ $1 \geq \sin x \geq -1$

(b) an oblique asymptote $y = x$ $\frac{1}{x} \geq \frac{\sin x}{x} \geq -\frac{1}{x}$

(c) a horizontal asymptote $y = 0$ $0 \quad \lim \frac{\sin x}{x} \geq 0$

(d) no asymptote

(10) The function $f(x) = x^3 - 3x^2 + 1$ on $[-1, 1]$ has an absolute minimum at $x =$

(a) 0 $f'(x) = 3x^2 - 6x$

(b) 2

(c) -1 $3x^2 - 6x = 0$

(d) 1 $x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0 \qquad x = 2$

$f(1) = 1 - 3 + 1 = -1$

$f(-1) = -1 - 3 + 1 = -3$

(11) If f, g are functions, such that f is odd, g is odd, then the function $f+g$ is

(a) odd

(b) even

(c) odd and even

(d) neither odd nor even

f odd $\Rightarrow f(-x) = -f(x)$
g odd $\Rightarrow g(-x) = -g(x)$
$z = f+g \Rightarrow z(-x) = f(-x) + g(-x) = -f(x) - g(x)$
$= -(f(x) + g(x))$.
$z(-x) = -z(x)$

(12) The range of the function $f(x) = \frac{1}{\sin^2 x}$ is

(a) $y \geq 0$

(b) $0 < y \leq 1$

(c) $y \geq 1$

(d) $|y| \geq 1$

$0 < \sin^2 x \leq 1$
$\frac{1}{0} > \frac{1}{\sin^2 x} \geq 1$
$\infty > y \geq 1 \Rightarrow [1, \infty)$

(13) The function $f(x) = -x^3 + 3x^2 - 4$ is increasing

(a) on the interval $(-1, 1)$

(b) for all $x > 0$

(c) for all $x > 2$, $x < 0$

(d) on the interval $(0, 2)$

$f'(x) = -3x^2 + 6x$
$-3x^2 + 6x = 0$
$x^2 - 2x = 0$
$x(x-2) = 0$
$x = 0, 2.$

$- - \; + + \; - -$ (0, 2)

$-27 + 18$

(14) Suppose that f, g are differentiable functions and $f(-1) = 2$, $f'(-1) = 3$, $g'(2) = -5$, $g'(-1) = -2$ then $(g \circ f)'(-1) =$

(a) -15

(b) 10

(c) 15

(d) 2

$(g(f(-1)))' = g'(f(-1)) \cdot f'(-1) =$
$= g'(2) \cdot (3) = (-5)(3) = -15$

(15) The equation of the normal line to the curve $x^2 + xy - y^2 = 11$ at the point $P(3, 1)$ is

(a) $y - 1 = \frac{1}{7}(x-3)$

(b) $y - 1 = 7(x-3)$

(c) $y - 1 = \frac{-1}{7}(x-3)$

(d) $y - 1 = -7(x-3)$

$2x + xy' + y - 2y\,y' = 0$
$6 + 3y' + 1 - (2)\,y' = 0$
$y' + 7 = 0 \Rightarrow \boxed{y' = -7}$

$y = y_0 + m(x - x_0)$
$y = 1 + m(x-3)$

m $\frac{1}{7}$

(16) If f is a function such that f' is negative and decreasing on an interval I, then f is

(a) increasing and concave up on I

(b) increasing and concave down on I

(c) decreasing and concave up on I

(d) decreasing and concave down on I

f' $- - -$

f'' $- - - -$

(17) The linearization of the function $y = x + \sin x$ at the point (π, π) is $y' = 1 + \cos x$

$y'(\pi) = 1 - 1 = 0$

(a) $y = 2x - \pi$ $L(x) = y(\pi) + y'(\pi)(x - \pi)$

(b) $y = 2x + \pi$ $L(x) = y(\pi) + 0$

(c) $y = \pi$ $= \pi$

(d) None of the above

(18) $\int_0^2 x\,|\,x-1\,|\,dx = \int_0^1 x - x^2\,dx + \int_1^2 x^2 - x\,dx$ $|x-1| = \begin{cases} x-1 & , x \geq 1 \\ 1-x & , x < 1 \end{cases}$

(a) $\frac{2}{3}$ $= \left(\frac{x^2}{2} - \frac{x^3}{3}\right]\Big|_0^1 + \left[\frac{x^3}{3} - \frac{x^2}{2}\right)\Big|_1^2$ $x\,|x-1| = \begin{cases} x^2 - x & , x \geq 1 \\ x - x^2 & , x < 1 \end{cases}$

(b) $\frac{-1}{3}$

(c) 1 $= \frac{1}{2} - \frac{1}{3} + \left[\left(\frac{8}{3} - 2\right) - \left(\frac{1}{3} - \frac{1}{2}\right)\right]$

(d) -1 $= \frac{1}{6} + \frac{2}{3} + \left(+\frac{1}{6}\right) = \frac{2}{6} + \frac{2}{3} = \frac{1}{3} + \frac{2}{3} = 1$

(19) If $f(x) = \begin{cases} x^2 + 2, & x \leq 1, \\ ax - b, & x > 1, \end{cases}$ is differentiable at $x = 1$, then

$1 + 2 = a - b$ $f'(x) = \begin{cases} 2x \\ a \end{cases}$

$3 = a - b$ $\boxed{2 = a}$

$a = 3 + b$

(a) $a = 2, \quad b = -1$

(b) $a = 2, \quad b = 5$

(c) $a = 1, \quad b = -2$

(d) $a = -1, \quad b = 2$

(20) Suppose that u and v are functions of x that are differentiable at $x = 1$ and that $u(1) = -3$, $u'(1) = -3$, $v(1) = 2$, $v'(1) = 4$. The value of $\frac{d}{dx}\left(\frac{u}{v}\right)$ at $x = 1$ is $\frac{u'v - uv'}{v^2}$

(a) $\frac{-2}{3}$ $= \frac{-3 \cdot 2 - (-3)\,4}{4}$

(b) $\frac{-9}{2}$

(c) $\frac{3}{2}$ $= \frac{-6 + 12}{4} = \frac{6}{4} = \frac{3}{2}$

(d) $\frac{-3}{4}$

(21) For the function $f(x) = \dfrac{x^2 + x - 2}{1 - x}$, the line $x = 1$ is

$\frac{(x+2)(x-1)}{(1-x)} = -x-2$

(a) a vertical asymptote

(b) a horizontal asymptote

(c) an oblique asymptote

(d) none of the above

(22) Suppose that $f(x)$ satisfies $\lim_{x \to 2} \dfrac{f(x) - 5}{x^2 - 4} = 3$, then $\lim_{x \to 2} f(x) = f(2)$.

$f(2) = 5$

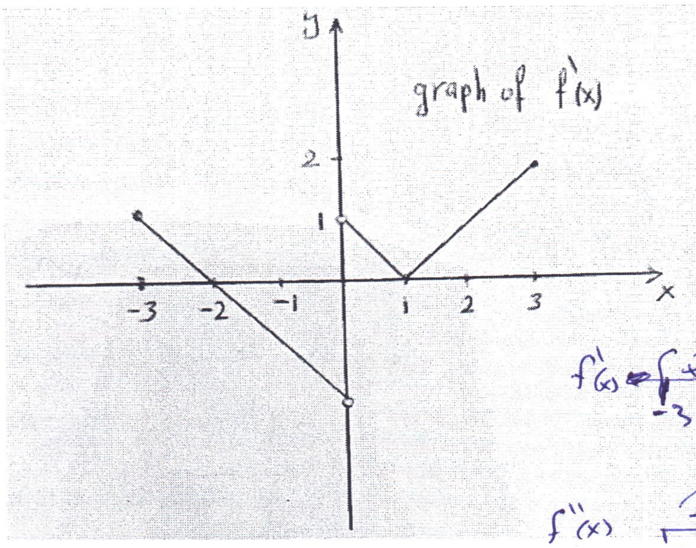
(a) 0

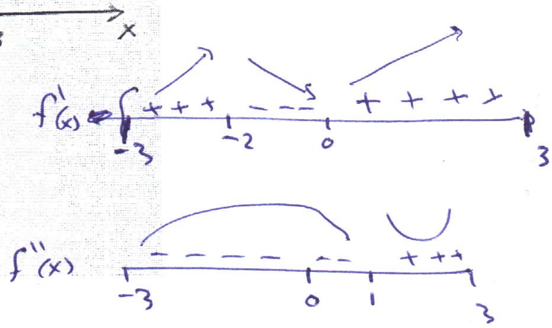
(b) 1

(c) 5

(d) 3

Question 2 (14%). Let $f(x)$ be continuous on $[-3, 3]$, the graph of its derivative $f'(x)$ is given below, Use the graph of f' to answer the following questions





1. $f(x)$ is increasing on: $[-3, -2] \cup [0, 3]$

2. $f(x)$ is decreasing on: $[-2, 0]$

3. graph of $f(x)$ is concave up on: $(1, 3)$

4. graph of $f(x)$ is concave down on: $[-3, 1]$

5. graph of $f(x)$ has inflection point(s) at $x =$ 1

6. $f(x)$ has a local maximum at $x = -2$

7. $f(x)$ has a local minimum at $x = 0$

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Question 3 (20%). Let $f(x) = \dfrac{x^2}{x+1}$. [$f'(x) = \dfrac{x(x+2)}{(x+1)^2}$, $f''(x) = \dfrac{2}{(x+1)^3}$]. Find

1. Domain of $f(x) =$ $\mathbb{R} \setminus \{-1\}$
2. $\lim\limits_{x\to\infty} f(x) =$ ∞
3. $\lim\limits_{x\to-\infty} f(x) =$ $-\infty$
4. $\lim\limits_{x\to-1^+} f(x) =$ ∞
5. $\lim\limits_{x\to-1^-} f(x) =$ $-\infty$
6. Horizontal asymptotes (if any) are : None
7. Vertical asymptotes (if any) are: $x = -1$
8. Oblique asymptotes (if any) are: $y = x - 1$
9. Intervals of increasing $(-\infty, -2] \cup [0, \infty)$
10. Intervals of decreasing $[-2, -1) \cup (-1, 0]$
11. local maximum points $(-2, f(-2)) = (-2, -4)$
12. local minimum points $(0, f(0)) = (0, 0)$
13. when is the graph concave up? $(-1, \infty)$
14. when is the graph concave down? $(-\infty, -1)$
15. inflection points None

$f'(x) = \dfrac{x(x+2)}{(x+1)^2} = 0$

$x = 0$ $\quad$ $x = -2$

$f'(x)$: $+ +$ | -2 | $- -$ | -1 | $- -$ | 0 | $+ +$

$x^2 \div (x+1) = x - 1$, remainder 1

$f''(x) = \dfrac{2}{(x+1)^3} = 0$

$- - -$ | -1 | $+ + +$

Graph the function using the above information.

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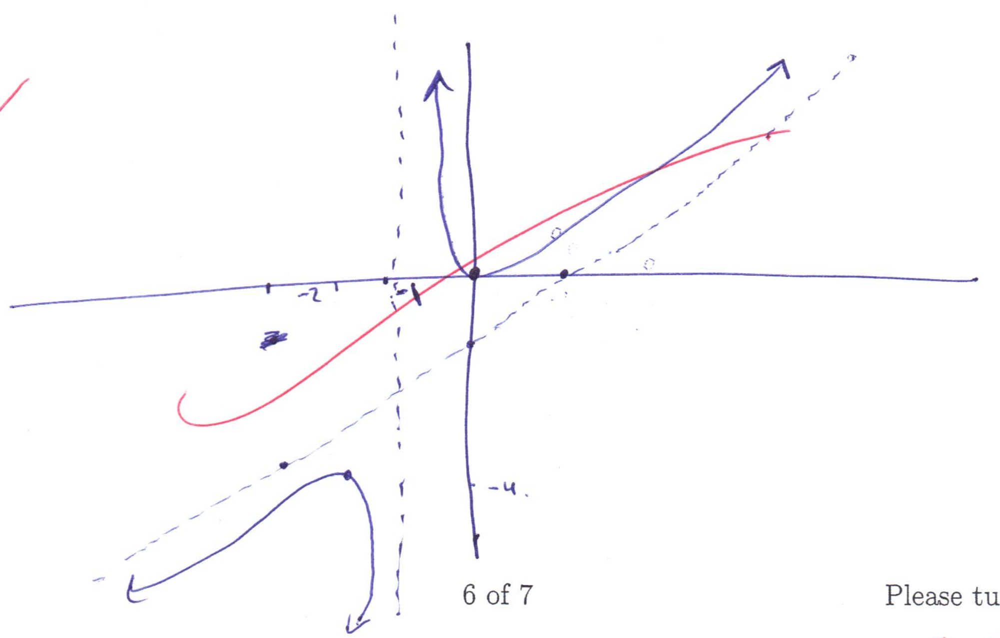

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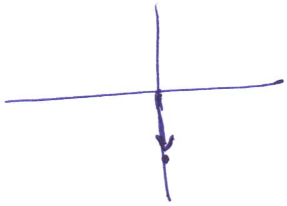
Question 4 (6%). Find the three cubic roots of $-8i$.

$z = -8i \qquad (0, -8)$

$r = 8$

$\theta = \frac{3\pi}{2}$



$n = 3$

$k = 0, 1, 2$

$$W_k = r^{\frac{1}{n}} \cdot e^{i\left[\frac{\theta}{n} + \frac{2\pi}{n} k\right]}$$

$$W_0 = (8)^{\frac{1}{3}} \cdot e^{i\left[\frac{\pi}{2} + \frac{2\pi}{3} \cdot 0\right]} = 2 \cdot e^{i\left[\frac{\pi}{2}\right]}$$

$$= 2\left[\cos\left(\frac{\pi}{2}\right) + i \sin\left(\frac{\pi}{2}\right)\right]$$

$$= 2\left[0 + i(1)\right]$$

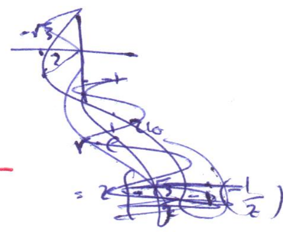
$$= \boxed{2i}$$

$$W_1 = 2 \cdot e^{i\left[\frac{\pi}{2} + \frac{2\pi}{3} 1\right]} = 2 \, e^{i\left[\frac{7\pi}{6}\right]}$$

$$= 2\left[\cos\left(\frac{7\pi}{6}\right) + i \sin\left(\frac{7\pi}{6}\right)\right]$$

$$= 2\left[-\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right]$$

$$= \boxed{-\sqrt{3} - i}$$



$$W_2 = 2 \cdot e^{i\left[\frac{\pi}{2} + \frac{2\pi}{3} 2\right]} = 2 \, e^{i\left[\frac{11\pi}{6}\right]}$$

$$= 2\left[\cos\left(\frac{11\pi}{6}\right) + i \sin\left(\frac{11\pi}{6}\right)\right]$$

$$= 2\left[\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right]$$

$$= \boxed{\sqrt{3} - i}$$

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