BIRZEIT UNIVERSITY

Mathematics Department

Calculus I – Math1411

First Exam **Time: 100 Minutes** First Semester 2021 – 2022

Name: Key B Number: Section:

- Write your full name and your number
- Choose your section from table below
- Turn off your mobile
- Calculator is not allowed
- The exam has 9 different pages. Answer all questions

Section	Instructor	Day	Time	Room
1	Farah Omar	T	14:15 - 15:05	SCI120
2	Areej Awawdah	S	08:00 - 08:50	SCI240
3	Muna Abu Alhalawa	R	09:00 - 09:50	S.Abdulhadi380
4	Areej Awawdah	R	11:25 - 12:15	S.Abdulhadi380
5	Ayah Sharsheer	S	11:25 - 12:15	Al-Juraysi002
6	Muna Abu Alhalawa	T	09:00 - 09:50	S.Abdulhadi380
7	Batool Raddad	T	14:15 - 15:05	SCI240
8	Ayah Sharsheer	S	08:00 - 08:50	S.Abdulhadi380
9	Farah Omar	R	13:00 - 13:50	SCI120
10	Ayah Sharsheer	S	13:00 - 13:50	O.Abdulhadi051
11	Ayah Sharsheer	T	08:00 - 08:50	S.Abdulhadi380
12	Ayah Sharsheer	S	09:00 - 09:50	SCI120
13	Alaeddin Elayyan	R	10:00 - 10:50	S.Abdulhadi380
14	Batool Raddad	S	08:00 - 08:50	Al-Juraysi002
15	Farah Omar	T	09:00 - 09:50	SCI240
16	Batool Raddad	R	14:15 - 15:05	SCI120
17	Ayah Sharsheer	T	10:00 - 10:50	SCI240
18	Farah Omar	S	09:00 - 09:50	O.Abdulhadi051
19	Areej Awawdah	R	08:00 - 08:50	SCI240
20	Batool Raddad	S	13:00 - 13:50	O.Abdulhadi052
21	Ayah Sharsheer	W	14:15 - 15:05	S.Abdulhadi380

Question One (60 points) Circle the most correct answer:

1. Assume $x^3 - 1 \leq f(x) \leq \frac{2x-1}{x^2+x+1}$ for all values of x. Then $\lim_{x\to 0} f(x) =$

   (a) 0

   (b) 1

   (c) -1

   (d) 2

   (e) -2

2. Assume the function $f(x) = \begin{cases} x^2 - bx + 3\,, & x \geq -1 \\ \frac{4}{x} - 2c\,, & x < -1 \end{cases}$ is differentiable on $\mathbb{R}$. Then the values of b and c are

   (a) $b = 2$ and $c = -2$

   (b) $b = 2$ and $c = 5$

   (c) $b = -2$ and $c = -2$

   (d) $b = 2$ and $c = -5$

3. A square has a side length x. Assume x increases from 1cm to 1.2cm. The estimated increase in the square's area is

   (a) 0.4

   (b) 0.44

   (c) 0.04

   (d) 0.044

4. If the function $f(x) = x^3 - \beta x^2$ has an inflection point at $x = 1$, then the value of β is

   (a) 0

   (b) 1

   (c) 2

   (d) 3

5. Using lineraization for the function $f(x)=\frac{x}{x+1}$ at $x=1$, the estimated value of $f(\frac{1}{3})$ is

(a) $\frac{1}{5}$

(b) $\frac{1}{4}$

(c) $\frac{1}{3}$

(d) $\frac{1}{2}$

6. If $g(x)=x-3$ and $f(x)=\frac{1}{x}$, then the domain of $(f\circ g)(x)$ is

(a) $\mathbb{R}$

(b) $\mathbb{R}\setminus\{0,3\}$

(c) $\mathbb{R}\setminus\{0\}$

(d) $\mathbb{R}\setminus\{3\}$

7. The range of the function $f(x)=\cos x+\sin x$ is

(a) $[-2\sqrt{2},2\sqrt{2}]$

(b) $[-\sqrt{2},\sqrt{2}]$

(c) $[-2,2]$

(d) $[-1,1]$

8. The domain of the function $f(x)=\frac{\sqrt{5-x}}{2-\sqrt{x-1}}$ is

(a) $(1,5)\cup(5,\infty)$

(b) $[1,5)\cup(5,\infty)$

(c) $[1,5]$

(d) $[1,5)$

9. If the **differential** of the function $f(x) = x^2 - x + 7$ when x changes from 2 to c is 6, then the value of c is

(a) 3
(b) 4
(c) 5
(d) 6

10. The function $f(x) = \frac{x^3}{x^2-2x}$ has

(a) oblique asymptote at $y = x$
(b) removable discontinuity at $x = 0$
(c) vertical asymptote at $x = 0$
(d) horizontal asymptote at $y = 1$

11. The period of the function $y = \tan\frac{x}{2}$ is

(a) π
(b) 2π
(c) $\frac{\pi}{2}$
(d) 2

12. The function $y = \frac{\cos x}{x - \sin x}$ is

(a) odd
(b) even
(c) neither odd nor even
(d) odd and even

13. Assume $f(x) = \sqrt{x+3}$ is defined on $[-2,6]$. The value of c in the conclusion of Mean Value Theorem is

(a) -1

(b) 0

(c) 1

(d) 2

14. If $y = 18 - 4x$ is the normal line to the curve $f(x) = \sqrt{x}$ at $x = c$, then the tangent line on this curve at $x = c$ is

(a) $y = \frac{x}{4} - 2$

(b) $y = \frac{x}{4} + 2$

(c) $y = \frac{x}{4} + 1$

(d) $y = \frac{x}{4} - 1$

15. If $y = \int_{\cot x}^{0} (1+t^2)dt$ then $y'(\frac{\pi}{4}) =$

(a) 4

(b) -4

(c) 2

(d) -2

16. The slope of the curve $x^2y - xy = 8$ at $x = -1$ is

(a) 6

(b) 5

(c) 4

(d) 3

17. $\lim_{x\to 1^+} \frac{|x-1|}{1-x} =$

(a) 0

(b) 1

(c) -1

(d) does not exist

18. The function $f(x) = \frac{x^2-2}{x-1}$

(a) has removable discontinuity at $x = 1$

(b) is odd

(c) is even

(d) has oblique asymptote at $y = x+1$

19. $\int_0^1 \frac{dx}{\sqrt{x}(1+\sqrt{x})^2} =$

(a) 2

(b) 1

(c) 0

(d) -1

20. $\int_0^{\frac{\pi}{4}} \frac{2}{1+\cos 2x}\, dx =$

(a) 0

(b) 1

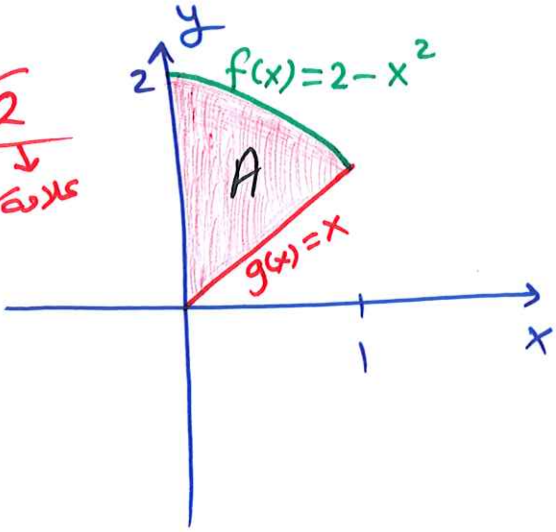
(c) -1

(d) 2

<u>Question Two</u> (9 points) Given the functions $f(x) = 2 - x^2$ and $g(x) = x$.

(i) Sketch the **region in the first quadrant** bounded between these functions and y-axis.

(ii) Find the **area** of this region.



(i) $f(x) = g(x)$

2 — علامة على الشكل، علامة على الكيرفز

1 — $2 - x^2 = x$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

1 — $x = -2$ or $\boxed{x = 1}$

2 — (ii) $A = \int_0^1 \left(f(x) - g(x)\right) dx = \int_0^1 (2 - x^2 - x)\, dx$

1 — $= 2x - \frac{x^3}{3} - \frac{x^2}{2} \Big|_0^1$

1 — $= 2 - \frac{1}{3} - \frac{1}{2}$

1 — $= \frac{7}{6}$

12

Question Three (11 points) Answer the following

(a) Find **continuous extension** $F(x)$ for the function $f(x) = \frac{2x}{x^2-x}$ at $x = 0$.

(1) $\lim_{x\to 0} \frac{2x}{x^2-x} = \lim_{x\to 0} \frac{2}{x-1} = -2$ (1)

(1) $F(x) = \begin{cases} \frac{2x}{x^2-x} & \text{if } x \neq 0 \\ -2 & \text{if } x = 0 \end{cases}$

(b) Find and classify the extreme values of $f(x) = 2\sqrt{x} - x$

(1) $D(f) = [0, \infty)$

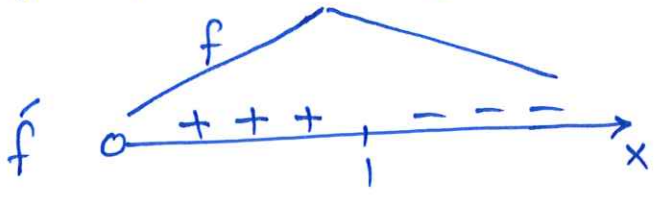
(1) $f'(x) = \frac{1}{\sqrt{x}} - 1 = \frac{1-\sqrt{x}}{\sqrt{x}}$

(1) $f'(x) = 0 \Leftrightarrow x = 1 \in D(f)$ and its interior

(1) $f'(x)$ is undefined at $x = 0 \in D(f)$ but not interior

(1) so $(1, f(1)) = (1, 1)$ is only critical point

(1) $(0, f(0)) = (0, 0)$ is only endpoint



(1)

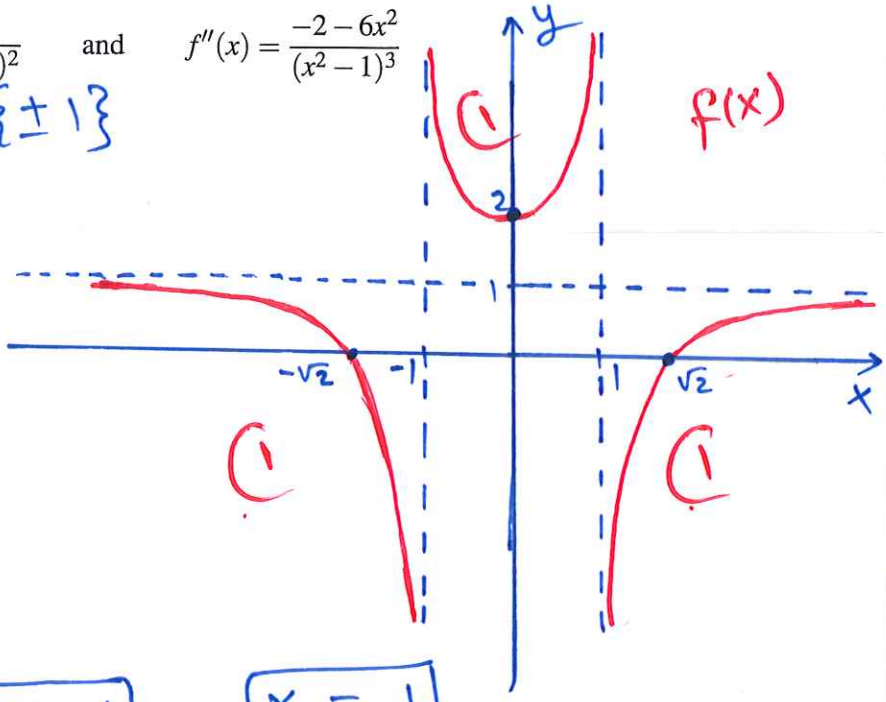
(1) f has L. Max of $f(1) = 1$ "Abs. Max"

(1) f has L. Min of $f(0) = 0$

Question Four (22 points) Find the following about the function $f(x) = \frac{x^2-2}{x^2-1}$ given that

$$f'(x) = \frac{2x}{(x^2-1)^2} \quad \text{and} \quad f''(x) = \frac{-2-6x^2}{(x^2-1)^3}$$

1. Domain of $f(x) =$ $\mathbb{R} \setminus \{\pm 1\}$
2. $\lim_{x\to -1^+} f(x) =$ $+\infty$
3. $\lim_{x\to -1^-} f(x) =$ $-\infty$
4. $\lim_{x\to \infty} f(x) =$ 1
5. $\lim_{x\to -\infty} f(x) =$ 1
6. $\lim_{x\to 1^+} f(x) =$ $-\infty$
7. $\lim_{x\to 1^-} f(x) =$ $+\infty$
8. Vertical asymptotes $x = -1$, $x = 1$
9. Horizontal asymptotes $y = 1$
10. Oblique asymptotes none
11. y-intercept $(0,2)$ or $y=2$
12. Critical points $(0,2)$ or $x=0$
13. Interval of increasing $[0,1) \cup (1,\infty)$
14. Interval of decreasing $(-\infty,-1) \cup (-1,0]$
15. Extreme Values $f(0) = 2$ is L. Min
16. Inflection points none
17. Intervals where f is concave up $(-1,1)$
18. Intervals where f is concave down $(-\infty,-1) \cup (1,\infty)$
19. Sketch the graph of f



Good Luck

-1 لمرة واحدة في حالة كانت صود القرات خطا

Asy. علامة 2

curves علامة 3