First+mid
Math 141

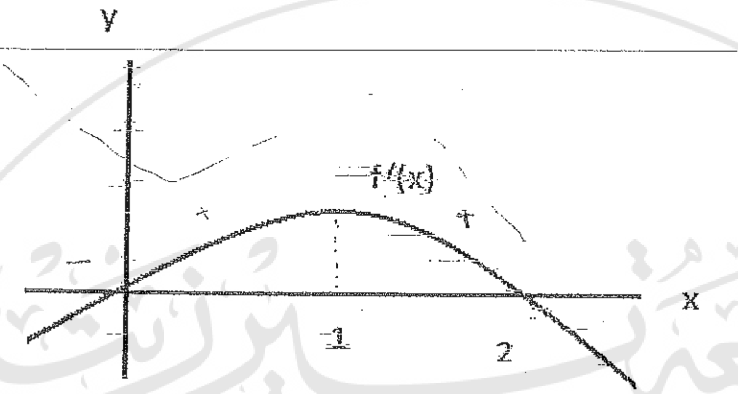
Key

جامعة بيرزيت
BIRZEIT UNIVERSITY
MATH. DEPARTMENT Summer Semester
2012/2013

Student Name (Arabic): ____________________ ID : __________
Section : ____________________ Short Exam(3) MATH141

Q1) Consider the following graph of $f'(x)$ on $(-\infty, \infty)$.

5



Answer question 1-7 below

1) The critical point(s) of f is/are $x=0$, $x=2$

2) f increasing on $[0,2]$ and decreasing on $(-\infty,0] \cup [2,\infty)$

3) f has a local minimum at x= 0 and a local maximum at x= 2

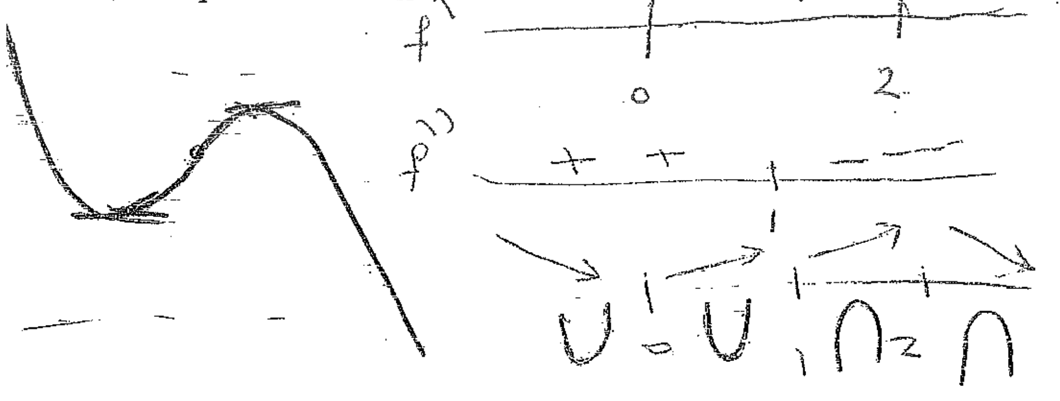
4) f is concave up on $(-\infty,1]$ and concave down on $[1,\infty)$

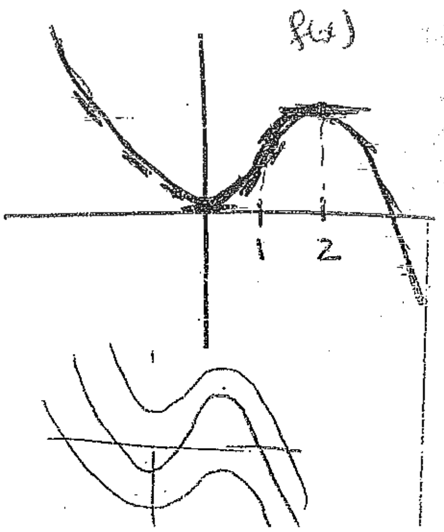
$f'(x)=0$
$f'(0)=0$
$f''(0)>0$

5) Are they absolute extreme values
a) Yes [✓ b) No]

6) f has inflection point at x= 1

7) Graph the function





Q(2) **Solve** the following questions then circle the correct answer.

(1) Applying the mean value theorem to the function $f(x) = x^{\frac{3}{4}}$ on the interval $[0,1]$ we get

[✓ a)] $c = \left(\frac{3}{4}\right)^{4}$

b) $c = \left(\frac{4}{3}\right)^{4}$

c) $c = 1$

d) We cannot apply the mean value theorem.

(2) The function $y = tanx - cotx - x,\ \ on\ \ (0,\frac{\pi}{2})$

(a) Has no zero.

(b) Has more than one zero.

[✓ (c)] Has exactly one zero.

(d) Has exactly two zero.

(3) $\lim_{x\to 0} \frac{\pi^{sinx}-1}{e^{x}-1}$

[✓ a)] $ln\pi$

b) 1

c) 0

d) ∞

a c a b

(4) $\lim_{x\to 0^+}(sinx)(lnx)$

a) 1

[✓ b)] 0

c) ∞

d) Does not exist.

(5) One of the following is always true

a) If f has a local maximum at $x = c$ then $f^{/}(c) = 0$.

b) If f has an inflection point $(c, f(c))$ then $f^{//}(c) = 0$.

c) If $f^{/}(c) = 0$ then f has a local max. or local min. at $x = c$.

[✓ d)] If f is continuous on a closed interval then f has both absolute maximum and absolute minimum.

(6) The absolute max. of $f(x) = \frac{1}{x^2-2x+5}$ $on\ [0,4]\ is$

a) 1/5
[✓ b)] ¼
c) 2/5
d) 1/13

d b d

(7) Suppose the radius of a sphere increases from 10 to 10.1 cm. The approximate change in the surface area of the sphere is

a) 2π
b) 4π
c) 6π
[✓ d)] 8π

(8) $\lim_{x\to 0^+} \sqrt{x}e^{\cos\left(\frac{\pi}{x}\right)} =$

$\lim_{x\to 0^+} \frac{e^{\cos\left(\frac{\pi}{x}\right)}}{\frac{1}{\sqrt{x}}}$ $x^{-1/2}$ $\left(\frac{\infty}{\infty}\right)$

$$\frac{e^{\cos\frac{\pi}{x}} \cdot \left(-\sin\frac{\pi}{x}\right)\left(\frac{-\pi}{x^2}\right)}{-\frac{1}{2}x^{-3/2}}$$

a) π
b) $-\pi$
[✓ c)] 0
d) Does not exist.

$$\sqrt{x}\, e^{-1} \le \left(e^{\cos\frac{\pi}{x}}\right)(\sqrt{x}) \le e\sqrt{x}$$

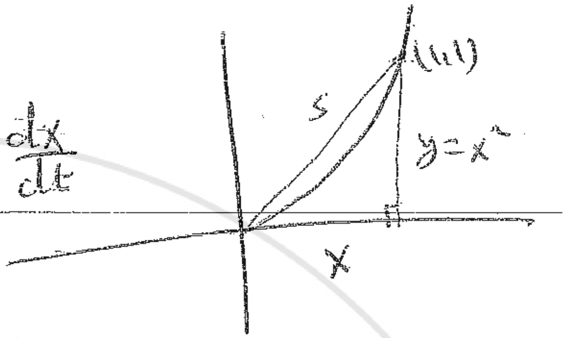
$\downarrow 0 \quad \downarrow 0 \quad \downarrow 0$

3

Q4 : A particle moves on the parabola $y = x^2$ in the first quadrant. Its distance from the origin increases at the rate of 1 cm/min. Find the rate at which its x-coordinate changes when it is at the point (1,1).

$$s^2 = x^2 + x^4$$

$$2s\frac{ds}{dt} = 2\frac{dx}{dt} + 4x^3\frac{dx}{dt}$$



at (1,1) , $s = \sqrt{2}$

$$\Rightarrow \sqrt{2}\frac{ds}{dt} = \frac{dx}{dt} + 2\frac{dx}{dt}$$

$$\Rightarrow \frac{dx}{dt} = \frac{\sqrt{2}}{3} \text{ cm/min.}$$

Good Luck

$\frac{22}{25}$

جامعة بيرزيت

BIRZEIT UNIVERSITY

Mathematics Department

Spring 2014

FIRST EXAM – MATH 141

Student's Name: Amin Saeb Al-Ajez

Student's Number: 1131965

Section: 4

Instructor:

1) Saddam Adnan Zaid
[✓ 2) Hiba Khalil Sharha]
3) Mahmoud Ghannam

Answer sheet for the multiple choice quastion:

Page 1	
1	d
2	a
3	b
4	b
5	c

Page 2	
6	b
7	d
8	c
9	b
10	b

Page 3	
11	d
12	a
13	b
14	d
15	c

Page 4	
16	d
17	d
18	[illegible]
19	b
20	b

$x^2+y^2=r^2$

$x^2+9=25$

$\frac{y}{r}$

$x^2=10$

$x=\pm4$

$\frac{y}{r}$

$x^2+y^2=r^2$

$3^2+y^2=5^2$

Question 1. (20%).Circle the most correct answer:

1. If $\sin x = \frac{3}{5}$, $x \in [\frac{\pi}{2}, \pi]$, then $\cos x =$

   (a) $\frac{3}{5}$.

   (b) $\frac{4}{5}$.

   (c) $\frac{-3}{5}$.

   [✓ (d)] $\frac{-4}{5}$.

$\sin x = \frac{y}{r}$ $9+x^2=25$

$-\frac{4}{5}$ $x^2=16$ $x=\pm4$

2. $\lim\limits_{x\to7}\sqrt{(11-x)} = 2$, given $\epsilon = 1$ find the largest δ such that $0 < |x-7| < \delta$, then $|\sqrt{(11-x)} - 2| < \epsilon$

   [✓ (a)] $\delta = 3$

   (b) $\delta = 5$

   (c) $\delta = 4$

   (d) $\delta = 2$

$-1 < \sqrt{11-x} - 2 < 1$

$+2 \quad +2 \quad +2$

$1 < \sqrt{11-x} < 3$

$1 < 11-x < 9$

$-11 \quad -11 \quad -11$

$-10 < -x < -2$

$2 < x < 10$

$5 \quad 3$

$2 \quad 7 \quad 10$

3. $\lim\limits_{x\to\infty}\frac{x^2-7x}{x+1} =$

   (a) $-\infty$

   [✓ (b)] ∞

   (c) 1

   (d) -7

$\frac{x^2(1-0)}{x(1+0)} = \infty$

✳ 4. Consider the function $f(x) = \frac{x^2-3x-4}{16-x^2}$, then the vertical asymptote/s is/are

   (a) $x = 4$

   [✓ (b)] $x = -4$

   (c) $x = 4, x = -4$

   (d) None of the above

$-x^2=-16$

$x=\pm4$

5. The domain of the function $f(x) = \frac{\sqrt{2-x}}{x+1}$ is

   (a) $[-2,1)\bigcup(1,\infty)$

   (b) $(-\infty, 2]$

   [✓ (c)] $(-\infty,-1)\bigcup(-1,2]$

   (d) $(-\infty,-1)\bigcup(-1,2)$

$(-\infty,-1)$

$x \to -4^-$ $\frac{(x-4)(x+1)}{(4-x)(4+x)}$

$\frac{\cancel{(x-4)}(x+1)}{-1\cancel{(x-4)}(x+4)} = \frac{x+1}{-(x+4)} = \frac{-(x+1)}{(4+x)}$

-4

-4^+

$+4$

6. $\lim_{x\to\infty}\sqrt{\frac{8x^2-3}{2x^2+x}} =$

$= \sqrt{\frac{x^2(8-0)}{x^2(2-0)}}$ $\sqrt{\frac{8}{2}}$ 4^2 -2

$\frac{x^2(8-\frac{3}{x^2})}{x^2(2+\frac{x}{x^2})}$ 0 $\frac{8}{2}\sqrt{4} =$

- (a) $-\infty$
- [✓ (b) 2]
- (c) ∞
- (d) 4

$-2+4+3$

X 7. The graphs of the functions $f(x) = x^2-5x+2$ and $g(x) = -2x^3+4x+3$ intersect over the interval

$\frac{5+2}{2}$

x^2-5x+2

$x^2+2x^3-5x+4x+2-3=0$

- (a) $(-2,-1)$
- (b) $(0,1)$
- (c) $(1,2)$
- [✓ (d) $(2,3)$]

8. Find value/s of b for which the function $f(x)$ is continuous for all values of x

$$f(x) = \begin{cases} \frac{\sin(3x)}{4x}, & x \neq 0; \\ b-4x, & x=0. \end{cases}$$

$\lim_{x\to 0}\frac{\sin 3x}{4x}$ $= \frac{\sin 3x}{3}\cdot 3x\cdot\frac{1}{4x}$

$\boxed{\frac{3}{4}=b}$

- (a) $\frac{1}{2}$.
- (b) $\frac{-1}{2}$.
- [✓ (c) $\frac{3}{4}$.]
- (d) $\frac{3}{2}$.

X 9. The range of $f(x) = \frac{1}{x-2}+3$ is

$\frac{1}{-1}$ $-1+3=2$

- (a) $(-\infty,3)\bigcup(3,\infty)$
- [✓ (b) $(0,\infty)$]
- (c) $(-\infty,\infty)$
- (d) $(-\infty,\infty)-2$

10. $\lim_{x\to 0}\frac{\cos^2 x-1}{x\sin(2x)} =$

$\frac{\frac{1+\cos 2x}{2}-1}{x\sin(2x)}$

$\frac{\frac{\cos x}{x}\cdot\frac{\cos x+1}{x}-1}{x\,\frac{\sin 2x}{2x}\times 2x}$

$-\frac{1}{2}$

- (a) 0
- [✓ (b) $\frac{-1}{2}$]
- (c) $\frac{1}{2}$
- (d) DNE

$$\frac{(x+\cancel{3})(x+4)}{x(\cancel{x+3})(x-3)}$$

$(x)(x-3)$

$x=0$
$x=3$

$\cancel{x}(x^2-9)$

$x(x+3)(x-3)$

$x=0$
$x=-3$
$x\neq+3$

11. The function $f(x)=\dfrac{x^2+7x+12}{x^3-9x}$ has a removable discontinuity at

(a) $x=-3$ and $x=3$

(b) $x=3$

(c) $x=0$

[✓ (d) $x=-3$]

12. The period of $y=\cos(\frac{\pi x}{2}-\frac{2}{\pi})$ is

[✓ (a) 4]

(b) $\frac{2}{\pi}$

(c) 2π

(d) $\frac{\pi}{2}$

$\frac{\pi}{2}=\frac{2\pi}{B}$

$\frac{\pi B}{\pi}=\frac{4\pi}{\pi}$

$B=4$

13. $\lim\limits_{x\to-\infty}\dfrac{\sin(x)}{x}$

(a) -1

[✓ (b) 0]

(c) 1

(d) DNE

$\cancel{x}(x+4)$

$(\quad)(\quad)$

14. $\lim\limits_{x\to0^+}\dfrac{\sqrt{x^2+4x+1}-1}{x}=$

$\Rightarrow$ ~~$\sqrt{4x}$~~

$$\frac{x^2+4x+\cancel{1}\cancel{-1}}{\cancel{(x)}(\sqrt{x^2+4x+1}+1)}$$

$$\frac{4}{\sqrt{1}+1}=\frac{4}{2}=2$$

(a) 4.

(b) -2.

(c) -4.

[✓ (d) 2.]

15. The horizontal asymptote of $f(x)=\frac{1-x^2}{x^2+1}$

(a) $y=0$

(b) $y=1$

[✓ (c) $y=-1$]

(d) None of the above.

$$\lim_{x\to\infty}\frac{x^2(0-1)}{x^2(1+0)}=-1$$

$x\to-\infty \quad = -1$

$\cancel{x}(x+4)$

$$\frac{x^2+4x+\cancel{1}\cancel{-1}}{\cancel{x}(\sqrt{x^2+4x+1}+1)}\quad\frac{4}{\sqrt{1}+1}\quad\frac{4}{2}$$

16. If the graph of the circle $x^2 + y^2 = 25$ is shifted upward by 3 units, and leftward by 4 units, then the new formula of the circle is

   (a) $(x+4)^2 + (y+3)^2 = 25$
   (b) $(x-4)^2 + (y+3)^2 = 25$
   (c) $(x-4)^2 + (y-3)^2 = 25$
   [✓ (d)] $(x+4)^2 + (y-3)^2 = 25$

17. The function $f(x) = |x| + x\cos x$ is

$f(-x) = |-x| + -x\cos x$

$= \cancel{x}\ x - (\cos x)$

   (a) odd function.
   (b) even function.
   (c) both even and odd function.
   [✓ (d)] neither even nor odd function.

18. Let $g(x) = \sqrt{x}$ and $(g \circ f)(x) = |x|$, then $f(x) =$

   [✓ (a)] $f(x) = -x^2$
   (b) $f(x) = \frac{1}{x^2}$
   (c) $f(x) = x^2$
   (d) $f(x) = \frac{-1}{x^2}$

$g(f(x)) = \sqrt{x} = |x|$

$\sqrt{f(x)} = |x|$

$f(x) = x^2$

$x = -1$

19. $\lim_{x\to -1^+} \frac{|x+1|}{2x+2} =$

$\frac{\cancel{x+1}}{2(\cancel{x+1})}$ $\frac{1}{2}$

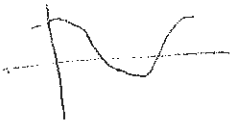
$|x+1| \begin{cases} x+1 & -1 < x \\ -(x+1) & x < -1 \end{cases}$

   (a) $\frac{-1}{2}$
   [✓ (b)] $\frac{1}{2}$
   (c) 0
   (d) DNE

20. $\lim_{x\to 0} \frac{2\cos x}{\sqrt{(x+1)} - 2} =$

2

$\frac{2\times 1}{\sqrt{1} - 2} = \frac{2}{-1}$



   (a) 0
   [✓ (b)] -2
   (c) $\frac{1}{2}$
   (d) DNE

$+ + + +$
x
-1

$\frac{(2\cos x)(\sqrt{x+1} + 2)}{x+1 - 4}$ -3

$(x-3)$

Question 2. *(5%) Let* $f(x) = \frac{x}{4-x^2}$ *Sketch the graph of* $f(x)$ *where;*

Domain of $f(x)$ *is* $(-\infty, \infty) - \{-2, 2\}$

$f(0) = 0$

$\lim_{x\to\infty} f(x) = 0,\ \lim_{x\to-\infty} f(x) = 0$

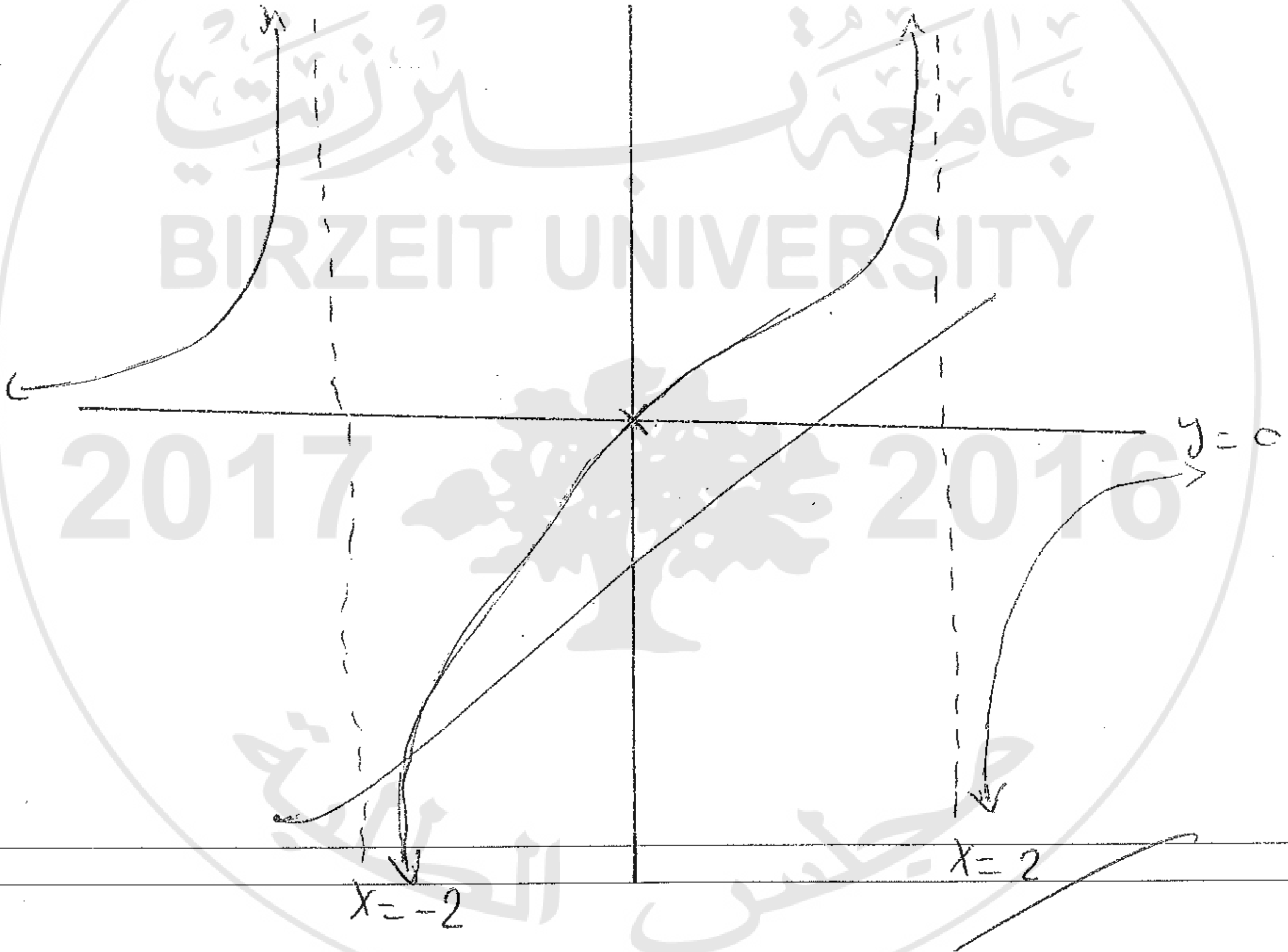
$\lim_{x\to 2^+} f(x) = -\infty,\ \lim_{x\to 2^-} f(x) = \infty$

$\lim_{x\to -2^+} f(x) = -\infty$ *and* $\lim_{x\to -2^-} f(x) = \infty.$

y=0 H.Asy

x=2 V.Asy

x=-2 V.Asy



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78

Birzeit University- Mathematics Department
Calculus I-Math 141

Midterm Exam
Name(Arabic):[redacted]
Instructor of Discussion(Arabic): أحمد ماهر

First Semester 2014/2015
Number:[redacted]
Section: 17

Time: 90 Minutes
There are 4 questions in 6 pages.

Question 1.(48%) Circle the correct answer:

(1) The domain of the function $f(x) = \sqrt{3-x} + \frac{1}{\sqrt{x^3+8}}$ is

(a) $[-2, 3)$
(b) $(-\infty, -2) \cup (-2, 3]$
[✓ (c)] $(-2, 3]$
(d) $(-\infty, -2) \cup [3, \infty)$

$X \leqslant -3$
$X \geqslant -2$
$-2 \quad 3$
$3-x > 0$
$-x > -3$
$x \leqslant 3$
$x^3+8 > 0$
$x > -2$

(2) The range of the function $g(x) = \frac{\sqrt{x}}{1+\sqrt{x}}$ is

(a) $(1, \infty)$
(b) $[0, \infty)$
[✓ (c)] $[0, 1)$
(d) $[0, 1]$

$\frac{4}{1+4}$
$\frac{1}{1+1}$

(3) If $f(x)$ is a differentiable function and $g(x) = \frac{1}{f(\sqrt{x})}$ then $g'(4) =$

(a) $-\frac{f'(2)}{2f^2(2)}$
[✓ (b)] $-\frac{f'(2)}{4f^2(2)}$
(c) $-\frac{f'(2)}{f^2(2)}$
(d) $-\frac{f'(4)}{4f^2(2)}$

$$\frac{-f(\sqrt{x})\,\frac{1}{2\sqrt{x}}}{f^2(\sqrt{x})}$$

$$\frac{-f(\sqrt{x}) * \frac{1}{2\sqrt{x}}}{f^2(\sqrt{x})}$$

$$\frac{-f(2)}{4f^2(2)}$$

$$\frac{-f'(2)\,1}{4f^2(2)}$$

(4) Let $\lfloor x \rfloor$ be the greatest integer of x. Then $\lim_{x\to 2^+} \frac{|x-1|}{\lfloor x \rfloor} =$

(a) 1
[✓ (b)] $\frac{1}{2}$
(c) 0
(d) Does not exist.

$x = 1$
$- - - \,|\, + +$
1
$\frac{x-1}{}$
$\frac{1}{2}$
$1 \quad 2$
$1 \quad 2 \quad 3 \quad 4$

$\frac{-2y\frac{dy}{dt}}{2\sqrt{1-y^2}}$ $\frac{dy}{dx}$

(5) If $\frac{dy}{dx}=\sqrt{1-y^2}$ then $\frac{d^2y}{dx^2}=$ $\frac{-2y\frac{dy}{dt}}{2\sqrt{1-y^2}}$

(a) y

[✓ (b)] $-y$

[✓ (c)] $-2y$

(d) $\frac{-y}{\sqrt{1-y^2}}$

$x^2\frac{dy}{dt}+y\,2x+y^2+2xy\frac{dy}{dt}=0$

$1- = \frac{-3}{3} = \frac{-2-1}{3}$ $\frac{-121-1}{1+211}$ $\frac{-y2x-y^2}{x^2+2xy}$

(6) The equation of the normal line to the curve $x^2y+y^2x=2$ at $(1,1)$ is

[✓ (a)] $y=x$

(b) $y=2-x$

(c) $y=3x-2$

(d) $y=2x-1$

$x^2\frac{dy}{dx}+y2x+y^2+x2y\frac{dy}{dt}=0$

$\frac{-y2x+y^2}{x^2+2xy}=$

$\frac{-1\times2\times1-(1)^2}{1+2}$ $\frac{-2-1}{3}$ $\frac{-3}{3}=-1$

1

$(y-1)=1(x-1)$ 1

$y-1=x-1$
$+1\quad +1$

$y-1=1(x-$

$y=x$ $y=1-x$

(7) One of the following statements is false

[✓ (a)] if f and g are odd then $f\circ g$ is odd.

[✓ (b)] if f is odd and g is even then $f\circ g$ is odd.

(c) if f is even and g is neither even nor odd then $g\circ f$ is even.

(d) if f and g are odd then fg is even.

(8) The point $(\frac{\pi}{4},\frac{\pi}{4})$ lies on the curve $\tan(x)+\sec(y)=1+\sqrt{2}$. At this point $y'=$.

(a) 2

(b) -2

(c) $\sqrt{2}$

[✓ (d)] $-\sqrt{2}$

$\sec^2\frac{\pi}{4}+\sec 0$ $2+\sqrt2$ $\frac{dy}{dx}$

$f(-x)=-f(x)$

$g(-x)=g(x)$

$f(g(x))$ $2+$ $\frac{-2\sqrt2}{\sqrt2\times\sqrt2}$

$f(g(-x))$ $f(-g(x))$

$-f(g(x))$ $-f(g(x))$

(9) $\lim_{x\to0}x\cot(3x)=$

(a) 0

(b) 3

[✓ (c)] $\frac{1}{3}$

(d) Does not exist.

$\frac{1}{1}$

$\sec^2x+\sec(y)\tan(y)\frac{dy}{dt}=0$

$\sec^2\frac{\pi}{4}+\sec(\frac{\pi}{4})\tan\frac{\pi}{4}\frac{dy}{dx}$

$\frac{1}{3}$ $\frac{\cancel{x}}{\frac{\tan 3x}{x}}$ $x\cdot\frac{1}{\tan(3x)}$

2

$\frac{1}{3}$ f $2+\sqrt2\times1\frac{dy}{dt}=\frac{-2}{\sqrt2}$

$f(g(-x))$ $f(g(x))$

$=-f(x)$ $=-\sqrt2$ $\frac{-2\sqrt2}{2}$ $\frac{-2}{\sqrt2\times\sqrt2}\times\sqrt2$

$v = 3t^2 - 24t + 45$ 25×3 − 24×5 + 45

$a = 6t - 24$ 12 − 48 + 45

18 − 12 + −45 +45

٧٥ ٣٠ ١٥٠ ٧٥ ٣٠

(10) Let $s(t) = t^3 - 12t^2 + 45t + 2$ be the position of an object moving in a straight line then

[✓ (a)] the object is at rest when $t = 5$ only. ×

(b) when $3 < t < 5$, the object is moving forward.

(c) the acceleration is zero when $t = 3$. ×

(d) when $t < 3$, the object is moving forward.

٤٨ − ٩٦ + ٤٥

٤٨ + ٤٥

(11) To shift the graph of f two units up and to compress it horizontally by a factor of 2 and then shift it one unit to the left, we use the function

(a) $f(2x-1)+2$

[✓ (b)] $f(2x+1)+2$

(c) $f(2x+2)+2$

(d) $f(2x-2)+2$

$f(2x+1) + 2$

٤٥

(12) $\lim_{x\to 1} \frac{\sqrt{x+3}-2}{x-1} =$

$\frac{x+3-4}{(x-1)(\sqrt{x+3}+2)}$ $\frac{①}{\sqrt{x+3}+2}$ $\frac{1}{1+4}$

$x-1$ $\frac{x-4}{}$ $\frac{1}{}$

(a) 2

(b) 4

(c) $\frac{1}{2}$

[✓ (d)] $\frac{1}{4}$

(13) Let $f(x) = \begin{cases} \lfloor x \rfloor & , \quad 0 \le x < 1 \\ ax+b & , \quad 1 \le x \le 3 \\ 3\ \lfloor x \rfloor & , \quad 3 < x < 4 \end{cases}$

The values of a and b that make the function continuous on the interval $[0, 4)$ are

(a) $a = 0, b = 1$

[✓ (b)] $a = \frac{3}{2}, b = -\frac{3}{2}$

(c) $a = -\frac{3}{2}, b = \frac{3}{2}$

(d) There are no values.

0 1 2 3

0 1 2 3 4

$3a + b = 3$

$a + b = 0$

$\frac{2a}{2} = \frac{3}{2}$

$a = \frac{3}{2}$

$a + b = 0 \to ①$

$3a + b = 3$

$\frac{2a}{2} = \frac{3}{}$

$a = \frac{3}{2}$

$\frac{(x-1)\ (x+1)}{x(x-1)}$ $\lim_{x\to 0} \frac{x+1}{x}$ 0^+

(14) Consider the function $f(x) = \frac{x^2-1}{x^2-x}$. One of the following statement is false

(a) f has a horizontal asymptote.

(b) f has a vertical asymptote.

[✓ (c) the range of f is $(-\infty, \infty)$.]

(d) f has a removable discontinuity.

~~$(-\infty, 0) \cup (0,1) \cup (1,\infty)$~~

~~$0 < x < \infty$~~

(15) The largest δ that satisfies $|\sqrt{x+1} - 2| < 1$ whenever $0 < |x-3| < \delta$ is

(a) 1

~~(b) 3~~

[✓ (c) 5]

(d) 8

$-1 < \sqrt{x+1} - 2 < 1$

$1 < \sqrt{x+1} < 3$

$1 < x+1 < 9$

$0-3 < x < 3-3$

$-3 < x-3 < 0$

$0 < x-3 < 8-3$

5

(16) Let a, b and c be the sides of a triangle with $a = b = 1$ and the angle between a and b is $\frac{2\pi}{3}$. Then $c =$

(a) $\sqrt{2}$

(b) 2

[✓ (c) $\sqrt{3}$]

(d) 3

$C^2 = a^2 + b^2 - Ab \cos\left(\frac{2\pi}{3}\right)$

$2 + \frac{1}{2}$

$2 + \frac{1}{2}$

$= 3$

$\frac{5}{2}$

$\frac{4}{2} + \frac{1}{2}$ $2 - 1 \times 1 \times \frac{1}{2}$

$C^2 = \frac{5}{2}$ $\frac{4}{2} + \frac{1}{2}$

$\frac{180}{3} \times 2 \times 60$ 60 120 $\frac{2 \times 180}{3}$ 120

9

Question 2 (20%) Answer by true or false

1. The function $f(x) = 1 + \sin x - x$ has a root in the interval $[0, \pi]$. (....T....)
2. A rational function can have an oblique and a horizontal asymptote. (....~~T~~....)
3. The function $f(x) = x^3 - x^2 - 1$ has a horizontal tangent at $x = 2/3$. (....T....)
4. The period of the function $\tan(2x)$ is π. (....F....)
5. If $\lim_{x \to c} g(x) = a$ then c belongs to the domain of g. (....~~T~~....)
6. The domain of the function $\tan(\pi \sin x)$ is $(-\infty, \infty)$. (....F....)
7. If f is differentiable at $x = c$ then f is continuous at $x = c$. (....T....)
8. The range of the function $\sec(x) + 1$ is $[2, \infty)$. (....F....)
9. The function $\frac{\tan x}{x}$ has a removable discontinuity at $x = 0$. (....T....)
10. If $y = \sec^2(\theta)$ then $y'(\frac{\pi}{4}) = 2\sqrt{2}$. (....F....)

16

$(1, 0)$ $(1, 0)$ $\frac{0}{1}$ $\frac{0}{0}$

$2 \sec\theta * \sec\theta \tan\theta$

$2 * 2 * \sqrt{2}$

4

$2\cos\theta \sec\theta \tan$

$2 \times 2 \times 1$

$1 + 0 - 0 = 1$

$1 + 0 - \pi$

$1 - \pi$ 3.14

$\sin(\pi$

$(-\infty, -1)$ $[2, \infty)$

$(-\infty, 0)$

$(-\infty, 2\pi) \cup (\infty,$

$y' = 3x^2 - 2x$

$3 \times \frac{4}{9} - 2 \times \frac{2}{3}$

$\frac{4}{3} - \frac{4}{3}$

$\frac{1}{\sin}$ 2π $\frac{1}{\sin}$ 2π

$\lim_{x \to 0} \frac{\tan}{0}$

$\frac{3}{4-}$

$\frac{(2-1)(2+1)}{2(\quad +}$

Question 3(18%) Consider the function $f(x) = \frac{x^2-1}{x^2-2x}$ $\frac{(x-1)(x+1)}{x(x-2)}$

$x(x-2)$

1. The domain of f is $\boxed{(-\infty, 0) \cup (0, 2) \cup (2, \infty)}$

2. $\lim_{x\to 2^+} f(x) =$ $\frac{(2-1)(2+1)}{2(2^+-2)} = \frac{3}{20^+} = \boxed{\infty}$

3. $\lim_{x\to 2^-} f(x) =$ $\frac{(2-1)(2+1)}{2(2-2)} = \frac{3}{20^-} = \boxed{-\infty}$

$\frac{-1}{0^+-0^+}$ $\frac{-1}{0^+}$

4. $\lim_{x\to 0^+} f(x) =$ $\frac{(0-1)(0+1)}{0(0-2)} = \frac{-1}{0^+} = \boxed{-\infty}$

-1

$\frac{1-1}{1-2}$ $\frac{0}{-1}$

5. $\lim_{x\to 0^-} f(x) =$ $\frac{(0-1)(0+1)}{0(0-2)} = \frac{-1}{0^-} = \boxed{\infty}$

6. $\lim_{x\to \infty} f(x) =$ $\frac{\frac{x^2}{x^2} - \frac{1}{x^2}}{\frac{x^2}{x^2} - \frac{2}{x}} = \frac{1-0}{1-0} = \boxed{1}$

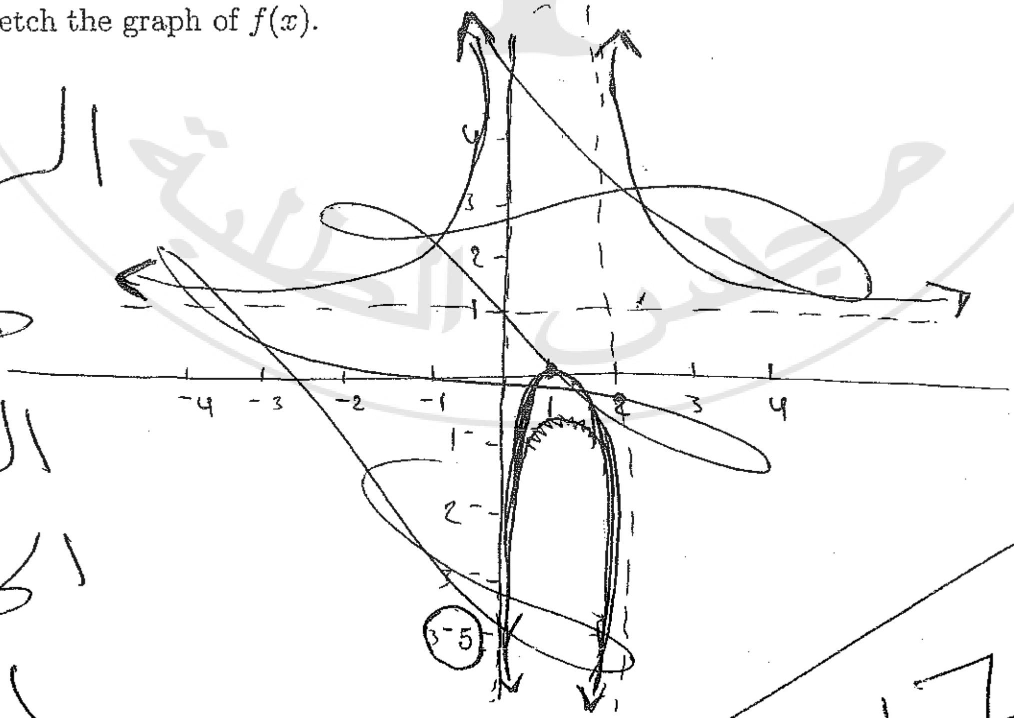
7. $\lim_{x\to -\infty} f(x) =$ $\frac{\frac{x^2}{x^2} - \frac{1}{x^2}}{\frac{x^2}{x^2} - \frac{2}{x}}$ $\frac{1}{1} = \boxed{1}$

8. Vertical asymptote(s) is/are $\boxed{x=2}$ $\boxed{x=0}$

9. Horizontal asymptote(s) is/are $\boxed{y=1}$

10. $x-$intercepts: $y = \frac{x^2-1}{x^2-2x} = 0 = x^2-1$ $x^2=1$ $x=\pm1$ $\boxed{x=1}$ $\boxed{x=-1}$

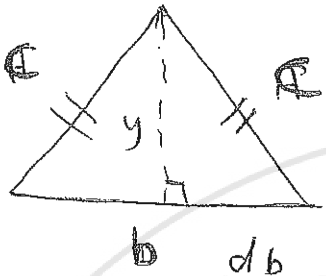
11. Sketch the graph of $f(x)$.



الرسمة على الورقة الخامسة

17

Question 4(14%) The two equal sides of an isosceles triangle (مثلث متساوي الساقين) with fixed base b are decreasing at the rate of 3 cm/min. How fast is the area decreasing when the two equal sides are equal to the base.



$\frac{db}{dt} = -3$ مطلوب $\frac{dA}{dt}$ when $C = b$

$$A = \frac{1}{2} b y \Longrightarrow A = \frac{1}{2} b \sqrt{C^2 - \frac{b^2}{4}}$$

$$y^2 + \left(\frac{b}{2}\right)^2 = C^2$$

$$y^2 + \frac{b^2}{4} = C^2 \Rightarrow y = \sqrt{C^2 - \frac{b^2}{4}}$$

$$\frac{dA}{dt} = \frac{1}{2} b * \frac{2b\frac{db}{dt}}{2\sqrt{C^2 - \frac{b^2}{4}}} + \sqrt{C^2 - \frac{b^2}{4}} * \frac{1}{2}\frac{db}{dt}$$

$$\frac{\frac{1}{\cancel{2}} * b * \cancel{2} * b * -3}{2\sqrt{\frac{4b^2 - b^2}{4}}} + \sqrt{\frac{4b^2 - b^2}{4}} * \frac{1}{2} * -3$$

$$\frac{-3\cancel{b^2}\,b}{\cancel{2}\frac{\sqrt{3}}{\cancel{2}}\cancel{b}\,1} + \frac{-3}{2}\frac{\sqrt{3^6}}{2} b \Longrightarrow \frac{-3b}{3\sqrt{3}} - \frac{3\sqrt{3}}{4} b$$

6

$$\frac{dA}{dt} = -\left(\frac{3}{\sqrt{3}} + \frac{3\sqrt{3}}{4}\right) b$$

MATHEMATICS DEPARTMENT
MATH141 -Midterm Exam-
Winter 2015/2016

- Name.[redacted]
- Number.[redacted]

- Circle your discussion's section number from the two tables below:

#	Discussion teacher	Time (T, R)	#	Discussion teacher	Time (T, R)
1	Leen Hethnawi	10:00 - 10:50	13	We'Am Abu Arqoub	13:00 - 13:50
2	Hiba Sharha	13:00 - 13:50	14	Leen Hethnawi	13:00 - 13:50
3	Hasan Yousef	14:00 - 14:50	15	Areej Awawdah	12:00 - 12:50
4	Aseil Altete	11:00 - 11:50	16	Areej Awawdah	08:00 - 08:50
5	Areej Awawdah	09:00 - 09:50	17	Maher Abdallatif	11:00 - 11:50
6	Leen Hethnawi	08:00 - 08:50	18	Aseil Altete	12:00 - 12:50
7	Areej Awawdah	10:00 - 10:50	19	Hiba Sharha	09:00 - 09:50
8	Leen Hethnawi	11:00 - 11:50	20	Mahmoud Ghannam	14:00 - 14:50
9	Hiba Sharha	12:00 - 12:50	[✓ 21]	Hiba Sharha	14:00 - 14:50
10	We'Am Abu Arqoub	09:00 - 09:50	22	Saddam Zaid	13:00 - 13:50
11	Hiba Sharha	10:00 - 10:50	23	Saddam Zaid	11:00 - 11:50
12	We'Am Abu Arqoub	11:00 - 11:50	24	Saddam Zaid	08:00 - 08:50

- Instructions

1. Write your name and number.
2. Choose your section from the above table.
3. There are three questions in the next 6 pages.
4. Answer all questions.
5. Turn off your mobile phones.
6. Calculators are not allowed.

72

Q1) [45 pts] Circle the correct answer.

(1) If $x^{2/3} + y^{2/3} = 8$, then $\dfrac{dy}{dx} =$

(a) $\dfrac{y^{1/3}}{x^{1/3}}$

[✓ (b)] $-\dfrac{y^{1/3}}{x^{1/3}}$ ← the answer

(c) $\dfrac{x^{-1/3}}{y^{-1/3}}$

(d) $-\dfrac{y^{-1/3}}{x^{-1/3}}$

$\frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}y^{-\frac{1}{3}}y' = 0$

$\frac{2}{3x^{\frac13}} + \frac{2}{3y^{\frac13}}y' = 0$

$\frac{2}{3x^{\frac13}} = -\frac{2}{3y^{\frac13}}y'$

$\left(-\frac{y^{\frac13}}{x^{\frac13}}\right)$

$-\frac{2}{3}x^{-\frac13} = \frac{2}{3}y^{-\frac13}y'$

$-\frac{y^{\frac13}}{x^{\frac13}} = -\frac{x^{-\frac13}}{y^{-\frac13}}$

$\frac{2}{3}x^{-\frac13} + \frac{2}{3}y^{-\frac13}y' = 0$

$-x^{-\frac13} = -y^{-\frac13}y'$

(2) If the population in Palestine increases exponentially since 2015 with a growth rate of 0.1 ln 4, then the population will double

(a) After 10 years.

(b) After 15 years.

[✓ (c)] In the year 2020

(d) In the year 2035

$P = P_0 e^{kt}$

$2P_0 = P_0 e^{\frac{\ln 4}{10}t}$

$\ln 2 = \frac{\ln 4}{10}t$

$\cancel{\ln 2} = \frac{2\cancel{\ln 2}}{10}t$

$\frac{10}{2} = 5 = t$

$\frac{1}{10}\ln(4)$

$\frac{\ln 4}{10}$

(3) If $g(x) = \dfrac{x^2+2x}{x^3-x^2-6x}$, then

$x(x^2-x-6)$

$\frac{x^2+2x}{x^3-x^2-6x} = \frac{\cancel{x}(x+2)}{\cancel{x}(x^2-x-6)} = \frac{\cancel{(x+2)}}{\cancel{(x+2)}(x-3)} = \frac{1}{(x-3)}$

(a) $y = 0$ is a horizontal asymptote and $x = 0$ is vertical asymptote for $g(x)$.

[✓ (b)] Both $x = -2$ and $x = 3$ are vertical asymptotes for $g(x)$.

[✓ (c)] The $x-$axis is a horizontal asymptote and $x = 3$ is a vertical asymptote for $g(x)$.

(d) The three lines $x = -2$, $x = 0$, and $x = 3$ are vertical asymptotes for $g(x)$.

(4) $\lim\limits_{x\to\infty} e^{-x}\ln(x) = \lim\limits_{x\to\infty}\frac{\ln(x)}{e^x}$

$\frac{\frac{1}{x}}{e^x}$

$\frac{1}{xe^x} =$

(a) 1

[✓ (b)] 0

(c) ∞

(d) Does not exist.

$\ln(\sin x) = -\ln 2$

$\ln\left(\frac{1}{2}\right) = -\ln 2$, $\sin x = \frac{1}{2}$

$\frac{\frac{\sqrt3}{2}}{\frac12}$

sin cos x

(5) If $f(x) = \ln(\sin x)$, $0 < x < \frac{\pi}{2}$, then $(f^{-1})'(-\ln 2) =$

(a) $\sqrt{3}$

[✓ (b)] $\frac{1}{\sqrt{3}}$

(c) 2

(d) $\frac{1}{2}$

$\frac{1}{f'(x)}\Big| = \frac{1}{\frac{\cos x}{\sin x}}\Big|$

$x = \frac{\pi}{6}$

$\frac{1}{\sqrt{}} = \frac{1}{\frac{\frac{\sqrt3}{2}}{\frac12}}$

$\sqrt{\frac{3}{2}}$

(6) The domain of the function $f(x) = \frac{\sqrt{x-2}}{\sqrt{16-x^2}}$ is

$16 - x^2 > 0$
$16 > x^2$
$4 > x$

- (a) $(-4, 4)$
- (b) $(2, \infty)$
- (c) $[0, 4)$
- [✓ (d) $[2, 4)$]

$x - 2 \geq 0$
$x \geq 2.$

(7) The function $y = xe^{-x}$ $\frac{x}{e^x}$

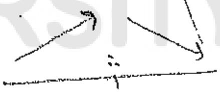
- (a) has absolute maximum at $x = 1$ and inflection point at $x = 2$
- (b) is concave up on $[2, \infty)$ and concave down on $(-\infty, 2]$
- (c) is increasing on $(-\infty, 1]$ and decreasing on $[1, \infty)$
- [✓ (d) All the above statements are true.]

$-xe^{-x} + e^{-x}$
$-\frac{x}{e^x} + \frac{1}{e^x}$
$x = 1 \quad \frac{1-x}{e^x}$
$\frac{e^x(-1) - (1-x)e^x}{(e^x)^2}$
$\frac{-e^x - e^x + xe^x}{(e^x)^2}$
$\frac{-2e^x + xe^x}{(e^x)^2}$
$-2e^x + xe^x = 0$
$1 \quad 2 \quad + + +$
$-2e^2 + 2e^2 = 0$
$-2e^2 + 3e^3$

(8) The function $f(x) = x \ln x - x$, $x > 0$, is increasing on

$x \cdot \frac{1}{x} + \ln x - 1$
$\ln x$

- (a) $(0, 1)$ x
- (b) $(0, \infty)$
- [✓ (c) $(1, \infty)$]
- (d) It is decreasing for all $x > 0$.



(9) The solution of the equation $e^{x-1} = 2^x$ is

- (a) $1 - \ln 2$
- [✓ (b) $\frac{1}{1-\ln 2}$]
- (c) $\frac{1}{1+\ln 2}$
- (d) $1 + \ln 2$

$\ln e^{x-1} = \ln 2^x$
$x - 2 = x \ln 2 + 2$
$x ~~= x \ln 2 + 2~~$
$x - x \ln 2 = 2$
$x(1 - \ln 2) = \frac{2}{1 - \ln 2}.$

$\ln e^{x-1} = \ln 2^x$
$x - 1 = x \ln 2$
$x - x \ln 2 = 1$
$x(1 - \ln 2) = 1$

(10) The derivative of the function $y = x^{2x}$ is

- (a) $2(1 + \ln x)$
- (b) $x^{2x}(1 + \ln x)$
- [✓ (c) $2x^{2x}(1 + \ln x)$]
- (d) $1 + \ln x$

$\ln y = \ln x^{2x}$
$\frac{y'}{y} = 2x \ln x$
$\frac{y'}{y} = ~~2x \ln~~$
$2x \cdot \frac{1}{x} + 2 \ln x$
$(2 + 2 \ln x)\, x^{2x}$
$2x^{2x}(1 + \ln x)$

(11) $\int_0^1 \frac{-\ln 2dx}{1+2^x} =$

[✓ (a)] $\ln\left(\frac{3}{4}\right)$.

[✓ (b)] $\ln 2 \ln\left(\frac{3}{4}\right)$

(c) $\ln\left(\frac{3}{2}\right)$

(d) $\ln 2 \ln 3$

$-\frac{\ln 2}{1+2}$ $2.$

$2x\frac{-\ln 2}{2\times 3} + \frac{\ln 2\times 3}{2\times 3}$

$\frac{-2\ln 2 + 3\ln 2}{6}$ $\frac{\ln 2}{6}$

$\int_0 \frac{}{1+2^x}$

$u = 2^x$

$\ln u = \ln 2^x$

$\ln u = x\ln 2$

$\frac{du}{u} = \ln 2\, dx$

$du = ~~2^{1+2^x}~~$

$-\ln 2 \int_0^1 \frac{\cdot}{1+2^x}\, dx$

$1+2^x = u$, $du = 2^x \ln 2\, dx$.

$1-u$

$x\ln 2$

$\int_0^1 \frac{~~\ln 2\cdot 2^x~~}{2^x\cdot u} du$

$\int_0^1 \frac{du}{(u-1)u}$

$u^2 - u$

(12) The area between the curves $y = \ln x$ and $y = \ln(4x)$, $1 \le x \le 2$ is

[✓ (a)] $2\ln 2$

[✓ (b)] $\ln 2$

(c) $1 + \ln 2$

(d) $1 + \ln 4$

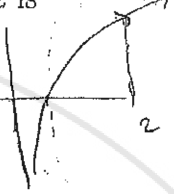
$\int_1^2 |\ln x - \ln 4x|$

$\frac{\ln x}{\ln 4x}$ $\frac{\ln 4x}{\ln x}$

$\ln 1 = 0$

$4(\frac{x}{})|^2$

$\frac{1}{8} = x$



$\int \frac{du}{u^2-u}$

$\ln 4x = 0$

(13) The derivative of the function $y = \log_2(\ln x)$ is $\frac{\ln(\ln x)}{\ln 2}$

~~log~~

$\frac{1}{\ln 2} \times \ln(\ln x)$

$\frac{1}{\ln 2} \times \frac{\frac{1}{x}}{\ln x}$

$\frac{1}{x\ln x \ln 2}$

(a) $\frac{1}{x\ln x}$

(b) $\frac{1}{\ln x}$

(c) $\frac{1}{\ln 2 \ln x}$

[✓ (d)] $\frac{1}{(\ln 2)x\ln x}$

$(113)\ 3 = 3^1$

(14) The linearization of the function 3^{x^2} at $x = 1$ is $y = 3^{x^2}$

$y' = 3x^2 \cdot 2x \cdot \ln 3$

$3 \cdot 2 \cdot \ln 3$

$(6\ln 3)$

$y - 3 = 6\ln 3(x-1)$

$y - 3 = 6\ln 3 x - 6\ln 3$.

$3 + 6\ln 3(x - 1)$

$3 + 6\ln 3 - x6\ln 3$

(a) $y = 3 + \ln 3(x-1)$

(b) $y = 3 + 6(x-1)$

[✓ (c)] $y = 3 + 6\ln 3(x-1)$

(d) $y = 9 + \ln 3(x-1)$

(15) $\lim_{x\to 0} \frac{\tan x - x}{x^3} =$

$\frac{\sec^2 - 1}{3x^2}$ $\frac{\tan^2}{3x^2}$

(a) 0

[✓ (b)] $\frac{1}{3}$

(c) $-\frac{1}{3}$

(d) ∞.

$2-y$

$\sqrt{2-x}=y^2 \quad 2 = x+y^2$

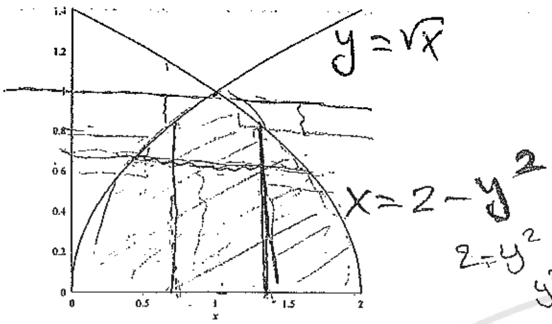
Q2) [16 pts] Find the volume of the solid generated by revolving the region enclosed between the curve $y=\sqrt{x}$, $x=2-y^2$ and the x−axis about

$\sqrt{x} = 2$

$x = 2-y^2$

$x+2=-y^2$

$x-2=y^2$



$2-y^2 = x+y^2$

$y^2 = x+2$

$y=\sqrt{2-x}$

Figure 1: Graph of $y=\sqrt{x}$ and $x=2-y^2$

(a) The x−axis **(Do NOT evaluate the integrals)**

(i) Use the shell method.

$$\int_0^2 (\sqrt{y^2}-y)(\sqrt{y^2}) \, dy$$

$$\int_0^2 y^4 - y^3 \, dy$$

①

(ii) Use the washers method.

$$\int_0^2 ((\sqrt{x})^2$$

$$\int_0^2 x \, dx$$

(1.5) dx

(b) The y−axis **(Do NOT evaluate the integrals)**

(i) Use the shell method.

$$\int_0^1 (\sqrt{2-x}-\sqrt{x})(\sqrt{2-x}-x) \, dx$$

$2x$

①

(ii) Use the washers method.

$$\int_0^1 (2-y^2-y)^2-(\sqrt{y})^2 \, dy$$

②

5.5

$x > 0$

$\frac{\sqrt{x}}{\sqrt{x}(1-\frac{3}{\sqrt{x}})} \qquad \frac{\sqrt{x}}{\sqrt{x}-3} \qquad \frac{1}{1-\frac{3}{\infty}}$

Q3) [24 pts] Consider the function $f(x) = \frac{\sqrt{x}}{\sqrt{x}-3}$

(a) The domain of f is $[0, 9) \cup (9, \infty)$

(b) $\lim_{x\to 9^+} f(x) = \infty$

(c) $\lim_{x\to 9^-} f(x) = -\infty$

(d) $\lim_{x\to +\infty} f(x) = 1$

(e) Horizontal asymptote is $y = 1$

(f) Vertical asymptote is $X = 9$

(g) Find $f^{-1}(x)$

$f(x) = \frac{\sqrt{x}}{\sqrt{x}-3}$

$y = \frac{\sqrt{x}}{\sqrt{x}-3}$

$y\sqrt{x} - 3y = \sqrt{x}$

$y\sqrt{x} - \sqrt{x} = 3y$

$\frac{\sqrt{x}(y-1)}{y-1} = \frac{3y}{y-1} \qquad \sqrt{x} = \frac{3y}{y-1}$

$x = \left(\frac{3y}{y-1}\right)^2 \qquad f^{-1} = \frac{9y^2}{(y-1)^2}$

$(\frac{1}{2}, 1.4)$

$\frac{4}{4-3} = 4$

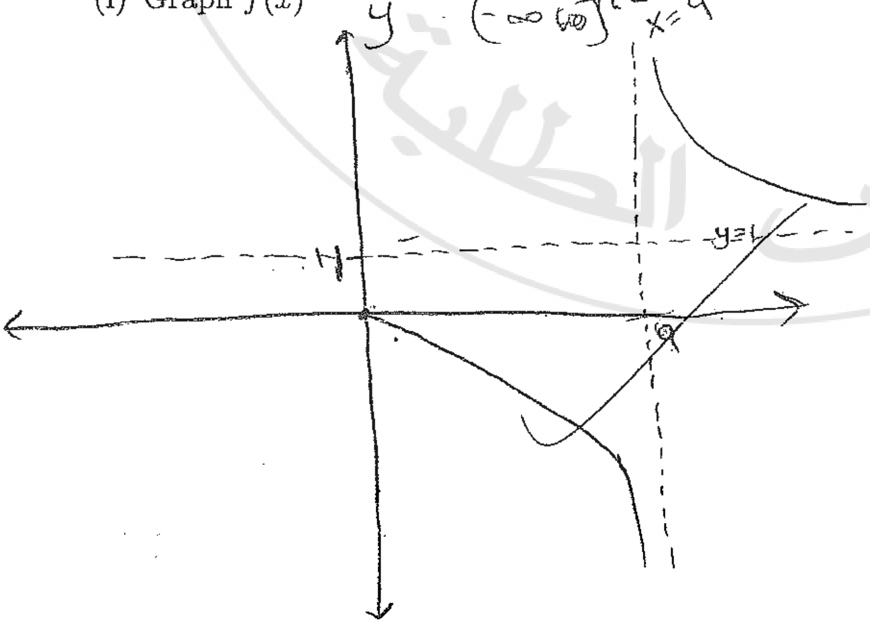
$16, 4$

$\frac{1}{1-3} = 1, -\frac{1}{2}$

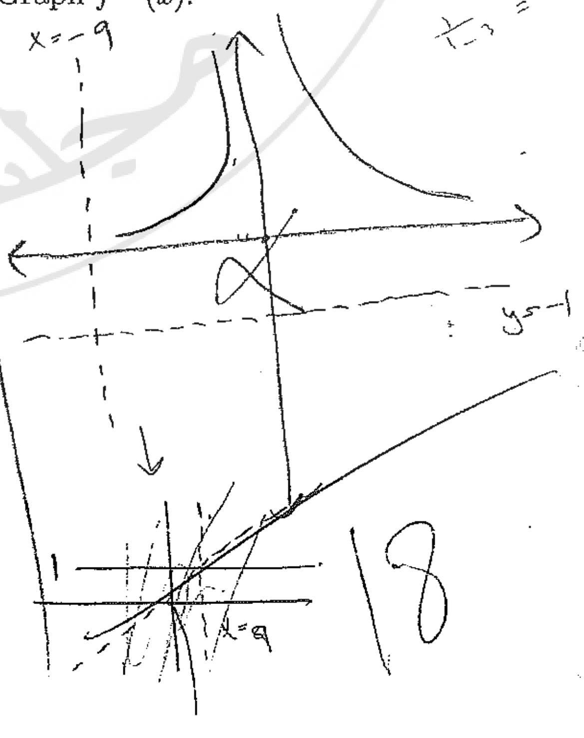
(h) Find the domain of f^{-1}

$=$ Range of $f(x) = [0, \infty)$

(i) Graph $f(x)$ $\qquad y \; (-\infty, 0] \cup [1, \infty)$.



Graph $f^{-1}(x)$.



18

[illegible]

Q3) [15 pts] Answer the following questions:

(a) Find the length of the curve $y = \ln(\cos x)$, $0 \le x \le \frac{\pi}{4}$.

$y' = \left(\frac{\cos x}{\sin x}\right)^2 \cdot \cot^2$

$L = \int_0^{\frac{\pi}{4}} \sqrt{1 + (f'(x))^2} \, dx$

$L = \int_0^{\frac{\pi}{4}} \sqrt{1 + \cot^2}$

$L = \int_0^{\frac{\pi}{4}} \sqrt{\csc^2}$

$L = \int_0^{\frac{\pi}{4}} \frac{\csc(\cancel{d}\csc + \cot)}{(\csc + \cot)} \, dx$

$L = \int_0^{\frac{\pi}{4}} \frac{\csc x^2 + \csc x \cot x}{\csc + \cot} \, \cancel{dx}$

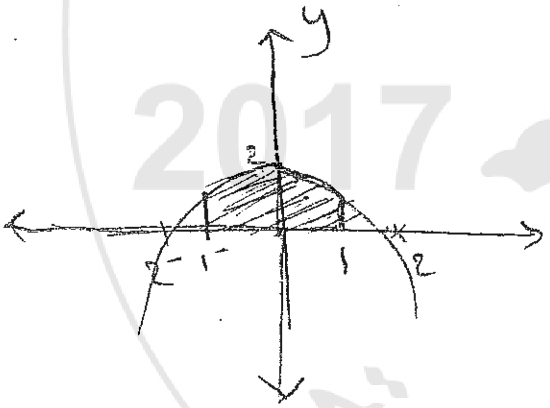
$L = \int_0^{\frac{\pi}{4}} = -\ln|\csc + \cot| \Big|_0^{\frac{\pi}{4}}$

$= -\ln(\sqrt{2} + 1) + \ln(\infty + 0)$

[illegible]

(b) Find the area of the surface generated by revolving the curve $y = \sqrt{4 - x^2}$, $-1 \le x \le 1$, about the x-axis.

$y = \sqrt{4 - x^2}$



~~symetric~~

$(4 - x^2)^2$

$16 - 8x^2 + x^4$

$\sqrt{4 - x^2}$

~~$Area = \int_{-1}^{1} \sqrt{4 - x^2} \, dx$~~

$\frac{1}{2}(4 - x^2)$

Area of surface =

$= \int_{-1}^{1} 2\pi y \sqrt{1 + f(x)^2} \, dx$

$=$ ~~$\int_{-1}^{1} 2\pi \sqrt{4 - x^2} \sqrt{1 + (4 - x^2)^2}$~~

$f(x) = \frac{1}{\cancel{2}}(4 - x^2)^{-\frac{1}{2}}(\cancel{2}x)$

$\left(\frac{x}{\sqrt{4 - x^2}}\right)^2 = \frac{x^2}{4 - x^2}$

$\boxed{8}$

$Area = \int_{-1}^{1} 2\pi \sqrt{4 - x^2} \sqrt{1 + \frac{x^2}{4 - x^2}} \, dx$

$\int_{-1}^{1} 2\pi \sqrt{(4 - x^2) + x^2}$

$\int_{-1}^{1} 2\pi \sqrt{4}$

$\int_{-1}^{1} 4\pi = 4\pi x \Big|_{-1}^{1} \text{unit}^2$

$= 4\pi + 4\pi = 8\pi$

12

MATHEMATICS DEPARTMENT
MATH141 -Midterm Exam-
First Semester 2015/2016

- Name. •Number.

- **Circle your discussion's section number from the two tables below:**

#	Discussion teacher	Time (T, R)
1	Leen Hethnawi	10:00 - 10:50
2	Hiba Sharha	13:00 - 13:50
3	Hasan Yousef	14:00 - 14:50
4	Aseil Altete	11:00 - 11:50
5	Areej Awawdah	09:00 - 09:50
6	Leen Hethnawi	08:00 - 08:50
7	Areej Awawdah	10:00 - 10:50
8	Leen Hethnawi	11:00 - 11:50
9	Hiba Sharha	12:00 - 12:50
10	We'Am Abu Arqoub	09:00 - 09:50
11	Hiba Sharha	10:00 - 10:50
12	We'Am Abu Arqoub	11:00 - 11:50

#	Discussion teacher	Time (T, R)
13	We'Am Abu Arqoub	13:00 - 13:50
14	Leen Hethnawi	13:00 - 13:50
15	Areej Awawdah	12:00 - 12:50
16	Areej Awawdah	08:00 - 08:50
17	Maher Abdallatif	11:00 - 11:50
18	Aseil Altete	12:00 - 12:50
19	Hiba Sharha	09:00 - 09:50
20	Mahmoud Ghannam	14:00 - 14:50
21	Hiba Sharha	14:00 - 14:50
22	Saddam Zaid	13:00 - 13:50
23	Saddam Zaid	11:00 - 11:50
24	Saddam Zaid	08:00 - 08:50

- **Instructions**

1. Write your name and number.
2. Choose your section from the above table.
3. There are three questions in the next 6 pages.
4. Answer all questions.
5. Turn off your mobile phones.
6. Calculators are not allowed.

Q1) [45 pts] **Circle the correct answer.**

(1) If $f(x) = \ln(\sin x)$, $0 < x < \frac{\pi}{2}$, then $(f^{-1})'(-\ln 2) =$

(a) $\sqrt{3}$

[✓ (b)] $\frac{1}{\sqrt{3}}$

(c) 2

(d) $\frac{1}{2}$

(2) The function $f(x) = x\ln x - x$, $x > 0$, is increasing on

(a) $(0, 1)$

(b) $(0, \infty)$

[✓ (c)] $(1, \infty)$

(d) It is decreasing for all $x > 0$.

(3) The solution of the equation $e^{x-1} = 2^x$ is

(a) $1 - \ln 2$

[✓ (b)] $\frac{1}{1-\ln 2}$

(c) $\frac{1}{1+\ln 2}$

(d) $1 + \ln 2$

(4) The derivative of the function $y = x^{2x}$ is

(a) $2(1 + \ln x)$

(b) $x^{2x}(1 + \ln x)$

[✓ (c)] $2x^{2x}(1 + \ln x)$

(d) $1 + \ln x$

(5) $\int_0^1 \frac{-\ln 2 dx}{1+2^x} =$

[✓ (a)] $\ln\left(\frac{3}{4}\right)$.

(b) $\ln 2 \ln\left(\frac{3}{4}\right)$

(c) $\ln\left(\frac{3}{2}\right)$

(d) $\ln 2 \ln 3$

(6) The area between the curves $y = \ln x$ and $y = \ln(4x)$, $1 \leq x \leq 2$ is

[✓ (a)] $2\ln 2$
(b) $\ln 2$
(c) $1 + \ln 2$
(d) $1 + \ln 4$

(7) The derivative of the function $y = \log_2(\ln x)$ is

(a) $\frac{1}{x \ln x}$
(b) $\frac{1}{\ln x}$
(c) $\frac{1}{\ln 2 \ln x}$
[✓ (d)] $\frac{1}{(\ln 2) x \ln x}$

(8) The linearization of the function 3^{x^2} at $x = 1$ is

(a) $y = 3 + \ln 3(x - 1)$
(b) $y = 3 + 6(x - 1)$
[✓ (c)] $y = 3 + 6\ln 3(x - 1)$
(d) $y = 9 + \ln 3(x - 1)$

(9) $\lim\limits_{x \to 0} \frac{\tan x - x}{x^3} =$

(a) 0
[✓ (b)] $\frac{1}{3}$
(c) $-\frac{1}{3}$
(d) ∞.

(10) If $x^{2/3} + y^{2/3} = 8$, then $\dfrac{dy}{dx} =$

(a) $\dfrac{y^{1/3}}{x^{1/3}}$
[✓ (b)] $-\dfrac{y^{1/3}}{x^{1/3}}$
(c) $\dfrac{x^{-1/3}}{y^{-1/3}}$
(d) $-\dfrac{y^{-1/3}}{x^{-1/3}}$

(11) If the population in Palestine increases exponentially since 2015 with a growth rate of $0.1 \ln 4$, then the population will double

(a) After 10 years.
(b) After 15 years.
[✓ (c)] In the year 2020
(d) In the year 2035

(12) If $g(x) = \dfrac{x^2 + 2x}{x^3 - x^2 - 6x}$, then

(a) $y = 0$ is a horizontal asymptote and $x = 0$ is vertical asymptote for $g(x)$.
(b) Both $x = -2$ and $x = 3$ are vertical asymptotes for $g(x)$.
[✓ (c)] The x–axis is a horizontal asymptote and $x = 3$ is a vertical asymptote for $g(x)$.
(d) The three lines $x = -2$, $x = 0$, and $x = 3$ are vertical asymptotes for $g(x)$.

(13) $\lim\limits_{x \to \infty} e^{-x} \ln(x) =$

(a) 1
[✓ (b)] 0
(c) ∞
(d) Does not exist.

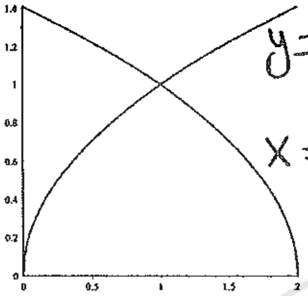
(14) The domain of the function $f(x) = \dfrac{\sqrt{x-2}}{\sqrt{16 - x^2}}$ is

(a) $(-4, 4)$
(b) $(2, \infty)$
(c) $[0, 4)$
[✓ (d)] $[2, 4)$

(15) The function $y = xe^{-x}$

(a) has absolute maximum at $x = 1$ and inflection point at $x = 2$
(b) is concave up on $[2, \infty)$ and concave down on $(-\infty, 2]$
(c) is increasing on $(-\infty, 1]$ and decreasing on $[1, \infty)$
[✓ (d)] All the above statements are true.

Q2) [16 pts] Find the volume of the solid generated by revolving the region enclosed between the curve $y = \sqrt{x}$, $x = 2 - y^2$ and the x−axis about



$$y = \sqrt{x} \Rightarrow x = y^2$$

$$x = 2 - y^2 \Rightarrow y = \sqrt{2 - x}$$

Figure 1: Graph of $y = \sqrt{x}$ and $x = 2 - y^2$

(a) The x−axis **(Do NOT evaluate the integrals)**

(i) Use the shell method.

$$V = 2\pi \int_0^1 (2 - 2y^2)\, y \, dy$$

(ii) Use the washers method.

$$V = \pi \int_0^1 x \, dx + \pi \int_1^2 (2 - x) \, dx$$

(b) The y−axis **(Do NOT evaluate the integrals)**

(i) Use the shell method.

$$V = 2\pi \int_0^1 x^{3/2} \, dx + 2\pi \int_1^2 x \sqrt{2 - x} \, dx$$

each integral

(ii) Use the washers method.

$$V = \pi \int_0^1 \left[(2 - y^2)^2 - y^4 \right] dy$$

Q3) [24 pts] Consider the function $f(x) = \frac{\sqrt{x}}{\sqrt{x}-3}$

(2pts)(a) The domain of f is $[0,\infty)\setminus\{9\}$

(2pts)(b) $\lim_{x\to 9^+} f(x) = +\infty$

(c) $\lim_{x\to 9^-} f(x) = -\infty$

(d) $\lim_{x\to +\infty} f(x) = 1$

(e) Horizontal asymptote is $y = 1$

(f) Vertical asymptote is $x = 9$

(g) Find $f^{-1}(x)$

$y(\sqrt{x}-3) = \sqrt{x}$

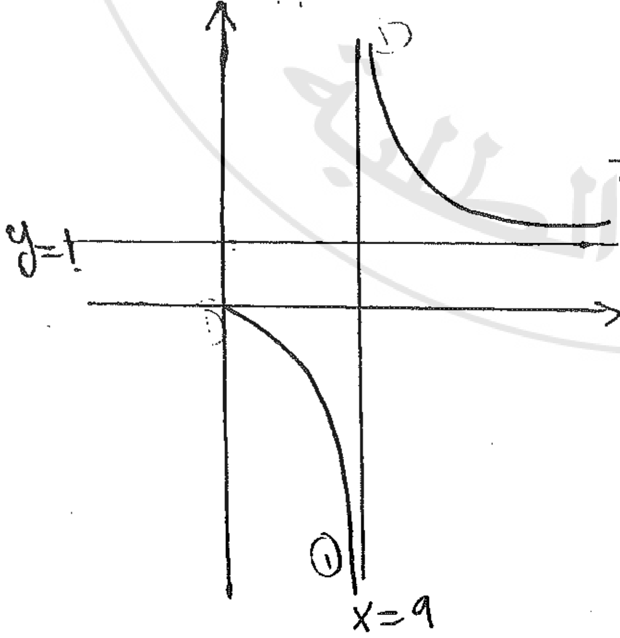
$\sqrt{x}(y-1) = 3y$

$\sqrt{x} = \frac{3y}{y-1} \geq 0 \Rightarrow y \in (-\infty,0] \cup (1,\infty)$

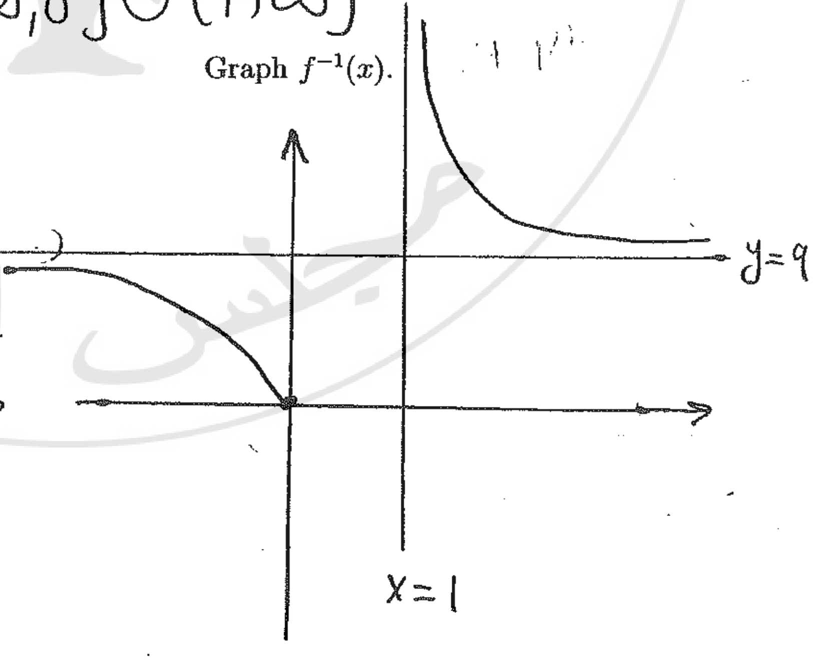
$f^{-1}(x) = \left(\frac{3x}{x-1}\right)^2$

(h) Find the domain of f^{-1} is $(-\infty,0] \cup (1,\infty)$

(i) Graph $f(x)$



Graph $f^{-1}(x)$.



Q3) [15 pts] Answer the following questions:

17 (a) Find the length of the curve $y = \ln(\cos x)$, $0 \le x \le \frac{\pi}{4}$.

$$\frac{dy}{dx} = -\frac{\sin x}{\cos x} = -\tan x$$

$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \tan^2 x = \sec^2 x$$

$$L = \int_0^{\pi/4} \sqrt{\sec^2 x}\, dx$$

$$= \int_0^{\pi/4} \sec x\, dx = \ln|\sec x + \tan x|\Big|_0^{\pi/4}$$

$$= \ln(\sqrt{2} + 1)$$

(b) Find the area of the surface generated by revolving the curve $y = \sqrt{4 - x^2}$, $-1 \le x \le 1$, about the x-axis.

$$\frac{dy}{dx} = \frac{-x}{\sqrt{4-x^2}}$$

$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{x^2}{4-x^2} = \frac{4}{4-x^2}$$

$$S = 2\pi \int_{-1}^{1} \sqrt{4-x^2}\sqrt{\frac{4}{4-x^2}}\; dx$$

$$= 2\pi \int_{-1}^{1} 2\, dx$$

$$= 4\pi x \Big|_{-1}^{1}$$

$$= 8\pi .$$