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Birzeit University- Mathematics Department
Calculus I-Math 141

Midterm Exam — First Semester 2014/2015

Name(Arabic):........[illegible]........ Number:........[illegible]........

Instructor of Discussion(Arabic):........[illegible]........ Section:........17........

Time: 90 Minutes — **There are 4 questions in 6 pages.**

Question 1.(48%) Circle the correct answer:

(1) The domain of the function $f(x) = \sqrt{3-x} + \frac{1}{\sqrt{x^3+8}}$ is

(a) $[-2, 3)$

(b) $(-\infty, -2) \cup (-2, 3]$

(c) $(-2, 3]$

(d) $(-\infty, -2) \cup [3, \infty)$

$x \leq -3$

$x \geq -2$

$-2 \quad 3$

$3 - x \geq 0$

$-x \geq -3$

$x \leq 3$

$x^3 + 8 > 0$

$x > -2$

(2) The range of the function $g(x) = \frac{\sqrt{x}}{1+\sqrt{x}}$ is

(a) $(1, \infty)$

(b) $[0, \infty)$

(c) $[0, 1)$

(d) $[0, 1]$

$\frac{4}{1+4}$

$\frac{1}{1+1} = \frac{1}{2}$

(3) If $f(x)$ is a differentiable function and $g(x) = \frac{1}{f(\sqrt{x})}$ then $g'(4) =$

(a) $-\frac{f'(2)}{2f^2(2)}$

(b) $-\frac{f'(2)}{4f^2(2)}$

(c) $-\frac{f'(2)}{f^2(2)}$

(d) $-\frac{f'(4)}{4f^2(2)}$

$\frac{-f'(\sqrt{x})\frac{1}{2\sqrt{x}}}{f^2(\sqrt{x})}$

$\frac{-f'(\sqrt{x}) * \frac{1}{2\sqrt{x}}}{f^2(\sqrt{x})}$

$\frac{-f'(2)}{4f^2(2)}$

$\frac{-f'(2)\,1}{4f^2(2)}$

(4) Let $\lfloor x \rfloor$ be the greatest integer of x. Then $\lim_{x\to 2^+} \frac{|x-1|}{\lfloor x \rfloor} =$

(a) 1

(b) $\frac{1}{2}$

(c) 0

(d) Does not exist.

$x = 1$

$- - - \,|\, + + +$

$\frac{x-1}{}$

$\frac{1}{2}$

$1 \quad 2 \quad 3 \quad 4$

(5) If $\frac{dy}{dx} = \sqrt{1-y^2}$ then $\frac{d^2y}{dx^2} =$

(a) y

(b) $-y$

(c) $-2y$

(d) $\frac{-y}{\sqrt{1-y^2}}$

(6) The equation of the normal line to the curve $x^2y + y^2x = 2$ at $(1,1)$ is

(a) $y = x$

(b) $y = 2 - x$

(c) $y = 3x - 2$

(d) $y = 2x - 1$

(7) One of the following statements is <u>false</u>

(a) if f and g are odd then $f \circ g$ is odd.

(b) if f is odd and g is even then $f \circ g$ is odd.

(c) if f is even and g is neither even nor odd then $g \circ f$ is even.

(d) if f and g are odd then fg is even.

(8) The point $(\frac{\pi}{4}, \frac{\pi}{4})$ lies on the curve $\tan(x) + \sec(y) = 1 + \sqrt{2}$. At this point $y' =$

(a) 2

(b) -2

(c) $\sqrt{2}$

(d) $-\sqrt{2}$

(9) $\lim_{x \to 0} x\cot(3x) =$

(a) 0

(b) 3

(c) $\frac{1}{3}$

(d) Does not exist.

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$v = 3t^2 - 24t + 45$

$a = 6t - 24$

$12 - 48 + 45$

(10) Let $s(t) = t^3 - 12t^2 + 45t + 2$ be the position of an object moving in a straight line then

(a) the object is at rest when $t = 5$ only. X

(b) when $3 < t < 5$, the object is moving forward.

(c) the acceleration is zero when $t = 3$. X

(d) when $t < 3$, the object is moving forward.

(11) To shift the graph of f two units up and to compress it horizontally by a factor of 2 and then shift it one unit to the left, we use the function

(a) $f(2x - 1) + 2$

(b) $f(2x + 1) + 2$

(c) $f(2x + 2) + 2$

(d) $f(2x - 2) + 2$

$f(2x+1) + 2$

(12) $\lim_{x \to 1} \frac{\sqrt{x+3}-2}{x-1} =$

$\frac{x+3-4}{(x-1)(\sqrt{x+3}+2)}$ $\quad \frac{1}{\sqrt{x+3}+2}$ $\quad \frac{1}{1+4}$

(a) 2

(b) 4

(c) $\frac{1}{2}$

(d) $\frac{1}{4}$

(13) Let $f(x) = \begin{cases} \lfloor x \rfloor & , \ 0 \le x < 1 \\ ax + b & , \ 1 \le x \le 3 \\ 3\ \lfloor x \rfloor & , \ 3 < x < 4 \end{cases}$

The values of a and b that make the function continuous on the interval $[0, 4)$ are

(a) $a = 0, b = 1$

(b) $a = \frac{3}{2}, b = -\frac{3}{2}$

(c) $a = -\frac{3}{2}, b = \frac{3}{2}$

(d) There are no values.

$3a + b = 3$

$a + b = 0$

$\frac{2a}{2} = \frac{3}{2}$

$a = \frac{3}{2}$

$a + b = 0$

$3a + b = 3$

$\frac{2a}{2} = \frac{3}{2}$

$a = \frac{3}{2}$

$\frac{(x-1)(x+1)}{x(x-1)} \lim_{x\to 0} \frac{x+1}{x}$ $\frac{1}{0^+}$

(14) Consider the function $f(x) = \frac{x^2-1}{x^2-x}$. One of the following statement is false

(a) f has a horizontal asymptote.

(b) f has a vertical asymptote.

(c) the range of f is $(-\infty, \infty)$.

(d) f has a removable discontinuity.

~~$(-\infty, 0) \cup (0,1) \cup (1, \infty)$~~

~~$0 < x < \infty$~~

(15) The largest δ that satisfies $|\sqrt{x+1} - 2| < 1$ whenever $0 < |x-3| < \delta$ is

(a) 1

(b) 3

(c) 5

(d) 8

$-1 < \sqrt{x+1} - 2 < 1$
$1 < \sqrt{x+1} < 3$
$1 < x+1 < 9$
$0-3 < x-3 < 8-3$
$(-3) < x-3 < 0$

$0 < x-3 < 8-3$
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(16) Let a, b and c be the sides of a triangle with $a = b = 1$ and the angle between a and b is $\frac{2\pi}{3}$. Then $c =$

(a) $\sqrt{2}$

(b) 2

(c) $\sqrt{3}$

(d) 3

$C^2 = a^2 + b^2 - ab\cos(\frac{2\pi}{3})$

$2 + \frac{1}{2}$ $= 3$ $\frac{5}{2}$ $\frac{4}{2} + \frac{1}{2}$ $2 - 1 \times 1 \times \frac{1}{2}$

$C^2 = \frac{5}{2}$, $\frac{4}{2} + \frac{1}{2}$

$\frac{180}{3} \times 2 \times 60$ 60 120 $\frac{2 \times 180}{3}$ 120

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Question 2 (20%) Answer by true or false

1. The function $f(x) = 1 + \sin x - x$ has a root in the interval $[0, \pi]$. (...T...)

2. A rational function can have an oblique and a horizontal asymptote. (...~~T~~...)

3. The function $f(x) = x^3 - x^2 - 1$ has a horizontal tangent at $x = 2/3$. (...T...)

4. The period of the function $\tan(2x)$ is π. (...F...)

5. If $\lim_{x \to c} g(x) = a$ then c belongs to the domain of g. (...~~T~~...)

6. The domain of the function $\tan(\pi \sin x)$ is $(-\infty, \infty)$. (...F...)

7. If f is differentiable at $x = c$ then f is continuous at $x = c$. (...T...)

8. The range of the function $\sec(x) + 1$ is $[2, \infty)$. (...F...)

9. The function $\frac{\tan x}{x}$ has a removable discontinuity at $x = 0$. (...T...)

10. If $y = \sec^2(\theta)$ then $y'(\frac{\pi}{4}) = 2\sqrt{2}$. (...F...)

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$2\sec\theta * \sec\theta \tan\theta$

$(-\infty, 2\pi) \cup (\infty,$ $\frac{1}{\sin}$ $\lim_{x\to 0} \frac{\tan}{\theta}$

$1 + \cdot -0 \cdot 1$

$1 + \cdot -\pi$

$1 - \pi$ 3.14

$\sin(\pi$

$y' = 3x^2 - 2x$

$3 \times \frac{4}{9} - 2 \times \frac{2}{3}$

$\frac{4}{3} - \frac{4}{3}$

2π $\frac{1}{\sin}$ 2π

$2 * 2 * \sqrt{2}$

$2\cos\theta \sec\theta \tan$

$2 \times 2 \times 1$

$(-\infty, -1)$ $[2, \infty)$

$(-\infty, 0)$

Question 3(18%) Consider the function $f(x) = \frac{x^2-1}{x^2-2x}$ $\frac{(x-1)(x+1)}{x(x-2)}$

1. The domain of f is $\boxed{(-\infty, 0) \cup (0, 2) \cup (2, \infty)}$

2. $\lim\limits_{x \to 2^+} f(x) = \frac{(2-1)(2+1)}{2(2^+-2)} = \frac{3}{2\,0^+} = \boxed{\infty}$

3. $\lim\limits_{x \to 2^-} f(x) = \frac{(2-1)(2+1)}{2(2-2)} = \frac{3}{2\,0^-} = \boxed{-\infty}$

4. $\lim\limits_{x \to 0^+} f(x) = \frac{(0-1)(0+1)}{0(0-2)} = \frac{-1}{0^+} = \boxed{-\infty}$

5. $\lim\limits_{x \to 0^-} f(x) = \frac{(0-1)(0+1)}{0(0-2)} = \frac{-1}{0^-} = \boxed{\infty}$

6. $\lim\limits_{x \to \infty} f(x) = \frac{\frac{x^2}{x^2} - \frac{1}{x^2}}{\frac{x^2}{x^2} - \frac{2}{x}} = \frac{1-0}{1-0} = \boxed{1}$

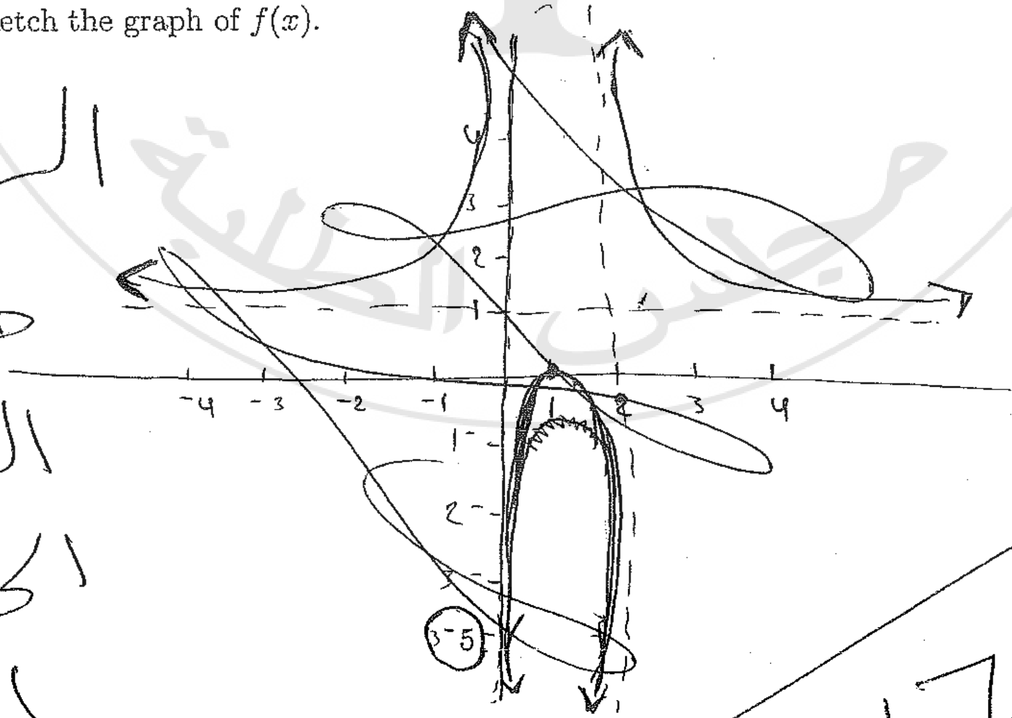
7. $\lim\limits_{x \to -\infty} f(x) = \frac{\frac{x^2}{x^2} - \frac{1}{x^2}}{\frac{x^2}{x^2} - \frac{2}{x}} = \frac{1}{1} = \boxed{1}$

8. Vertical asymptote(s) is/are $\boxed{x = 2}$ $\boxed{x = 0}$

9. Horizontal asymptote(s) is/are $\boxed{y = 1}$

10. x–intercepts: $y = \frac{x^2-1}{x^2-2x} = 0 = x^2 - 1$ $x^2 = 1$ $x = \pm 1$ $\boxed{x = 1}$ $\boxed{x = -1}$

11. Sketch the graph of $f(x)$.



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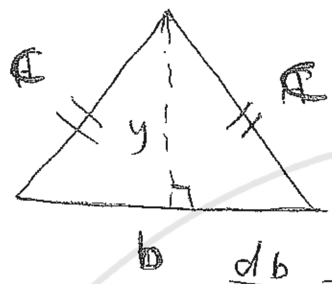
Question 4(14%) The two equal sides of an isosceles triangle (مثلث متساوي الساقين) with fixed base b are decreasing at the rate of 3 cm/min. How fast is the area decreasing when the two equal sides are equal to the base.



$\frac{db}{dt} = -3$ مطلوب $\frac{dA}{dt}$ when $c = b$

$$A = \frac{1}{2} b y \Longrightarrow A = \frac{1}{2} b \sqrt{c^2 - \frac{b^2}{4}}$$

$$y^2 + \left(\frac{b}{2}\right)^2 = c^2$$

$$y^2 + \frac{b^2}{4} = c^2 \Rightarrow y = \sqrt{c^2 - \frac{b^2}{4}}$$

$$\frac{dA}{dt} = \frac{1}{2} b * \frac{2b\frac{db}{dt}}{2\sqrt{c^2 - \frac{b^2}{4}}} + \sqrt{c^2 - \frac{b^2}{4}} * \frac{1}{2}\frac{db}{dt}$$

$$\frac{\frac{1}{2} * b * 2 * b * -3}{2\sqrt{\frac{4b^2 - b^2}{4}}} + \sqrt{\frac{4b^2 - b^2}{4}} * \frac{1}{2} * -3$$

$$\frac{-3b^2}{2\frac{\sqrt{3}}{2}b} + \frac{-3}{2}\frac{\sqrt{3}}{2}b \Longrightarrow \frac{-3b}{\sqrt{3}} - \frac{3\sqrt{3}}{4}b$$

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$$\frac{dA}{dt} = -\left(\frac{3}{\sqrt{3}} + \frac{3\sqrt{3}}{4}\right) b$$