MATHEMATICS DEPARTMENT
MATH141 -Midterm Exam-
First Semester 2015/2016

- Name.........KEY..................... •Number........................

- **Circle your discussion's section number from the two tables below:**

#	Discussion teacher	Time (T, R)
1	Leen Hethnawi	10:00 - 10:50
2	Hiba Sharha	13:00 - 13:50
3	Hasan Yousef	14:00 - 14:50
4	Aseil Altete	11:00 - 11:50
5	Areej Awawdah	09:00 - 09:50
6	Leen Hethnawi	08:00 - 08:50
7	Areej Awawdah	10:00 - 10:50
8	Leen Hethnawi	11:00 - 11:50
9	Hiba Sharha	12:00 - 12:50
10	We'Am Abu Arqoub	09:00 - 09:50
11	Hiba Sharha	10:00 - 10:50
12	We'Am Abu Arqoub	11:00 - 11:50

#	Discussion teacher	Time (T, R)
13	We'Am Abu Arqoub	13:00 - 13:50
14	Leen Hethnawi	13:00 - 13:50
15	Areej Awawdah	12:00 - 12:50
16	Areej Awawdah	08:00 - 08:50
17	Maher Abdallatif	11:00 - 11:50
18	Aseil Altete	12:00 - 12:50
19	Hiba Sharha	09:00 - 09:50
20	Mahmoud Ghannam	14:00 - 14:50
21	Hiba Sharha	14:00 - 14:50
22	Saddam Zaid	13:00 - 13:50
23	Saddam Zaid	11:00 - 11:50
24	Saddam Zaid	08:00 - 08:50

- **Instructions**

1. Write your name and number.
2. Choose your section from the above table.
3. There are three questions in the next 6 pages.
4. Answer all questions.
5. Turn off your mobile phones.
6. Calculators are not allowed.

Q1) [45 pts] **Circle the correct answer.**

(1) If $f(x) = \ln(\sin x)$, $0 < x < \frac{\pi}{2}$, then $(f^{-1})'(-\ln 2) =$

(a) $\sqrt{3}$

(b) $\frac{1}{\sqrt{3}}$

(c) 2

(d) $\frac{1}{2}$

(2) The function $f(x) = x\ln x - x$, $x > 0$, is increasing on

(a) $(0, 1)$

(b) $(0, \infty)$

(c) $(1, \infty)$

(d) It is decreasing for all $x > 0$.

(3) The solution of the equation $e^{x-1} = 2^x$ is

(a) $1 - \ln 2$

(b) $\frac{1}{1-\ln 2}$

(c) $\frac{1}{1+\ln 2}$

(d) $1 + \ln 2$

(4) The derivative of the function $y = x^{2x}$ is

(a) $2(1 + \ln x)$

(b) $x^{2x}(1 + \ln x)$

(c) $2x^{2x}(1 + \ln x)$

(d) $1 + \ln x$

(5) $\int_0^1 \frac{-\ln 2 dx}{1+2^x} =$

(a) $\ln\left(\frac{3}{4}\right)$.

(b) $\ln 2 \ln\left(\frac{3}{4}\right)$

(c) $\ln\left(\frac{3}{2}\right)$

(d) $\ln 2 \ln 3$

(6) The area between the curves $y = \ln x$ and $y = \ln(4x)$, $1 \leq x \leq 2$ is

(a) $2\ln 2$

(b) $\ln 2$

(c) $1 + \ln 2$

(d) $1 + \ln 4$

(7) The derivative of the function $y = \log_2(\ln x)$ is

(a) $\frac{1}{x \ln x}$

(b) $\frac{1}{\ln x}$

(c) $\frac{1}{\ln 2 \ln x}$

(d) $\frac{1}{(\ln 2) x \ln x}$

BZU-HUB

(8) The linearization of the function 3^{x^2} at $x = 1$ is

(a) $y = 3 + \ln 3(x - 1)$

(b) $y = 3 + 6(x - 1)$

(c) $y = 3 + 6\ln 3(x - 1)$

(d) $y = 9 + \ln 3(x - 1)$

(9) $\lim\limits_{x \to 0} \frac{\tan x - x}{x^3} =$

(a) 0

(b) $\frac{1}{3}$

(c) $-\frac{1}{3}$

(d) ∞.

(10) If $x^{2/3} + y^{2/3} = 8$, then $\dfrac{dy}{dx} =$

(a) $\dfrac{y^{1/3}}{x^{1/3}}$

(b) $-\dfrac{y^{1/3}}{x^{1/3}}$

(c) $\dfrac{x^{-1/3}}{y^{-1/3}}$

(d) $-\dfrac{y^{-1/3}}{x^{-1/3}}$

(11) If the population in Palestine increases exponentially since 2015 with a growth rate of $0.1 \ln 4$, then the population will double

(a) After 10 years.

(b) After 15 years.

(c) In the year 2020

(d) In the year 2035

(12) If $g(x) = \dfrac{x^2 + 2x}{x^3 - x^2 - 6x}$, then

(a) $y = 0$ is a horizontal asymptote and $x = 0$ is vertical asymptote for $g(x)$.

(b) Both $x = -2$ and $x = 3$ are vertical asymptotes for $g(x)$.

(c) The $x-$axis is a horizontal asymptote and $x = 3$ is a vertical asymptote for $g(x)$.

(d) The three lines $x = -2$, $x = 0$, and $x = 3$ are vertical asymptotes for $g(x)$.

(13) $\lim\limits_{x \to \infty} e^{-x} \ln(x) =$

(a) 1

(b) 0

(c) ∞

(d) Does not exist.

(14) The domain of the function $f(x) = \dfrac{\sqrt{x-2}}{\sqrt{16 - x^2}}$ is

(a) $(-4, 4)$

(b) $(2, \infty)$

(c) $[0, 4)$

(d) $[2, 4)$

(15) The function $y = xe^{-x}$

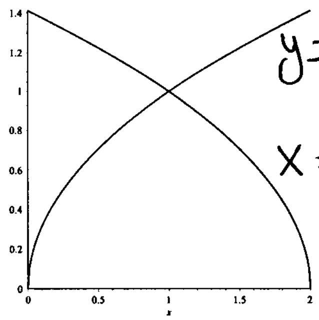
(a) has absolute maximum at $x = 1$ and inflection point at $x = 2$

(b) is concave up on $[2, \infty)$ and concave down on $(-\infty, 2]$

(c) is increasing on $(-\infty, 1]$ and decreasing on $[1, \infty)$

(d) All the above statements are true.

Q2) [16 pts] Find the volume of the solid generated by revolving the region enclosed between the curve $y = \sqrt{x}$, $x = 2 - y^2$ and the x-axis about



$$y = \sqrt{x} \Rightarrow x = y^2$$

$$x = 2 - y^2 \Rightarrow y = \sqrt{2 - x}$$

Figure 1: Graph of $y = \sqrt{x}$ and $x = 2 - y^2$

(a) The x-axis **(Do NOT evaluate the integrals)**

(i) Use the shell method.

$$V = 2\pi \int_0^1 (2 - 2y^2)\, y \, dy$$

(ii) Use the washers method.

$$V = \pi \int_0^1 x \, dx + \pi \int_1^2 (2 - x) \, dx$$

(b) The y-axis **(Do NOT evaluate the integrals)**

(i) Use the shell method.

$$V = 2\pi \int_0^1 x^{3/2} \, dx + 2\pi \int_1^2 x\sqrt{2 - x} \, dx$$

(ii) Use the washers method.

$$V = \pi \int_0^1 \left[(2 - y^2)^2 - y^4 \right] dy$$

Q3) [24 pts] Consider the function $f(x) = \frac{\sqrt{x}}{\sqrt{x}-3}$

(2pts) (a) The domain of f is $[0, \infty) \setminus \{9\}$

(2pts) (b) $\lim_{x \to 9^+} f(x) = +\infty$

(c) $\lim_{x \to 9^-} f(x) = -\infty$

(d) $\lim_{x \to +\infty} f(x) = 1$

(e) Horizontal asymptote is $y = 1$

(f) Vertical asymptote is $x = 9$

(g) Find $f^{-1}(x)$

$$y(\sqrt{x} - 3) = \sqrt{x}$$

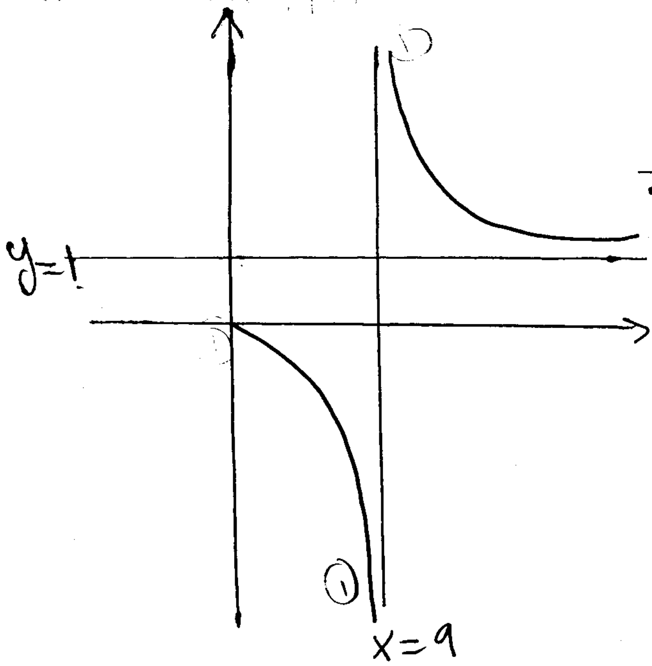
$$\sqrt{x}(y - 1) = 3y$$

$$\sqrt{x} = \frac{3y}{y-1} \geq 0 \Rightarrow y \in (-\infty, 0] \cup (1, \infty)$$

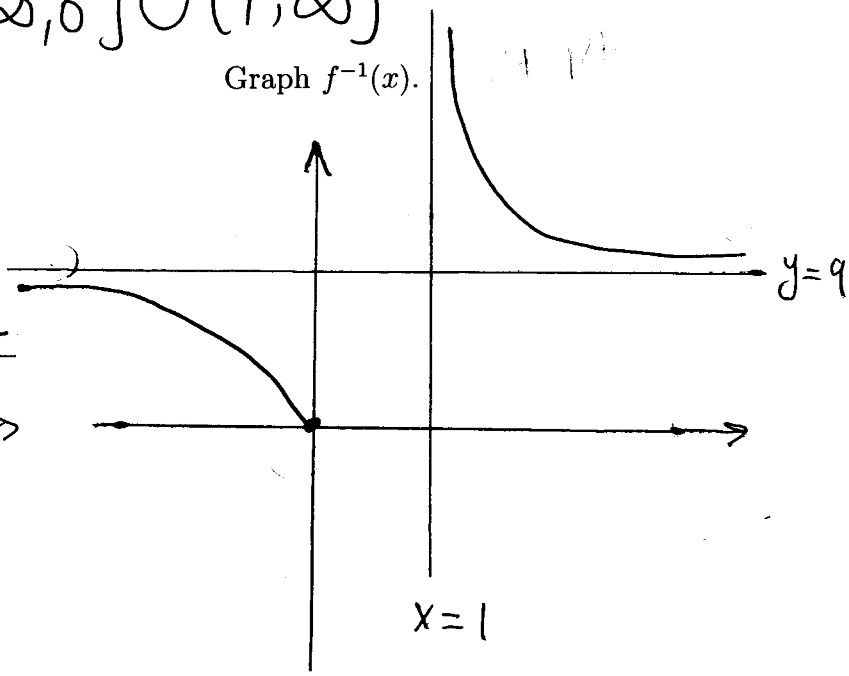
$$f^{-1}(x) = \left(\frac{3x}{x-1}\right)^2$$

(h) Find the domain of f^{-1} is $(-\infty, 0] \cup (1, \infty)$

(i) Graph $f(x)$



Graph $f^{-1}(x)$.



Q3) [15 pts] Answer the following questions:

(a) Find the length of the curve $y = \ln(\cos x)$, $0 \le x \le \frac{\pi}{4}$.

$$\frac{dy}{dx} = -\frac{\sin x}{\cos x} = -\tan x$$

$$1+\left(\frac{dy}{dx}\right)^2 = 1+\tan^2 x = \sec^2 x$$

$$L = \int_0^{\pi/4} \sqrt{\sec^2 x}\, dx$$

$$= \int_0^{\pi/4} \sec x\, dx = \ln|\sec x+\tan x| \Big|_0^{\pi/4}$$

$$= \ln(\sqrt{2}+1)$$

(b) Find the area of the surface generated by revolving the curve $y = \sqrt{4-x^2}$, $-1 \le x \le 1$, about the $x-$axis.

$$\frac{dy}{dx} = \frac{-x}{\sqrt{4-x^2}}$$

$$1+\left(\frac{dy}{dx}\right)^2 = 1+\frac{x^2}{4-x^2} = \frac{4}{4-x^2}$$

$$S = 2\pi \int_{-1}^{1} \sqrt{4-x^2}\sqrt{\frac{4}{4-x^2}}\, dx$$

$$= 2\pi \int_{-1}^{1} 2\, dx$$

$$= 4\pi x \Big|_{-1}^{1}$$

$$= 8\pi.$$