key

5

MATHEMATICS DEPARTMENT
MATH 1411 -Quiz 2-
First Semester 2021/2022

Section.......14D.......

• Name (Arabic).......................... Number..................

Q1) Choose the most correct answer. $f' < 0$ $f'' > 0$
If f is a function such that f' is negative and incresing on an interval I, then f is

1. Increasing and concave down on I
2. Decreasing and concave up on I
3. Decreasing and concave down on I
4. Increasing and concave up on I

Q2) Let $f(x) = x^2 - 2x, x \in [1, 2]$, find the value of c in the conclusion of the Mean value theorem .

$f'(x) = 2x - 2$

$$f'(c) = \frac{f(2) - f(1)}{2-1}$$

$$2c - 2 = \frac{(2)^2 - 2(2) - [(1)^2 - 2(1)]}{1}$$

$$2c - 2 = 1 \Rightarrow 2c = 3 \Rightarrow \boxed{c = \frac{3}{2}}$$

Q3) Let $f(x) = \frac{x^2}{x+1}$ where $f' = \frac{x(x+2)}{(x+1)^2}$ and $f'' = \frac{2}{(x+1)^3}$.
Find the intervals in which $f(x)$ is concave up and the intervals in which $f(x)$ is concave down , then find the inflection points(if any).

$D: (-\infty, \infty) / \{-1\}$

$$f'' = \frac{2}{(x+1)^3}$$

$- - -$ -1 $+ +$ sign of f''

$\cap$ $\cup$

- Concave up on $(-1, \infty)$
- Concave down on $(-\infty, -1)$
- no inflection points

Key

MATHEMATICS DEPARTMENT
MATH 1411 -Quiz 2-
First Semester 2021/2022

• Name (Arabic)........................... Number.................. Section.......14D.......

Q1) If f is a function such that f' is positive and incresing on an interval I, then f is

$f'>0$ $f''>0$

1. Increasing and concave down on I
2. Decreasing and concave up on I
3. Decreasing and concave down on I
4. Increasing and concave up on I

Q2) Let $f(x) = x^2 - x, x \in [0, 2]$, find the value of c in the conclusion of the Mean value theorem

$f' = 2x - 2$

$$f'(c) = \frac{f(2) - f(0)}{2 - 0}$$

$$2c - 2 = \frac{(2)^2 - 2 - (0^2 - 0)}{2}$$

$$2c - 2 = 0 \Rightarrow 2c = 2 \Rightarrow \boxed{c = 1}$$

Q3) Let $f(x) = \frac{-x^2}{x+1}$ where $f' = \frac{-x(x+2)}{(x+1)^2}$ and $f'' = \frac{-2}{(x+1)^3}$.
Find the intervals in which $f(x)$ is concave up and the intervals in which $f(x)$ is concave down, then find the inflection points(if any).

$D: (-\infty, \infty) / \{-1\}$

$$f'' = \frac{-2}{(x+1)^3}$$

$+ + +$ -1 $- - -$ sign of f''

$\cup$ $\cap$

Concave up on $(-\infty, -1)$

concave down on $(-1, \infty)$

no inflection points

MATHEMATICS DEPARTMENT
MATH 1411 -Quiz 2-
First Semester 2021/2022

• Name (Arabic)........................... Number.................. Section.......16D.......

Q1) Choose the most correct answer.
If the radius of a circle changes from 2 to 2.1, then the area of the circle changes approximately by

1. 0.4π
2. 0.2π
3. 0.3π
4. 0.6π

$A = r^2\pi \longrightarrow dA = 2r\pi\,dr$

$= 2(2)(\pi)(0.1)$

$= 0.4\pi$

1

Q2) Find the linearization of the function $y = x + \sin x$ at the point (π, π) .

$L(x) = f(a) + f'(a)(x-a)$

$L(x) = f(\pi) + f'(\pi)(x - \pi)$

$= \pi + 0(x-\pi)$

$\boxed{L(x) = \pi}$

1

$f'(x) = 1 + \cos x$

$f'(\pi) = 1 + \cos\pi = 0$

Q3) Let $f(x) = \frac{x^2}{x+1}$ where $f' = \frac{x(x+2)}{(x+1)^2}$.

Find the intervals in which $f(x)$ is increasing and the intervals in which $f(x)$ is decreasing , then find the extreme values.

$D: (-\infty, \infty) / \{-1\}$

1

$f' = \frac{x(x+2)}{(x+1)^2}$

+ | − | − | + sign of f'

−2   −1   0

1 increasing on $(-\infty, -2] \cup [0, \infty)$

decreasing on $[-2, 0] / \{-1\}$

max at $x = -2$, $f(-2) = \frac{(-2)^2}{-2+1} = \boxed{-4}$

min at $x = 0$, $f(0) = \boxed{0}$ 1

Key

$\overline{\quad 5 \quad}$

MATHEMATICS DEPARTMENT
MATH 1411 -Quiz 2-
First Semester 2021/2022

• Name (Arabic)........................... Number.................. Section.......16D.......

Q1) Choose the most correct answer.
If the radius of a circle changes from 3 to 3.1, then the area of the circle changes approximately by

$A = r^2\pi$ $\qquad dA = 2r\pi\, dr$

$dA = 2(3)\pi(0.1)$

$= 0.6\pi$

1. 0.4π
2. 0.2π
3. 0.3π
4. 0.6π

Q2) Find the linearization of the function $y = x - \sin x$ at the point $(\overset{a}{\pi}, \pi)$

$L(x) = f(\pi) + f'(\pi)(x - \pi)$ $\qquad f(\pi) = \pi$

$L(x) = \pi + 0(x - \pi)$ $\qquad f'(x) = 1 - \cos x$

$f'(\pi) = 1 - \cos(\pi)$

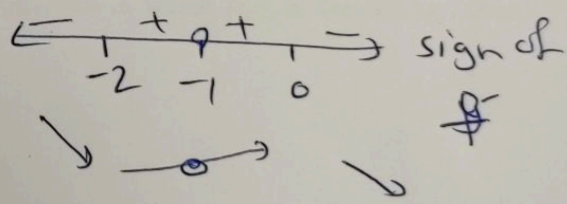
$= 0$

$\boxed{L(x) = \pi}$

Q3) Let $f(x) = \frac{-x^2}{x+1}$ where $f' = \frac{-x(x+2)}{(x+1)^2}$.
Find the intervals in which $f(x)$ is increasing and the intervals in which $f(x)$ is decreasing , then find the extreme values.

$D: (-\infty, \infty) / \{-1\}$

$f' = \dfrac{-x(x+2)}{(x+1)^2}$



increasing on $[-2, 0] / \{-1\}$

or $[-2, -1) \cup (-1, 0]$

decreasing on $(-\infty, -2] \cup [0, \infty)$

Local max at $x = 0 \rightarrow f(0) = \boxed{0}$

Local min at $x = -2 \rightarrow f(-2) = \dfrac{-(-2)^2}{-2+1} = \boxed{4}$

5

key

MATHEMATICS DEPARTMENT
MATH 1411 -Quiz 2-
First Semester 2021/2022

- Name (Arabic)........................ Number................ Section.......20D.......

Q1) If the radius of a circle changes from 3 to 3.05, then the area of the circle changes approximately by

$A = r^2 \pi \rightarrow dA = 2r\pi \, dr$

$dA = 2(3)(\pi)(0.05)$

$= 0.3\pi$

1. 0.6π
2. 0.2π
3. 0.3π
4. 0.4π

1

Q2) Find the linearization of the function $y = x - \cos x$ at the point $(\frac{\pi}{2}, \frac{\pi}{2})$

a

$L(x) = f(\frac{\pi}{2}) + f'(\frac{\pi}{2})(x - \frac{\pi}{2})$

$= \frac{\pi}{2} + 2(x - \frac{\pi}{2})$

$= \frac{\pi}{2} + 2x - \frac{2\pi}{2} = \boxed{-\frac{\pi}{2} + 2x}$

$f' = 1 - -\sin x$

$f'(\frac{\pi}{2}) = 1 + 1 = 2$

Q3) Let $f(x) = \frac{-x^2+4x}{(x+4)^2}$ where $f' = \frac{4(4-3x)}{(x+4)^3}$ and $f'' = \frac{24(x-4)}{(x+4)^4}$, then find the intervals in which $f(x)$ is increasing and the intervals in which $f(x)$ is decreasing , then find the extreme values.

$D = (-\infty, \infty) / \{-4\}$

$f' = \frac{4(4-3x)}{(x+4)^3}$

$- - - \quad -4 \quad + + + \quad \frac{4}{3} \quad - - -$ sign of f'

$\searrow \quad \nearrow \quad \searrow$

increasing on $(-4, \frac{4}{3}]$

decreasing on $(-\infty, -4) \cup [\frac{4}{3}, \infty)$

max at $\frac{4}{3}$

$f(\frac{4}{3}) = \frac{-(\frac{4}{3})^2 + 4(\frac{4}{3})}{(\frac{4}{3}+4)^2} = \frac{-\frac{16}{9} + \frac{16}{3}}{(\frac{16}{3})^2} = \frac{-\frac{16}{9} + \frac{48}{9}}{(\frac{16}{3})^2} = \frac{32}{(16)^2} = \frac{2}{16} = \boxed{\frac{1}{8}}$

MATHEMATICS DEPARTMENT
MATH 1411 -Quiz 2-
First Semester 2021/2022

• Name (Arabic)........................... Number.................. Section.......20D.......

Q1) Choose the most correct answer.
If the radius of a circle changes from 2 to 2.05, then the area of the circle changes approximately by

1. 0.2π
2. 0.6π
3. 0.3π
4. 0.4π

$A = r^2\pi \rightarrow dA = 2r\pi\, dr$
$= 2(2)(\pi)(0.05)$
$= 0.2\pi$

Q2) Find the linearization of the function $y = x + \cos x$ at the point $(\frac{\pi}{2}, \frac{\pi}{2})$

a

$L(\cancel{x}) = f(\frac{\pi}{2}) + f'(\frac{\pi}{2})(x - \frac{\pi}{2})$

$= \frac{\pi}{2} + 0(x - \frac{\pi}{2})$

$L(x) = \frac{\pi}{2}$

$f' = 1 - \sin x$
$f'(\frac{\pi}{2}) = 1 - 1$
$= 0$

Q3) Let $f(x) = \frac{x^2-4x}{(x+4)^2}$ where $f' = \frac{4(3x-4)}{(x+4)^3}$ and $f'' = \frac{24(4-x)}{(x+4)^4}$ find the intervals in which $f(x)$ is decreasing , then find the intervals in which $f(x)$ is increasing and the intervals in which $f(x)$ is decreasing , then find the extreme values.

$D: (-\infty, \infty) / \{-4\}$

f is increasing on
$(-\infty, -4) \cup [\frac{4}{3}, \infty)$

$+ + \quad - - - \quad + +$ f'
$-4 \quad \frac{4}{3}$

f is decreasing on $(-4, \frac{4}{3}]$

f has local min at $x = \frac{4}{3}$

$-\frac{2}{16} = \boxed{-\frac{1}{8}}$

$f(\frac{4}{3}) = \frac{(\frac{4}{3})^2 - 4(\frac{4}{3})}{(\frac{4}{3}+4)^2} = \frac{\frac{16}{9} - \frac{16}{3}}{(\frac{16}{3})^2} = \frac{\frac{16-48}{9}}{\frac{(16)^2}{9}} = -\frac{32}{(16)^2} =$

Q2) [24 %] Consider the function $f(x)$ and its first and second derivatives

$$f(x) = \frac{x^2 - 4x}{(x+4)^2}, \quad f'(x) = \frac{4(3x-4)}{(x+4)^3}, \quad f''(x) = \frac{24(4-x)}{(x+4)^4}$$

1 (1) The domain of f is $(-\infty, \infty) \setminus \{-4\}$

1 (2) $\lim_{x \to -4^+} f(x) = +\infty$

1 (3) $\lim_{x \to -4^-} f(x) = +\infty$

1 (4) $\lim_{x \to +\infty} f(x) = 1$

1 (5) $\lim_{x \to -\infty} f(x) = 1$

1 (6) Horizontal asymptote is $y = 1$

1 (7) Vertical asymptote is $x = -4$

2 (8) the graph of f crosses the x-axis at the point(s) $(0,0), (4,0)$

2 (9) f is increasing on $(-\infty, -4) \cup [\frac{4}{3}, \infty)$

1 (10) f is decreasing on $(-4, \frac{4}{3})$

1 (11) is $f(\frac{4}{3})$ an absolute maximum or minimum of $f(x)$? abs. min

2 (12) f is concave up on $(-\infty, -4) \cup (-4, 4]$

1 (13) f is concave down on $[4, \infty)$

1 (14) f has inflection point(s) $(4,0)$

1 (15) the range of f is $(-\frac{1}{8}, \infty)$

6 (16) Sketch the graph of f