Section 14

BIRZEIT UNIVERSITY
MATHEMATICS DEPARTMENT
MATH1411 - QUIZ 2

Key

Name :..... Student Number..... Section ..... 10/10.

Question #1:Circle the correct answer 4.5 Pt.

1) The solution of the equation $\log_3 2 + 2\log_3 x = \log_3(7x - 3)$

a) $x = 3$

b) $x = \frac{1}{2},\ x = 3$

c) $x = \frac{1}{3}$

d) $x = \frac{1}{2}$

2) Let $f(x) = x^2,\ x \leq 0$. Find the derivative of $f^{-1}(x)$ at x=4

a) $\frac{1}{2}$

b) $\frac{1}{8}$

c) $\frac{1}{4}$

d) $\frac{1}{-4}$

3) $\int \sec(5x)dx$,

a) $\frac{1}{5}\ln|tan(5x)| + c$

b) $\frac{1}{5}\ln|sec(5x) + tan(5x)| + c$

c) $5\ln|sec^2(5x)| + c$

d) $5\ln|sec(5x) + tan(5x)| + c$

Question #2: Find the derivative of the inverse function of $f(x) = x + \sin x$ at $x = \pi$ 3 Pt.

$f(x) = x + \sin x$

$f'(x) = 1 + \cos x$

$$\left.\frac{df^{-1}}{dx}\right|_{x=\pi} = \frac{1}{\left.\frac{df}{dx}\right|_{f^{-1}(\pi)}} = \frac{1}{\left.\frac{df}{dx}\right|_{x=\pi}} = \frac{1}{1+\cos\pi} = \text{Undefined}$$

$f^{-1}(\pi) = ??$

$\pi = x + \sin x$

$\boxed{x = \pi}$

Question #3: Selected values of a strictly increasing function $g(x)$ and $g'(x)$ are shown on the table below. Find 25Pt.

x	-3	-1	1	4
$g(x)$	5	1	0	-3
$g'(x)$	-4	$\frac{-1}{5}$	$\frac{-1}{6}$	-2

1) $(g^{-1})'(1)$

$$\left.\frac{dg^{-1}}{dx}\right|_{x=1} = \frac{1}{\left.\frac{dg}{dx}\right|_{x=-1}} = \frac{1}{g'(-1)} = \frac{1}{-\frac{1}{5}} = -5$$

2) $(g^{-1})'(-3)$

$$\left.\frac{dg^{-1}}{dx}\right|_{x=-3} = \frac{1}{\left.\frac{dg}{dx}\right|_{x=4}} = \frac{1}{g'(4)} = \frac{1}{-2}$$

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Section 1 (10/10)

Name :..... Student Number..... Section .....

Question #1:Circle the correct answer 4.5 pt.

1) The solution of the equation $\ln(x^3 - 2x^2 - x) - \ln x = \ln 2$

a) $x = 3$

b) $x = 3,\ x = -1,\ x = 0$

c) $x = -1$

d) $x = 3,\ x = -1$

2) Let $f(x) = x^3 + 2x - 3$. Find the value of a if $f^{-1}(a) = 2$

a) 9

b) 1

c) 2

d) Does not exist

3) If $y = x^{\ln x}$, then $y'(e) =$

a) 0

b) 1

c) 2

d) e

Question #2:Given the following function

$$f(x) = 1 - \sqrt{x}$$

→ Is f 1-1?? Yes

Since $f'(x) = \frac{-1}{2\sqrt{x}} < 0$

→ f is dec on Dom(f)

2 Pt a) Find $f^{-1}(x)$

$y = 1 - \sqrt{x} \longrightarrow \sqrt{x} = 1 - y$

$x = (1-y)^2 \longrightarrow y = (1-x)^2$

$f^{-1}(x) = (1-x)^2$

2 Pt. b) Find $\frac{df^{-1}}{dx}$ at $x = -1$

$\frac{df^{-1}}{dx} = 2(1-x)(-1)$

$\left(\frac{df^{-1}}{dx}\right)\Big|_{x=-1} = -2(2) = \boxed{-4}$

OR

$\left(\frac{df^{-1}}{dx}\right)\Big|_{x=-1} = \frac{1}{\left(\frac{df}{dx}\right)\Big|_{x=4}}$

$= \frac{1}{\frac{-1}{4}} = \boxed{-4}$

$f(x) = 1 - \sqrt{x}$

$f'(x) = \frac{-1}{2\sqrt{x}}$

1.5 Pt c) $f^{-1}(2)$

$f^{-1}(2)$ Undefined

Since $2 \notin Dom(f^{-1})$

$Dom(f^{-1}) = (-\infty, 1]$

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Section 6

Key

Name :..... Student Number..... Section 10/10

Question #1:Circle the correct answer 4.5 pts

1) If $y = ln\sqrt{\tan x}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$

a) 10

b) 1

c) 0

d) $\frac{3\pi}{4}$

2) The solution of the equation

$$\ln(x) + \ln(x-2) = \ln(x+10)$$

is

a) $\{-2, 5\}$

b) 5

c) $\{-5, 2\}$

3) $tan^{-1}(tan\frac{3\pi}{4})$

a) $\frac{-3\pi}{4}$

b) $\frac{3\pi}{4}$

c) $\frac{\pi}{4}$

d) Does not exist

Question #2:Given the following function

$f(x) = 1 - \sqrt{x}$ → Is f 1-1 ?? Yes

since $f'(x) = \frac{-1}{2\sqrt{x}} < 0$

→ f(x) dec on Dom(f)

1.5 Pt. a) Find $f^{-1}(x)$

$y = 1 - \sqrt{x}$

$\sqrt{x} = 1 - y$

$x = (1-y)^2$

$y = (1-x)^2$ → $\boxed{f^{-1}(x) = (1-x)^2}$

2 Pt. b) Find $\frac{df^{-1}}{dx}$ at $x = -1$

$\frac{df^{-1}}{dx} = 2(1-x)\cdot(-1) = -2(1-x)$

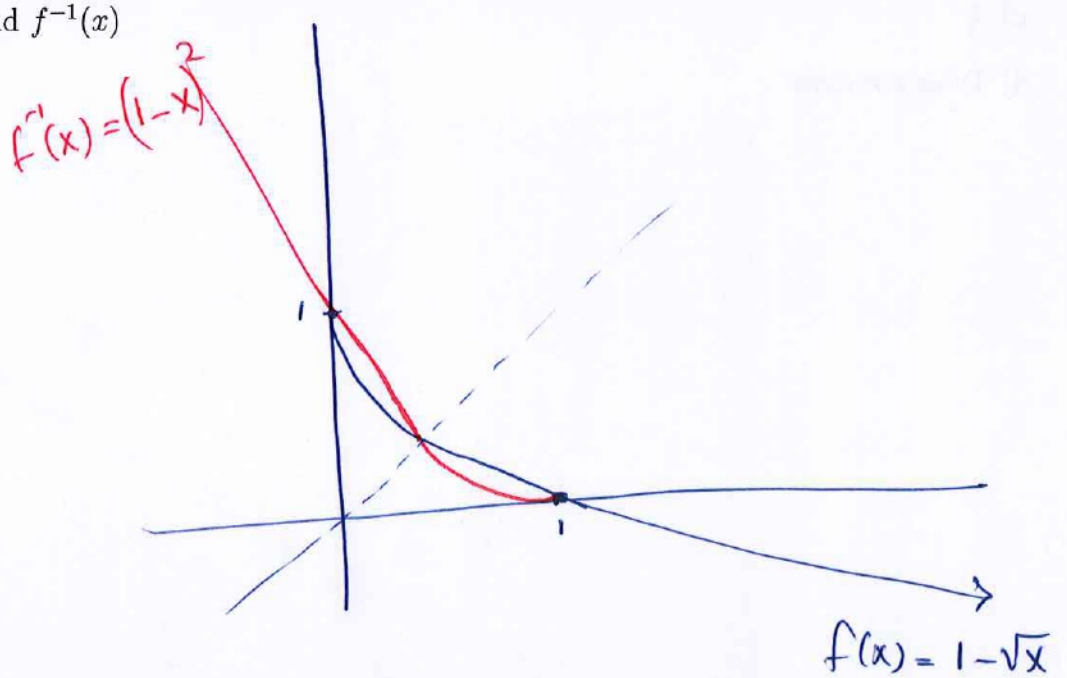
$\left.\frac{df^{-1}}{dx}\right|_{x=-1} = -2(2) = \boxed{-4}$

OR $\left(\frac{df^{-1}}{dx}\right)\Big|_{x=-1} = \frac{1}{\left(\frac{df}{dx}\right)\Big|_{x=4}} = \frac{1}{\frac{-1}{4}} = \boxed{-4}$

$f(x) = 1 - \sqrt{x}$

$f'(x) = \frac{-1}{2\sqrt{x}}$

2. Pt. c) Sketch $f(x)$ and $f^{-1}(x)$



Section 21

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Key

Name :..... Student Number..... Section 10/10

Question #1:Circle the correct answer P1

1) The solution of the equation $\ln x - \ln(x-1) = 1$

a) $x = 0$

b) $x = 1,\ x = 0$

c) $x = \frac{-e}{e-1}$

d) $x = \frac{e}{e-1}$

2) Let $f(x) = (x-4)^3 - 1$. Find the solution of $f^{-1}(x) = 6$

a) 9

b) 7

c) 2

d) Does not exist

3) If $\int \frac{sec^2(2x)}{tan(2x)}$,

a) $\frac{1}{2}\ln|tan(2x)| + c$

b) $\frac{1}{2}\ln|sec(2x) + tan(2x)| + c$

c) $\frac{1}{2}\ln|sin(2x)| + c$

Question #2:Let $f(x) = \frac{5}{x} - 2$ ⟶ Is f 1-1 ?? Yes

2 Pt. a) Determine $f^{-1}(x)$

Since $f'(x) = \frac{-5}{x^2} < 0$

f(x) dec. on $\mathbb{R} \setminus \{0\}$

$y = \frac{5}{x} - 2$

$y + 2 = \frac{5}{x}$

$x = \frac{5}{y+2}$

$y = \frac{5}{x+2}$ ⟶ $\boxed{f^{-1}(x) = \frac{5}{x+2}}$

2 Pt b) Find and simplify $(f^{-1} \circ f)(x)$

$f^{-1}(f(x)) = f^{-1}\left(\frac{5}{x} - 2\right)$

$= \frac{5}{\left(\frac{5}{x} + 2\right) - 2} = \frac{5}{\frac{5}{x}} = 5 \cdot \frac{x}{5}$

$= x$

$(f^{-1} \circ f)(x) = x, \quad \forall x \in Dom(f)$