

Section 1

Key

Name : .....

Student Number .....

Section 10/10

Question #1: Circle the correct answer: 3 Pt. + 1 Bonus.

1)  $\lim_{x \rightarrow 0} \frac{\sqrt{2+x}-\sqrt{2}}{\cos x - e^x}$

a)  $\frac{-1}{\sqrt{2}}$

b)  $\frac{-1}{2\sqrt{2}}$

c)  $\frac{1}{\sqrt{2}}$

2) If  $y = 10^{10^x}$ , then  $\frac{dy}{dx}$  is:

a)  $10^{10^x} \ln 10$

b)  $10^{10^x} (\ln 10)^2$

c)  $10^{10^x} 10^x (\ln 10)^2$

3) If  $y = xe^y$ , then  $\frac{dy}{dx}$  is

a)  $\frac{y}{1-y}$

b)  $\frac{y}{x(1-y)}$

c)  $\frac{x}{y(1-y)}$

Question #2: Find the integral.

3pt.

$$a) \int_1^4 \frac{1}{4x\sqrt{16x^2-5}} dx = \frac{1}{4} \int \frac{du}{u\sqrt{u^2-(\sqrt{5})^2}}$$

$$u = 4x \\ du = 4 dx$$

$$= \frac{1}{4} \frac{1}{\sqrt{5}} \left[ \sec^{-1} \left( \left| \frac{u}{\sqrt{5}} \right| \right) \right]_4^{16}$$

$$x=1 \rightarrow u=4$$

$$x=4 \rightarrow u=16$$

$$= \frac{1}{4\sqrt{5}} \left[ \sec^{-1} \frac{|u|}{\sqrt{5}} \right]_4^{16}$$

$$= \frac{1}{4\sqrt{5}} \left[ \sec^{-1} \frac{16}{\sqrt{5}} - \sec^{-1} \frac{4}{\sqrt{5}} \right]$$

3pt

$$b) \int \frac{12}{1+9x^2} dx$$

$$\int \frac{\frac{4}{3}}{1+(u)^2} \frac{du}{3}$$

$$\text{let } u = 3x \\ du = 3 dx$$

$$\int \frac{4}{1+u^2} du = 4 \tan^{-1}(u) + C \\ = 4 \tan^{-1}(3x) + C$$

Section 6

BIRZEIT UNIVERSITY  
MATHEMATICS DEPARTMENT  
MATH1411 - QUIZ 5

Key

Name : .....

Student Number.....

Section 10/10

Question #1: Circle the correct answer: 3pt. + 1 Bonus.

1) If  $y = e^{\ln(x+\sqrt{x^2+16})}$ , then  $\frac{dy}{dx}$  is:

a)  $e^{\ln(x+\sqrt{x^2+16})}$

b)  $1 + \frac{x}{\sqrt{x^2+16}}$

c)  $e^{\ln(x+\sqrt{x^2+16})} \left(1 + \frac{x}{\sqrt{x^2+16}}\right)$

2)  $2^{3 \log_2 4} =$

a) 16

b) 8

c) 64

3)  $\lim_{x \rightarrow 0^+} x^{\sqrt{x}} =$

a) 0

b) 1

c) e

Question #2: Find the integral.

$$\text{let } U = x - 3 \\ dU = dx$$

3 pt a)  $\int \frac{1}{4+(x-3)^2} dx$

$$\begin{aligned} \int \frac{1}{4+U^2} dU &= \int \frac{1}{(2)^2+(U)^2} dU \\ &= \frac{1}{2} \tan^{-1}\left(\frac{U}{2}\right) + C \\ &= \frac{1}{2} \tan^{-1}\left(\frac{x-3}{2}\right) + C \end{aligned}$$

3 pt. b)  $\int \frac{x-2}{(x+1)^2+4} dx$

$$\text{let } U = x + 1 \\ dU = dx$$

$$\begin{aligned} \int \frac{U-1-2}{U^2+4} dU \\ = \frac{1}{2} \int \frac{2U}{U^2+4} dU - \int \frac{3}{U^2+4} dU \\ \frac{1}{2} \ln|U^2+4| - 3 \cdot \frac{1}{2} \tan^{-1}\left(\frac{U}{2}\right) + C \end{aligned}$$

$$\frac{1}{2} \ln|(x+1)^2+4| - \frac{3}{2} \tan^{-1}\left(\frac{x+1}{2}\right) + C$$

# Section 21

Key

Name : .....

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Section 10/10

Question #1: Find the integral. +1 Bonus

3 pt. a)  $\int_0^1 \frac{1}{\sqrt{4-x^2}} dx = \int_0^1 \frac{1}{\sqrt{(2)^2 - (x)^2}} dx$

$$= \left[ \sin^{-1}\left(\frac{x}{2}\right) \right]_0^1$$

$$= \sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1}(0)$$

$$= \frac{\pi}{6} - 0$$

$$= \frac{\pi}{6}$$

3 pt. b)  $\int \frac{4x+3}{\sqrt{1-x^2}} dx = \int \frac{4x}{\sqrt{1-x^2}} dx + \int \frac{3}{\sqrt{1-x^2}} dx$

$$= -4(1-x^2)^{1/2} + 3 \sin^{-1}(x) + C$$

let  $u = 1-x^2$   
 $du = -2x dx$

$$\int \frac{4x}{\sqrt{1-x^2}} dx = \int \frac{-2 du}{\sqrt{u}}$$
$$= -2 \frac{u^{1/2}}{1/2} + C$$
$$= -4(1-x^2)^{1/2} + C$$

Question #2: Evaluate 3 Pt

a)  $\lim_{x \rightarrow 0^-} x \cot x$ .

0,  $-\infty$

$$\lim_{x \rightarrow 0^+} \frac{x}{\tan x} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0^-} \frac{1}{\sec^2 x} = \frac{1}{1} = \boxed{1}$$

# Section H

Name : .....

Student Number.....

Section 10/10.

Question #1: Find the integral. + 1 Bonus.

3 Pt.

a)  $\int \frac{1}{4x\sqrt{16x^2-5}} dx$

let  $u=4x$   
 $du=4dx$

$$\int \frac{1}{u \sqrt{u^2 - (\sqrt{5})^2}} \cdot \frac{du}{4}$$

$$= \frac{1}{4} \int \frac{du}{u \sqrt{u^2 - (\sqrt{5})^2}} = \frac{1}{4} \cdot \frac{1}{\sqrt{5}} \sec^{-1} \left| \frac{u}{\sqrt{5}} \right| + C$$

$$= \frac{1}{4\sqrt{5}} \sec^{-1} \left| \frac{4x}{\sqrt{5}} \right| + C$$

3 Pt.

b)  $\int_{\sqrt{3}}^3 \frac{6}{9+x^2} dx$

$$\int_{\sqrt{3}}^3 \frac{6}{(3)^2 + x^2} dx$$

$$= 6 \cdot \frac{1}{3} \left[ \tan^{-1} \left( \frac{x}{3} \right) \right]_{\sqrt{3}}^3 = 2 \left[ \tan^{-1}(1) - \tan^{-1} \left( \frac{1}{\sqrt{3}} \right) \right]$$

$$= 2 \left[ \frac{\pi}{4} - \frac{\pi}{6} \right]$$

$$= \frac{\pi}{6}$$

Question #2: Evaluate

3 pt

$$a) \lim_{x \rightarrow 0^+} x^{\sin x} = 0^0$$

$$\text{let } f(x) = x^{\sin x}$$

$$\ln f(x) = \sin x \ln x$$

$$\lim_{x \rightarrow 0^+} \ln f(x) = \lim_{x \rightarrow 0^+} \sin x \ln x = 0 \cdot -\infty$$

$$= \lim_{x \rightarrow 0^+} \frac{\ln x}{\csc x} = \frac{-\infty}{\infty}$$

$$= \lim_{x \rightarrow 0^+} \frac{\frac{1}{x}}{-\csc x \cot x}$$

$$= \lim_{x \rightarrow 0^+} \frac{-\sin^2 x}{x \cos x}$$

$$= \lim_{x \rightarrow 0^+} \frac{-2 \sin x \cos x}{x(-\sin x) + \cos x}$$

$$= 0$$

$$\text{Then } \lim_{x \rightarrow 0^+} x^{\sin x} = e^0 = 1$$