Chapter 5 :

Q1: (a) $\int \sin(5x)\,dx = \frac{-\cos 5x}{5} + c$

(b) $\int \tan^2 x\,dx = \int (\sec^2 x - 1)\,.dx$

$= \tan x - x + c$

(c) $\int (1 + \cot^2\theta)\,d\theta = \int \csc^2\theta\,.d\theta$

$= -\cot\theta + c$

(d) $\int \frac{\csc\theta}{\csc\theta - \sin\theta}\,.d\theta =$

$\int \frac{\frac{1}{\sin\theta}}{\frac{1}{\sin\theta} - \sin\theta}\,.d\theta = \int \frac{\cancel{\frac{1}{\sin\theta}}}{\frac{1-\sin^2\theta}{\cancel{\sin\theta}}}\,.d\theta$

$= \int \frac{1}{1-\sin^2\theta}\,.d\theta = \int \frac{1}{\cos^2\theta}\,.d\theta$

$\int \sec^2\theta\,.d\theta = \tan\theta + c$

Q2 (a) $y = \int_1^x \frac{dt}{t}$

$y' = \frac{1}{x}$

(b) $y = \int_0^{\sqrt{x}} \cos t \, dt \Rightarrow$

$y' = \frac{1}{2\sqrt{x}} \cos \cancel{x}$

(c) $y = \int_{\tan x}^{0} \frac{dt}{1+t^2}$

$y = -\int_0^{\tan x} \frac{dt}{1+t^2}$

$y' = -\frac{\sec^2 x}{1+t^2} = \frac{\sec^2 x}{-1-t^2}$

Q3 Find the Linearization of $g(x) = 3 + \int_1^{x^2} \sec(t-1)\,dt$ at $x = -1$ $(-1)^2$

$g(x) = 3 + \int_1^{x^2} \sec(t-1)\,dt \Rightarrow g(1) = 3 + \int_1^{1} \sec(t-1)\,dt \Rightarrow 3$

$g'(x) = 2x * \sec(x^2-1) \Rightarrow g'(1) = 2\sec 0 \Rightarrow 2$

$L(x) = g(a) + g'(a)(x-a)$

$= 3 + 2(x-1)$

$= 3 + 2x - 2$

$= 1 + 2x$

Q4 (a) $\int_1^{\sqrt{2}} \left(\frac{s^2 + \sqrt{s}}{s^2} \right) ds = \int_1^{\sqrt{2}} \left(1 + s^{-\frac{3}{2}} \right) ds$

$s + \frac{s^{-\frac{1}{2}}}{-\frac{1}{2}} \Big|_1^{\sqrt{2}} =$

$\left(\sqrt{2} - \frac{2}{\sqrt{2}} \right) - \left(1 - 2 \right)$

$\left(0 \right) - \left(-1 \right)$

$= 1$

(b) $$\int_0^{\pi/6} (\sec x + \tan x)^2 \, dx$$

$$\int_0^{\pi/6} (\sec^2 x + \sec x \tan x + \tan^2 x) \, dx$$

$$\int_0^{\pi/6} (\sec^2 x + \sec x \tan x + \sec^2 x - 1) \, dx$$

$$\int_0^{\pi/6} (2\sec^2 x + \sec x \tan x - 1) \, dx$$

$$2 \tan x + \sec x - x \Big|_0^{\pi/6}$$

$$\left(2 \tan \frac{\pi}{6} + \sec \frac{\pi}{6} - \frac{\pi}{6}\right) - (2\tan 0 + \sec 0 - 0)$$

$$= \left(\frac{2}{\sqrt{3}} + \frac{2}{\sqrt{3}} - \frac{\pi}{6}\right) - (0 + 1 - 0)$$

$$= \frac{4}{\sqrt{3}} - \frac{\pi}{6} - 1$$

c) $\int_0^{\pi} (\cos x + |\cos x|)\, dx$

$\int_0^{\pi} \cos x \, dx + \int_0^{\pi} |\cos x| \, dx$

$\int_0^{\pi} \cos x \, dx + \int_0^{\frac{\pi}{2}} \cos x \, dx + \int_{\frac{\pi}{2}}^{\pi} -\cos x \, dx$

$\left(\sin x \Big|_0^{\pi} \right) + \left(\sin x \Big|_0^{\frac{\pi}{2}} \right) + \left(-\sin \Big|_{\frac{\pi}{2}}^{\pi} \right)$

$(0-0) + (1-0) + (-0 - -1)$

$1 + 1 = 2$

Q5 (a) $\int \frac{dx}{\sqrt{x}(1+\sqrt{x})^2}$

$\sqrt{x} = y$

$dy = \frac{1}{2\sqrt{x}} dx$

$\boxed{dx = dy \, 2\sqrt{x}}$ *

$= \int \frac{dy \, 2\sqrt{x}}{y(1+y)^2} = \int \frac{2\cancel{y}\, dy}{\cancel{y}(1+y)^2}$

$= \int 2(1+y)^{-2} \cdot dy = \frac{2(1+y)^{-3}}{-3} + c$

$= \frac{2(1+\sqrt{x})^{-3}}{-3} + c$

(b) $\int \frac{\sec z \tan z}{\sqrt{\sec z}} \cdot dz$

$y = \sec z$

$dy = \sec z \tan z \, dz$

$\int \frac{\cancel{\sec z \tan z}}{\sqrt{y}} \cdot \frac{dy}{\cancel{\sec z \tan z}} = \int dy \, (y)^{-\frac{1}{2}}$

$\frac{y^{\frac{1}{2}}}{\frac{1}{2}} + c = 2\sqrt{y} + c = 2\sqrt{\sec z} + c$

(c) $\int \sqrt{\left(\frac{x-1}{x^5}\right)} \cdot dx = \int \sqrt{\frac{x}{x^5} - \frac{1}{x^5}} \cdot dx$

$= \int \sqrt{\frac{1}{x^4} - \frac{1}{x^5}} \cdot dx = \int \sqrt{\frac{1}{x^4}\left(1 - \frac{1}{x}\right)} \cdot dx$

$= \int \frac{1}{x^2} \sqrt{1 - \frac{1}{x}} \cdot dx$

$y = \frac{1}{x}$

$dy = \frac{-1}{x^2} dx$

$= \int \frac{1}{\cancel{x^2}} \sqrt{1-y} \cdot -\cancel{x^2} \, dy = \int -\sqrt{1-y} \, dy$

$-\frac{2}{3}(1-y)^{\frac{3}{2}} + c = -\frac{2}{3}\left(1 - \frac{1}{x}\right)^{\frac{3}{2}} + c$

(d) $\int x^3 \sqrt{x^2+1}\, dx$ $\qquad y = x^2+1$

$dy = 2x\, dx$

$\int x^3 \sqrt{y}\ .\frac{dy}{2x}$

$\frac{1}{2}\int x^2 \sqrt{y}\ .dy$

$\frac{1}{2}\int (y-1)\sqrt{y}\ .dy = \frac{1}{2}\int \left(y^{3/2} - y^{1/2}\right).dy$

$= \frac{1}{2} \times \frac{2}{5}\, y^{5/2} - \frac{2\times 3}{2} y^{3/2} + C$

$\frac{1}{5} y^{5/2} - 3y^{3/2} + C$

$\frac{1}{5}(x^2+1)^{5/2} - 3(x^2+1)^{3/2} + C$

Q6 (a) $y = x^2 - 2x$, $y = x$

$x = x^2 - 2x$

$\rightarrow 0 = x^2 - 3x$

$0 = x(x-3)$

$x = 0, 3$

$\int_0^3 \left(x - (x^2 - 2x)\right).dx$

$\int_0^3 (x - x^2 + 2x)\, dx = \int_0^3 (3x - x^2).dx$

$\frac{3x^2}{2} - \frac{x^3}{3}\Big|_0^3 = \left(\frac{3\times 9}{2} - \frac{27}{3}\right) - (0-0)$

$\frac{27}{2} - \frac{27}{3} = \frac{27}{6}$

(b) $y = x^2 \quad , \quad y = -x^2 + 4x$

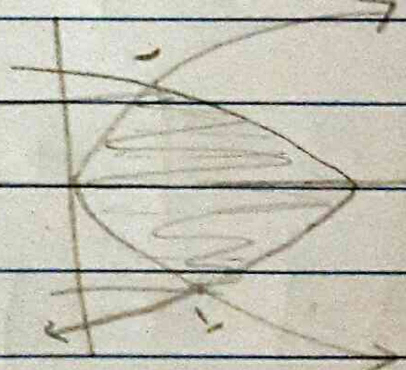
$$x^2 = -x^2 + 4x \Rightarrow 2x^2 - 4x = 0$$
$$x^2 - 2x = 0$$
$$x(x-2) = 0$$
$$x = 0 \ , \ x = 2$$

$$\int_0^2 \left(-x^2 + 4x - x^2\right) dx$$

$$= \int_0^2 -2x^2 + 4x \ . dx = \frac{-2x^3}{3} + \frac{4x^2}{2} \Big|_0^2$$

$$= -\frac{2}{3} * 8 + 2 \times 4$$
$$= \frac{-16}{3} + \frac{8*3}{3} = \frac{8}{3}$$

(c) $x = y^2 \quad , \quad x = 3 - 2y^2$



$$y^2 = 3 - 2y^2$$
$$0 = 3 - 3y^2$$
$$y = \pm 1$$

$$\int_{-1}^{1} 3 - 2y^2 - y^2 \ . dy = \int_{-1}^{1} \left(3 - 3y^2\right) . dy$$

$$3y - y^3 \Big|_{-1}^{1}$$

$$= (3-1) - (-3+1)$$
$$2 - -2 = 4$$