A.I. Complex number

Q2 : a. $(3+4i)^2 - 2(x-iy) = x+iy$

$9 + 24i - 16 \cancel{-2x} + \cancel{2iy} = x + iy$
$\cancel{+2x} \quad \cancel{-2iy} \quad +2x \quad -2iy$

$-7 + 24i = 3x - iy$

$3x = -7 \Rightarrow x = \frac{-7}{3}$

$-\cancel{i}y = 24\cancel{i} \Rightarrow y = -24$

b. $\left(\frac{1+i}{1-i}\right)^2 + \frac{1}{x+iy} = 1+i$

$\left(\frac{(1+i)(1+i)}{(1-i)(1+i)}\right)^2 + \frac{1(x-iy)}{(x+iy)(x-iy)} = 1+i$

$\left(\frac{\cancel{1}+2i-\cancel{1}}{1+1}\right)^2 + \frac{x-iy}{x^2+y^2} = 1+i$

$(i)^2 + \frac{x-iy}{x^2+y^2} = 1+i$

$-1 + \frac{x}{x^2+y^2} - \frac{iy}{x^2+y^2} = 1+i$

$-1 + \frac{x}{x^2+y^2} = 1$ — ① $\quad x = 2(x^2+y^2)$ — ①

$-\frac{iy}{x^2+y^2} = i$ — ② $\quad y = -(x^2+y^2)$ — ②

$\frac{①}{②} \Rightarrow \frac{x}{y} = -2 \Rightarrow x = -2y$

$-2y = 2((-2y)^2 + y^2) \Rightarrow -\cancel{2}y = \cancel{2}(4y^2+y^2)$

$-y = 5y^2 \Rightarrow 5y^2 + y = 0 \Rightarrow y(5y+1) = 0$

$y = 0 \quad , \quad y = \frac{-1}{5}$

$x = -2 \times -\frac{1}{5} = \frac{2}{5}$

(c) $(3-2i)(x+iy) = 2(x-2iy)+2i-1$

$3x + \cancel{3iy} - 2ix + 2y = \cancel{2x} - 4iy + 2i - 1$

$-2x \quad -3iy \qquad -2x \quad -3iy$

$x - 2ix + 2y = -7iy + 2i - 1$

$+7iy \qquad +7iy$

$x - 2ix + 2y + 7iy = 2i - 1$

$7iy - 2ix = 2i$ — ① $\quad 7y - 2x = 2$ — ①

$x + 2y = -1$ — ② $\quad x + 2y = -1$ — ②

$7y - 2x = 2$

$4y + 2x = -2 \quad +$

$11y = 0$

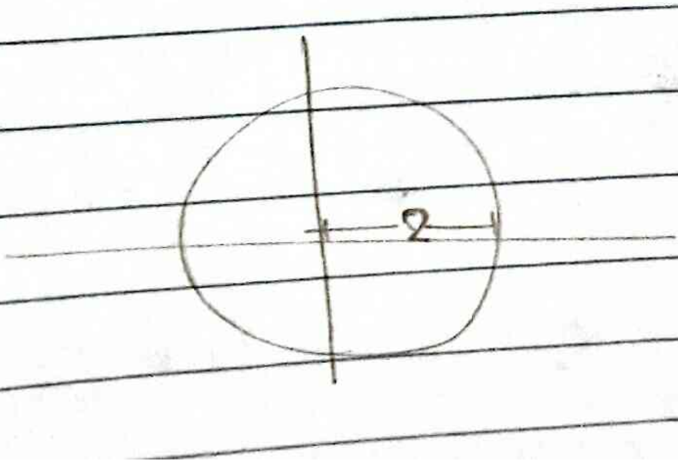
$\boxed{y = \text{zero}}$

$x + 2\times 0 = -1 \Rightarrow \boxed{x = -1}$

Q5: a. $|z| = 2 \qquad r = 2$

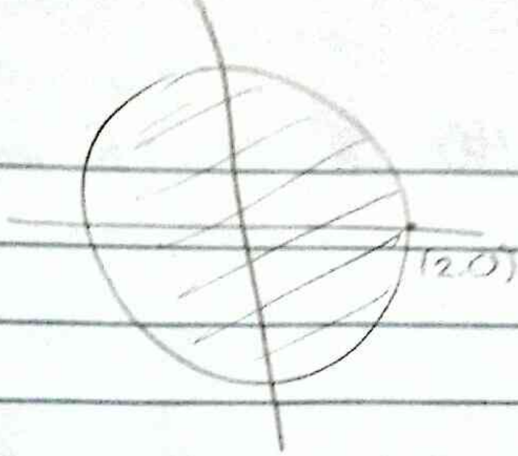
$|x+iy| = 2$

$\sqrt{x^2+y^2} = 2 \Rightarrow 4 = x^2 + y^2$



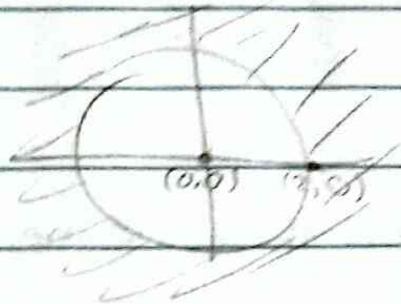
b. $|z| < 2$

$x^2 + y^2 < 4$



c. $|z| > 2 \implies$

$x^2 + y^2 > 4$

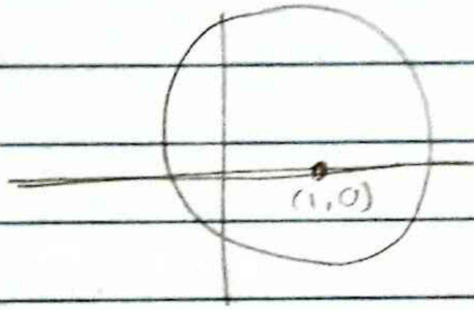


6. $|z-1| = 2$

$|x - 1 + iy| = 2$

$\sqrt{(x-1)^2 + y^2} = 2$

$(x-1)^2 + y^2 = 4$



8: $|z+1| = |z-1|$

$|x+1+iy| = |x-1+iy|$

$\sqrt{(x+1)^2 + y^2} = \sqrt{(x-1)^2 + y^2}$

$(x-1)^2 + \cancel{y^2} = (x+1)^2 + \cancel{y^2}$

$(x-1)^2 = (x+1)^2$

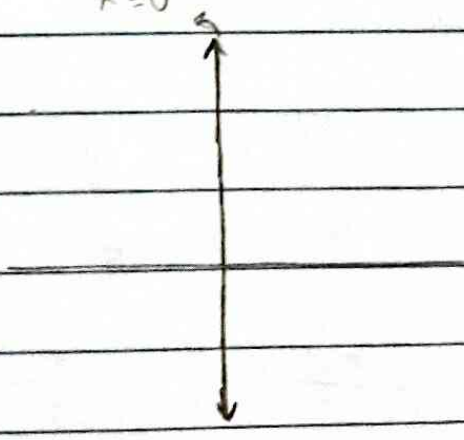
$\cancel{x^2} - 2x + \cancel{1} = \cancel{x^2} + 2x + \cancel{1}$

$-2x = 2x$

$4x = 0$

$x = 0$

$x = 0 \implies$ y-axis



Q15: $\sin 4\theta$, $\cos 4\theta$.

$$\cos 4\theta + i\sin 4\theta = (\cos\theta + i\sin\theta)^4$$ Demaviors

$$(\cos^2\theta + 2i\cos\theta\sin\theta - \sin^2\theta)(\cos^2\theta + 2i\cos\theta\sin\theta - \sin^2\theta)$$

$$= \cos^4\theta + 2i\sin\theta\cos^3\theta - \boxed{\cos^2\theta\sin^2\theta} + 2i\sin\theta\cos^3\theta - \boxed{4\cos^2\theta\sin^2\theta} - 2i\cos\theta\sin^3\theta$$

$$- \boxed{\sin^2\theta\cos^2\theta} - 2i\cos\theta\sin^3\theta + \sin^4\theta$$

$$= \cos^4\theta + 4i\sin\theta\cos^3\theta - 6\cos^2\theta\sin^2\theta - 4i\cos\theta\sin^3\theta + \sin^4\theta$$

$$= \cos^4\theta + \sin^4\theta - 6\cos^2\theta\sin^2\theta + 4i\sin\theta\cos^3\theta - 4i\cos\theta\sin^3\theta$$

$$= \cos^4\theta + \sin^4\theta - 6\cos^2\theta\sin^2\theta + i(4\sin\theta\cos^3\theta - 4\cos\theta\sin^3\theta)$$

$$\cos 4\theta = \cos^4\theta + \sin^4\theta - 6\cos^2\theta\sin^2\theta$$

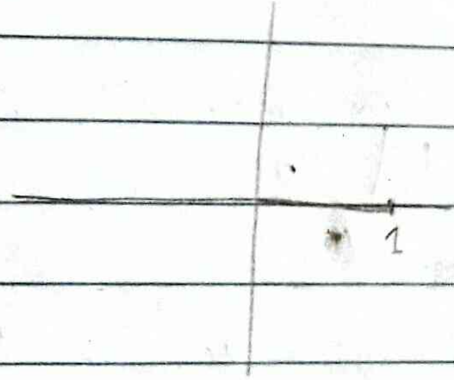
$$\sin 4\theta = 4(\sin\theta\cos^3\theta - \cos\theta\sin^3\theta)$$

17. find the three cube roots of 1

$\sqrt[3]{1}$

$z = 1 = re^{i\theta}$

$r = |1| = 1$



$\theta = 0 + 2\pi k \quad , \quad k = 0, \pm 1, \pm 2, \dots$

$z = 1 \, e^{i(\theta + 2\pi k)}$

$z^{\frac{1}{3}} = 1^{\frac{1}{3}} \, e^{i(\theta + 2\pi k)\frac{1}{3}}$

$\sqrt[3]{z} = 1 \, e^{i\frac{2\pi k}{3}}$

$k = 0 \; , \quad w_1 = 1e^{i\frac{2\pi \times 0}{3}} = 1e^{0} = 1$

$k = 1 \; , \quad w_2 = 1e^{i\frac{2\pi \times 1}{3}} = 1e^{i\frac{2\pi}{3}} = 1\left[\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right]$

$= -\frac{1}{2} + \frac{\sqrt{3}}{2} i$

$k = 2 \; , \quad w_3 = 1e^{i\frac{2\pi \times 2}{3}} = 1e^{i\frac{4\pi}{3}} = 1\left[\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right]$

$= -\frac{1}{2} - \frac{\sqrt{3}}{2} i$

18. Find the two square roots of i.

$\sqrt[2]{-1}$

$z = -1 = re^{i\theta}$

$r = |-1| = 1$

$\theta = 180 + 2\pi k, \; k = 0, +1$

1^{-}

$z = 1 = re^{i\theta}$

$r = |-1| = 1$

$\theta = 2\pi + 2\pi k, \; k = 0, +1$

$z = 1 e^{i(2\pi + 2k\pi)}$

$z^{1/2} = 1^{1/2} e^{i(\pi + 2\pi k) \times \frac{1}{2}}$

$k = 0, \; w_1 = 1 e^{i(\pi)} = 1 [\cos \pi + i \sin \pi] = 1 [-1 + i \times 0] = -1$

$k = 1, \; w_2 = 1 e^{2\pi} = 1 [\cos 2\pi + i \sin 2\pi] = 1$

Q20 find the Sixth roots of 64.

$Z = 64 \Rightarrow$

$|Z| = 64$

$r = 64$ $\quad \theta + 2k\pi$

$\theta = 0 \Rightarrow 0 + 2\pi k$

$x + iy \Rightarrow \overset{x}{64} + i\overset{y}{0}$

$\tan\theta = \frac{0}{64}$

$\tan\theta = 0$

$\theta = 0$

$\sqrt[6]{Z} = \sqrt[6]{64} \exp i(0 + 2\pi k)$

$\sqrt[6]{Z} = 2 \exp\left(\frac{i\pi k}{3}\right)$

$w_1 \Rightarrow k=0 \Rightarrow 2\exp i0 \Leftrightarrow 2[\cos 0 + i\sin 0] = 2[1+0] = 2$

$w_2 \Rightarrow k=1 \Rightarrow 2\exp i\frac{\pi}{3} \Rightarrow 2[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}] = 2[\frac{1}{2} + i\frac{\sqrt{3}}{2}] = 1 + i\sqrt{3}$

$w_3 \Rightarrow k=2 \Rightarrow 2\exp i\frac{\pi 2}{3} \Rightarrow 2[\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}] = 2[\frac{-1}{2} + i\frac{\sqrt{3}}{2}] = -1 + i\sqrt{3}$

$w_4 \Rightarrow k=3 \Rightarrow 2\exp i\pi \Rightarrow 2[\cos\pi + i\sin\pi] = 2[-1 + i0] = -2$

$w_5 \Rightarrow k=4 \Rightarrow 2\exp i\frac{4\pi}{3} \Rightarrow 2[\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}] = 2[\frac{-1}{2} + -i\frac{\sqrt{3}}{2}] = -1 - i\sqrt{3}$

$w_6 \Rightarrow k=5 \Rightarrow 2\exp i\frac{\pi 5}{3} \Rightarrow 2[\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}] = 2[\frac{1}{2} - i\frac{\sqrt{3}}{2}] = 1 - i\sqrt{3}$

Q21 $\quad z^4 - 2z^2 + 4 = 0 \quad$ Let $x = z^2$

$$x^2 - 2x + 4 = 0$$

$$x = \frac{--2 \pm \sqrt{4 - 4 \times 1 \times 4}}{2} = \frac{2 \pm \sqrt{4-16}}{2}$$

$$x = \frac{2 \pm \sqrt{-12}}{2} \Rightarrow x = 1 \mp \sqrt{-3}$$

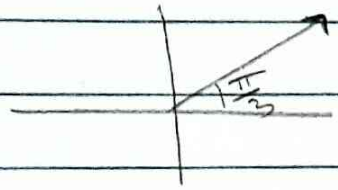
$$x = 1 + \sqrt{-3}, \quad 1 - \sqrt{-3}$$

$$x = 1 + i\sqrt{3}, \quad 1 - i\sqrt{3}$$

$$z^2 = 1 + i\sqrt{3}, \quad 1 - i\sqrt{3}$$

$$|r| = \sqrt{1^2 + \sqrt{3}^2} = 2$$

$$\tan\theta = \frac{\sqrt{3}}{1} \Rightarrow \theta = \frac{\pi}{3} + 2k\pi$$



$$\sqrt{z} = \sqrt{2} \quad \exp i\left(\frac{\pi}{6} + k\pi\right)$$

$$w_1 \Rightarrow k = 0 \Rightarrow \sqrt{2} \quad \exp i \frac{\pi}{6}$$

$$w_1 = \sqrt{2}\left[\cos\frac{\pi}{6} + i \sin\frac{\pi}{6}\right]$$

$$= \sqrt{2}\left[\frac{\sqrt{3}}{2} + \frac{i}{2}\right]$$

$$= \frac{\sqrt{6}}{2} + \frac{i}{\sqrt{2}}$$

$$w_2 \Rightarrow k = 1 \Rightarrow \sqrt{2} \quad \exp i \frac{7\pi}{6}$$

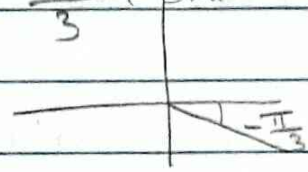
$$= \sqrt{2}\left[\cos\frac{7\pi}{6} + i \sin\frac{7\pi}{6}\right]$$

$$\sqrt{2}\left[\frac{-\sqrt{3}}{2} + \frac{i}{2}\right]$$

$$= -\frac{\sqrt{6}}{2} - \frac{i}{\sqrt{2}}$$

$$|r| = 2$$

$$\theta = \frac{-\pi}{3} + 2k\pi$$



$$\sqrt{z} = \sqrt{2} \quad \exp i\left(\frac{-\pi}{6} + k\pi\right)$$

$$w_1 \Rightarrow k = 0 \Rightarrow \sqrt{2} \exp i \frac{-\pi}{6}$$

$$= \sqrt{2}\left[\cos\frac{-\pi}{6} + i \sin\frac{-\pi}{6}\right]$$

$$= \sqrt{2}\left[\frac{\sqrt{3}}{2} - \frac{i}{2}\right]$$

$$= \frac{\sqrt{6}}{2} - \frac{i}{\sqrt{2}}$$

$$w_2 \Rightarrow k = 1 \Rightarrow \sqrt{2} \quad \exp i \frac{5\pi}{6}$$

$$= \sqrt{2}\left[\cos\frac{+5\pi}{6} + i \sin\frac{+5\pi}{6}\right]$$

$$= \sqrt{2}\left[\frac{-\sqrt{3}}{2} + \frac{i}{2}\right]$$

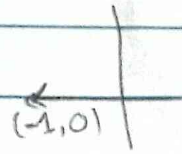
$$= \frac{-\sqrt{6}}{2} + \frac{i}{\sqrt{2}}$$

Q 24) $x^4 + 1 = 0 \Rightarrow x^4 = -1$

$|x| = 1$

$\theta = \pi + 2k\pi$

$\sqrt[4]{x^4 = \exp i(\pi + 2k\pi)}$



$\therefore x = \exp i \dfrac{(\pi + 2k\pi)}{4} = \exp i \left(\dfrac{\pi}{4} + \dfrac{k\pi}{2}\right)$

$w_1 \Rightarrow k=0 \Rightarrow \exp i \dfrac{\pi}{4} \Rightarrow \left[\cos\dfrac{\pi}{4} + i\sin\dfrac{\pi}{4}\right] = \dfrac{1}{\sqrt{2}} + i\dfrac{1}{\sqrt{2}}$

$w_2 \Rightarrow k=1 \Rightarrow \exp i\left(\underbrace{\dfrac{\pi}{4} + \dfrac{\pi}{2}}_{\frac{3\pi}{4}}\right) \Rightarrow \left[\cos\dfrac{3\pi}{4} + i\sin\dfrac{3\pi}{4}\right] = -\dfrac{1}{\sqrt{2}} + i\dfrac{1}{\sqrt{2}}$

$w_3 \Rightarrow k=2 \Rightarrow \exp i\left(\dfrac{5\pi}{4}\right) \Rightarrow \left[\cos\dfrac{5\pi}{4} + i\sin\dfrac{5\pi}{4}\right] = -\dfrac{1}{\sqrt{2}} - i\dfrac{1}{\sqrt{2}}$

$w_4 \Rightarrow k=3 \Rightarrow \exp i\left(\dfrac{7\pi}{4}\right) \Rightarrow \left[\cos\dfrac{7\pi}{4} + i\sin\dfrac{7\pi}{4}\right] = \dfrac{1}{\sqrt{2}} - i\dfrac{1}{\sqrt{2}}$

Chapter 1 (Function)

1.4 Exercises:

(1) (a) $f(x) = \frac{1}{\sqrt{x}}$ $D = (0, \infty)$ $R = (0, \infty)$

(2) (b) $f(x) = \tan \pi x$ $D = \mathbb{R} \setminus x = \frac{1}{2} + n$, $n = 0, \pm 1, \ldots$

$R = \mathbb{R}$

* $\tan \pi x = \frac{\sin \pi x}{\cos \pi x}$

$\cos \pi x \neq 0$

$\pi x \neq \frac{\pi}{2} \Rightarrow x \neq \frac{1}{2} + n$

(c) $f(x) = 1 + |x|$ $D = \mathbb{R}$

$R = [1, \infty)$

(d) $f(x) = \sec^2 x$ $D = \mathbb{R} / \{x = \frac{1}{2} + n$, $n = 0, \pm 1, \ldots\}$

$R = [1, \infty)$

* $\sec x = \frac{1}{\cos x}$

$\cos x \neq 0$

$x = \frac{\pi}{2} + n\pi$

(e) $f(x) = \frac{1}{x^2}$ $D = \mathbb{R} \setminus 0$

$R = (0, \infty)$

(f) $f(x) = \frac{1}{\sqrt{1 - x^2}}$ $D =]-1, 1[$

$R = (1, \infty)$

$1 - x^2 > 0$

$1 > x^2$

$1 > x > 1-$

* Note:- $g = \cot (\pi x + \frac{\pi}{3})$ $D = \mathbb{R} \setminus \{x = \frac{-1}{3} + n\}$ $n = 0$

$\Rightarrow \cot (\pi x + \frac{\pi}{3}) = \frac{\cos (\pi x + \frac{\pi}{3})}{\sin (\pi x + \frac{\pi}{3})}$

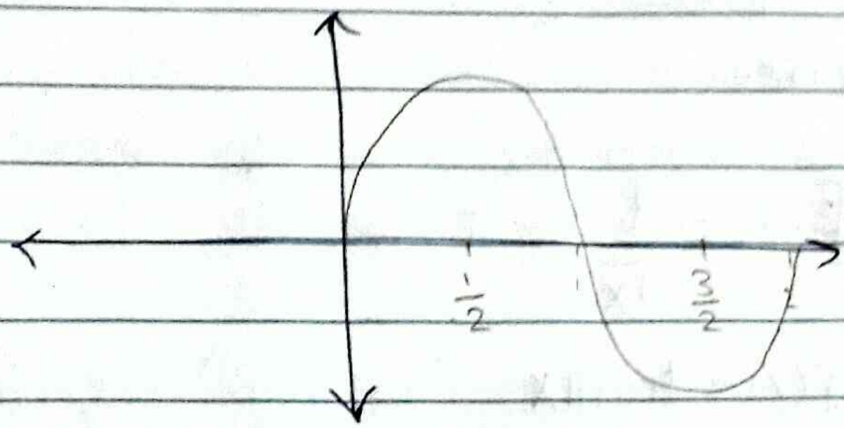
$\sin (\pi x + \frac{\pi}{3}) \neq 0$

$\pi y + \frac{\pi}{3} = 0$

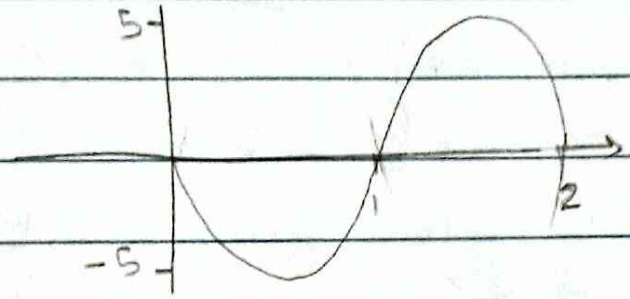
$\pi y = -\frac{\pi}{3}$

$x = -\frac{1}{3}$

(2) (a) $y = \sin(\pi x)$



extra * $y = -5\sin(\pi x)$

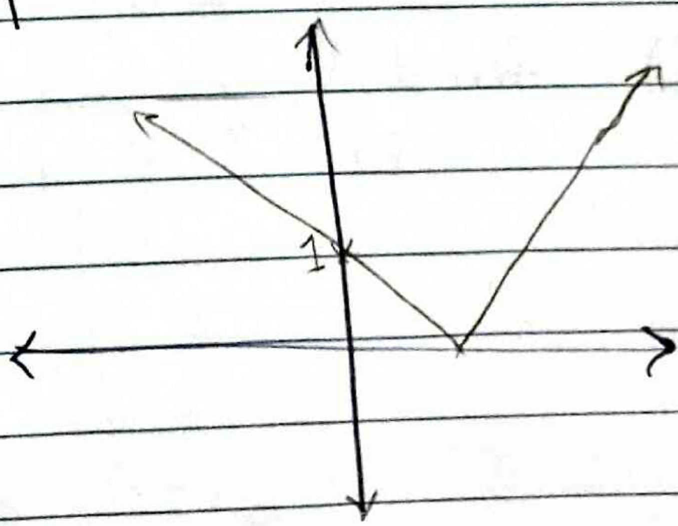


* $y = a\sin(bx)$ or $y = a\cos(bx)$

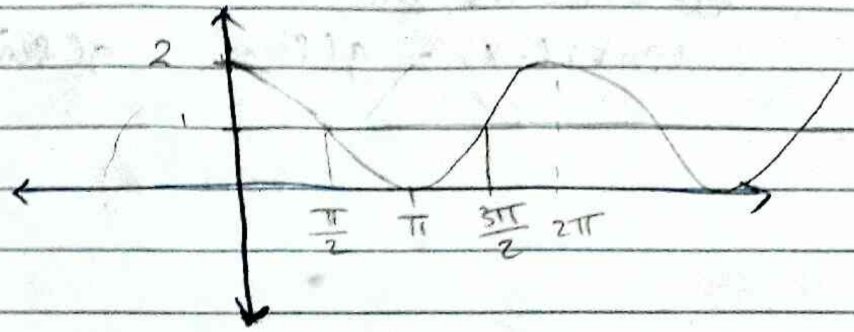
period $= p = \frac{2\pi}{|b|}$

Range $= [-|a|, |a|]$.

(b) $y = |x-1|$



(c) $y = \cos(\pi) + 1$



(3) (a) $f(x) = x^2 + 1$

$f(-x) = (-x)^2 + 1 = x^2 + 1 = f(x)$ even

(b) $f(x) = x^3 + x$

$f(-x) = (-x)^3 - x = -(x^3 + x) = -f(x)$

odd

(c) $g(t) = \frac{1}{t-1}$

$g(-t) = \frac{1}{-t-1}$ neither odd nor even

(d) $h(x) = \frac{x}{x^2 - 1}$

$h(-x) = \frac{-x}{(-x)^2 - 1} = \frac{-x}{x^2 - 1} = -f(x)$ odd

(4) (a) $(\overset{\text{even}}{f} \circ \overset{\text{odd}}{g})(x)$

$f(g(-x)) = f(-g(x)) = f(g(x)) = (f \circ g)(x)$

even

(b) $h(x) = \frac{\overset{\text{even}}{f(x)}}{\underset{\text{odd}}{g(x)}}$

$h(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -h(x)$ odd

Q 4) a) If $f(x)$ is even and $g(x)$ is odd then: $(g \circ f)(x)$ is even.

$f(x)$ even
$f(x) = f(-x)$
$g(x)$ odd
$f(-x) = -f(x)$

$$(g \circ f)(-x) = g(f(-x)) = g(f(x)) = (g \circ f)(x)$$

b) If $f(x)$ even and $g(x)$ odd then $\frac{f(x)}{g(x)}$ is odd.

$f(x)$ even $\rightarrow$ $f(x) = f(-x)$

$g(x)$ odd $\rightarrow$ $g(x) = -g(-x)$

$$R(x) = \frac{f(x)}{g(x)}$$

$$R(-x) = \frac{f(-x) \rightarrow \text{even}}{g(-x) \rightarrow \text{odd}} = \frac{f(x)}{-g(x)} = -R(x)$$

odd

Chapter 2:

1) (a) $\lim_{t\to -1} \frac{t^2+3t+2}{t^2-t-2} = \lim \frac{(t+2)(t+1)}{(t+1)(t-2)} = \frac{1}{-3}$

(b) $\lim_{x\to 1} \frac{1-\sqrt{x}}{1-x} = \lim \frac{1-\sqrt{x}}{(1-\sqrt{x})(1+\sqrt{x})} = \frac{1}{2}$

(c) $\lim_{\theta\to 1} \frac{\theta^4-1}{\theta^3-1} = \lim_{\theta\to 1} \frac{4\theta^3}{3\theta^2} = \frac{4}{3}$

(d) $\lim_{\theta\to 0} \frac{2}{2}\times\frac{\sin 2\theta}{3\theta} = \frac{2}{3}$

(e) $\lim_{\theta\to 0} \frac{1-\cos\theta}{\sin 2\theta}\frac{(1+\cos\theta)}{(1+\cos\theta)} = \lim_{\theta\to 0} \frac{1^2-\cos^2\theta}{2\sin\theta\underbrace{\cos\theta}_{\to 1}}$

$= \lim_{\theta\to 0} \frac{\sin^2\theta}{2\sin\theta} = 0$

(f) $\lim_{x\to\infty} \frac{1+\sqrt{x}}{1-\sqrt{x}} = \lim_{x\to\infty} \frac{\cancel{\sqrt{x}}\left(\frac{1}{\sqrt{x}}+1\right)}{\cancel{\sqrt{x}}\left(\frac{1}{\sqrt{x}}-1\right)} = -1$

(g) $\lim_{x\to -\infty} \frac{\overset{|x|}{\sqrt{x^2+1}}}{x+1} = \lim_{x\to-\infty} \frac{\overset{-x}{|x|}\sqrt{1+\frac{1}{x^2}}}{x\left(1+\frac{1}{x}\right)} = \frac{-\cancel{x}}{\cancel{x}}\frac{\left(\sqrt{1+\frac{1}{x^2}}\right)}{\left(1+\frac{1}{x}\right)} = -1$

(h) $\lim_{x\to\infty} \frac{\sqrt[3]{x}-\sqrt[5]{x}}{\sqrt[3]{x}+\sqrt[5]{x}} = 1$

Q (i) $\lim\limits_{x \to \infty} \left(\sqrt{x^2+1} - \sqrt{x^2-x}\right) * \dfrac{\left(\sqrt{x^2+1} + \sqrt{x^2-x}\right)}{\left(\sqrt{x^2+1} + \sqrt{x^2-x}\right)}$

$\lim\limits_{x \to \infty} \dfrac{x^2+1 - (x^2-x)}{\sqrt{x^2+1} + \sqrt{x^2-x}}$

$\lim\limits_{x \to \infty} \dfrac{\cancel{x^2} + x}{\sqrt{x^2+1} + \sqrt{x^2-x}} = \lim\limits_{x \to \infty} \dfrac{x\left(1 + \frac{1}{x}\right)}{|x|\left(\sqrt{1+\frac{1}{x^2}} + \sqrt{1-\frac{1}{x}}\right)}$

$= \dfrac{1}{1+1} = \dfrac{1}{2}$ //

(j) $\lim_{t \to 3^+} \frac{\lfloor t \rfloor}{t} = \lim_{t \to 3^+} \frac{3}{t} = 1$

(K) $\lim_{x \to 0} x \sin\left(\frac{1}{x}\right) = 0 \Rightarrow$ sandwich theorem

$$X * \left(-1 \leq \sin \frac{1}{X} \leq 1 \right)$$

$$\lim_{x \to 0} \left(-X \leq \left(\sin \frac{1}{x}\right) X \leq X \right)$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$0 \qquad 0 \qquad 0$$

Q2 find the asymptotes of the following functions then sketch their graphs

(a) $f(x) = \frac{x+1}{x-1}$, $x \neq 1$

$\lim_{x \to 1^+} f(x) = \left(\frac{+}{0^+}\right) = +\infty$

$x = 1$ V. asy

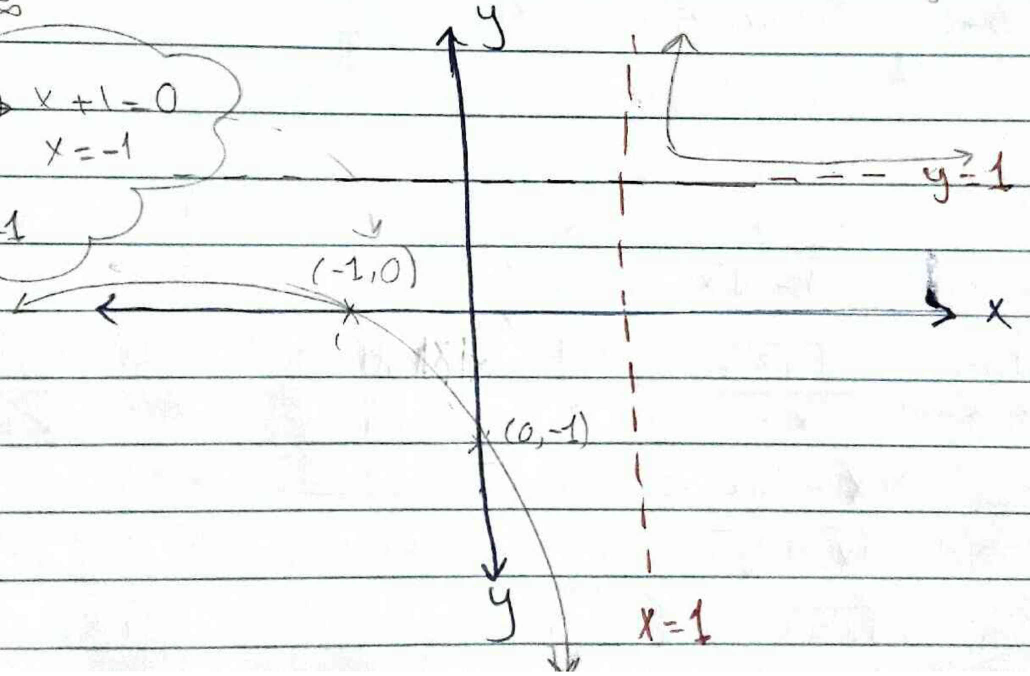
$\lim_{x \to 1^-} f(x) = \left(\frac{+}{0^-}\right) = -\infty$

$\lim_{x \to \pm\infty} f(x) = 1$ $y = 1$ H. Asy

$0 = \frac{x+1}{x-1} \Rightarrow x+1=0$

$x=-1$

$x=0 \Rightarrow y=-1$



(b) $y = \frac{x^3+1}{x^2}$ $\quad x \neq 0$

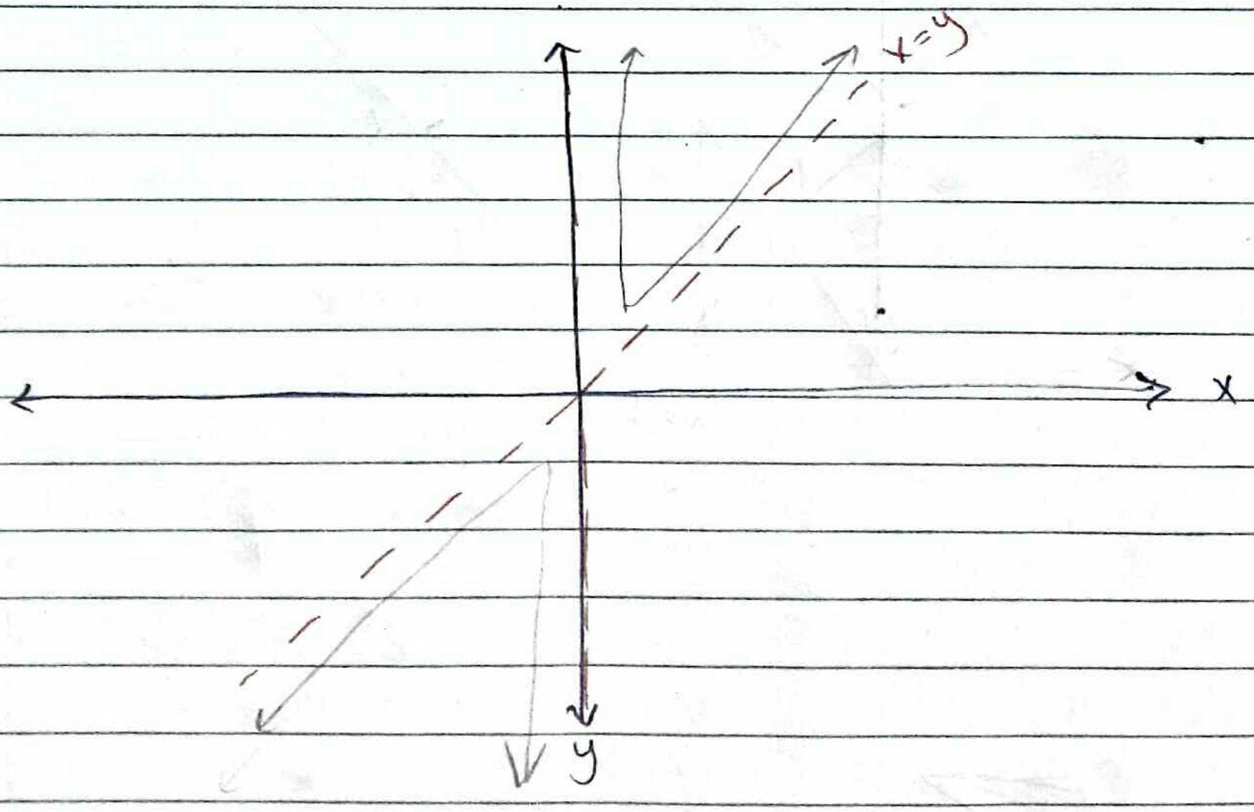
$$\begin{array}{r} x \\ x^2 \overline{\big) \, x^3 + 1} \\ \underline{-x^3 \quad} \\ 1 \end{array}$$

$y = x + \frac{1}{x^2}$ $\quad y = x$ oblique asy

$\lim_{x \to 0^+} f(x) = \infty$

$\lim_{x \to 0^-} f(x) = -\infty$ $\quad \underbrace{x = 0}_{y\text{-axis}}$ V. asy

No H. Asy



Q2) c) $f(x) = \frac{x^2+1}{x-1}$ $x \neq 1$

No. H.asyptots.

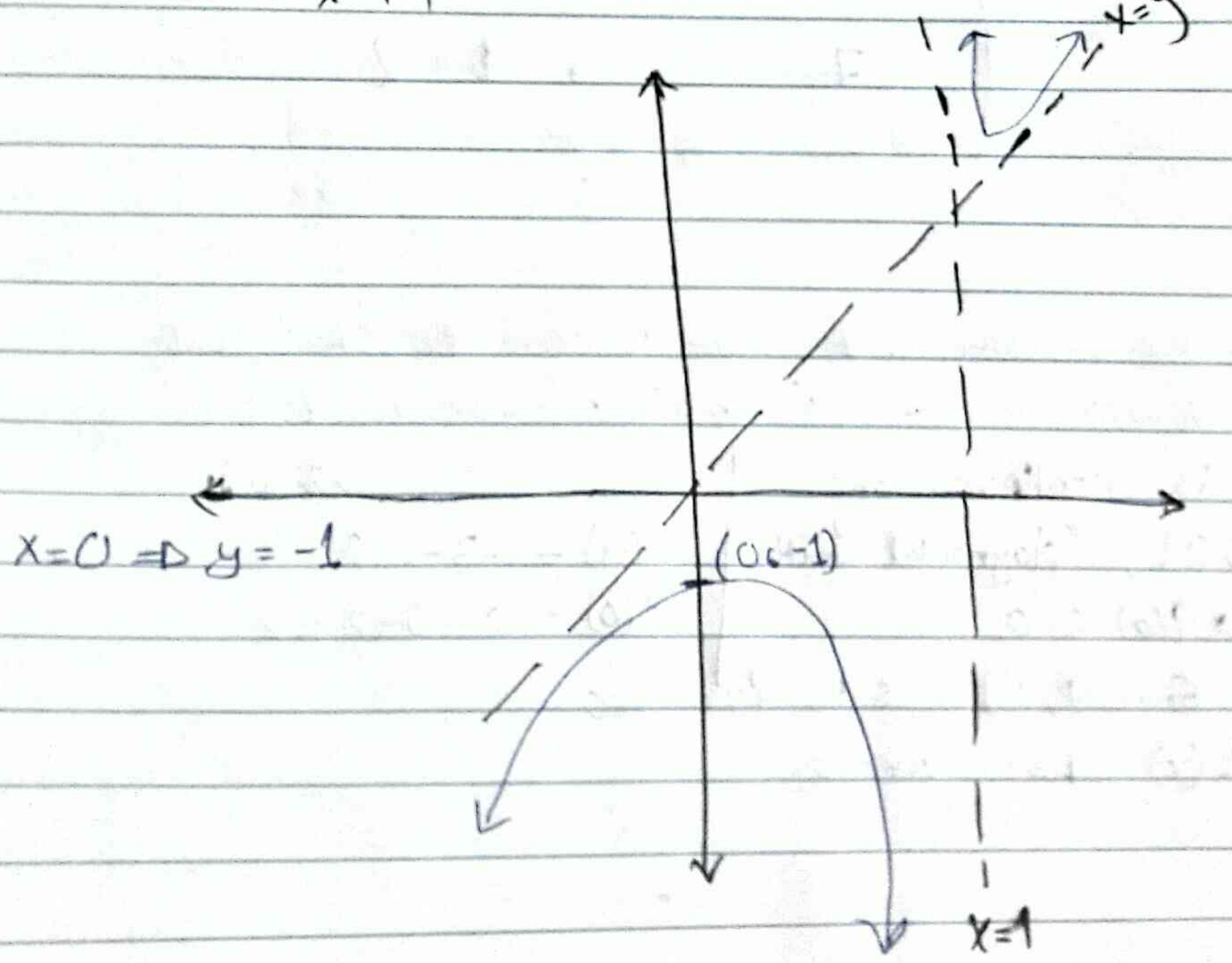
$\lim_{x \to 1^+} f(x) = \left(\frac{+}{0^+}\right) = +\infty$

$\lim_{x \to 1^-} f(x) = \left(\frac{+}{0^-}\right) = -\infty$

$\Rightarrow x = 1$ V.Asy

$$\begin{array}{r|l} & x \\ \hline x-1 & x^2+1 \\ & -x^2 \pm x \\ \hline & x+1 \end{array}$$

$\Rightarrow x = y$ obliq.asy



(d) $f(x) = \dfrac{x^3+1}{x^2-1}$ $\quad x \neq -1, 1$

$$\begin{array}{r|l} & x \\ x^2-1 & x^3+1 \\ & -x^3 \pm x \\ \hline & 1+x \end{array}$$

$\Rightarrow f(x) = x + \dfrac{1+x}{x^2-1}$

$f(x) = x + \dfrac{1+x}{(x+1)(x-1)}$

$\Rightarrow f(x) = x + \dfrac{1}{x-1}$

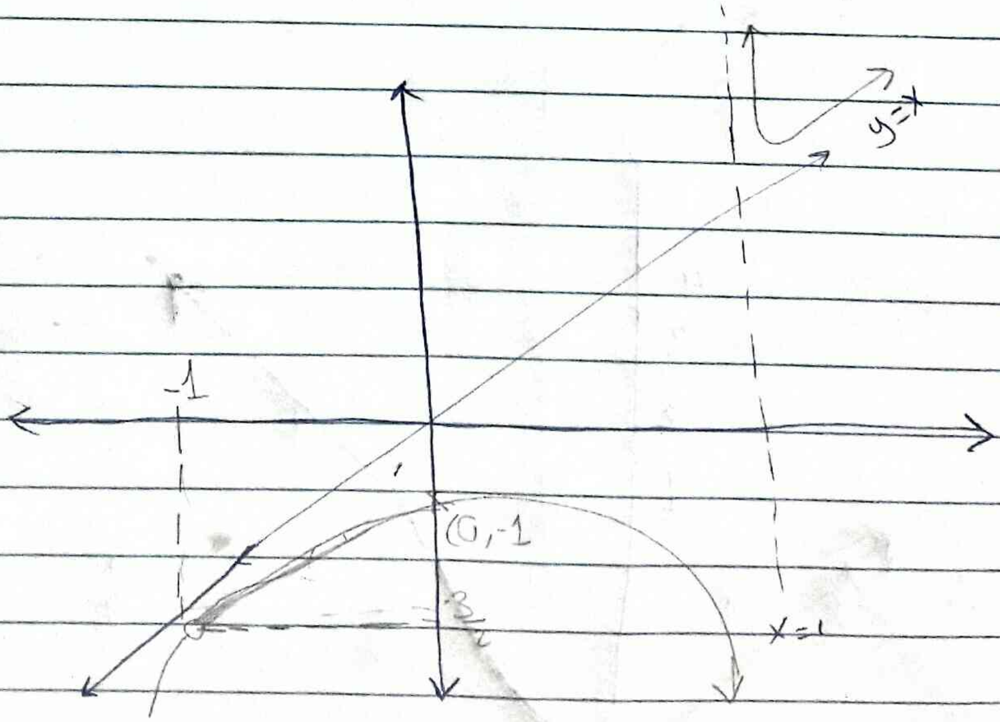
$\lim\limits_{x \to -1} f(x) = \lim\limits_{x \to -1} -1 + \dfrac{1}{-1-1} = \dfrac{-3}{2}$ No V. Asy

$\lim\limits_{x \to +1} f(x) = \lim\limits_{x \to +1^-} \left(\dfrac{+}{0^-}\right) = -\infty$

$\lim\limits_{x \to +1} f(x) = \lim\limits_{x \to 1^+} \left(\dfrac{+}{0^+}\right) = +\infty$

$(x = 1)$ V. Asy

$f(x) = x + \dfrac{1}{x-1} \Rightarrow x = y$ oblique



Q3.

$$g(x) = \begin{cases} ax + 2b, & x \leq 0 \\ x^2 + 3a - b, & 0 < x \leq 2 \\ 3x - 5, & x > 2 \end{cases}$$

$$\lim_{x \to 0^-} g(x) = \lim_{x \to 0^+} g(x)$$

$$\lim_{x \to 0} ax + 2b = \lim_{x \to 0^+} x^2 + 3a - b$$

$$2b = 3a - b \Rightarrow \boxed{b = a}$$

$$\lim_{x \to 2^-} g(x) = \lim_{x \to 2^+} g(x)$$

$$4 + 3a - b = 3 \times 2 - 5$$

$$4 + 2a = 6 - 5$$

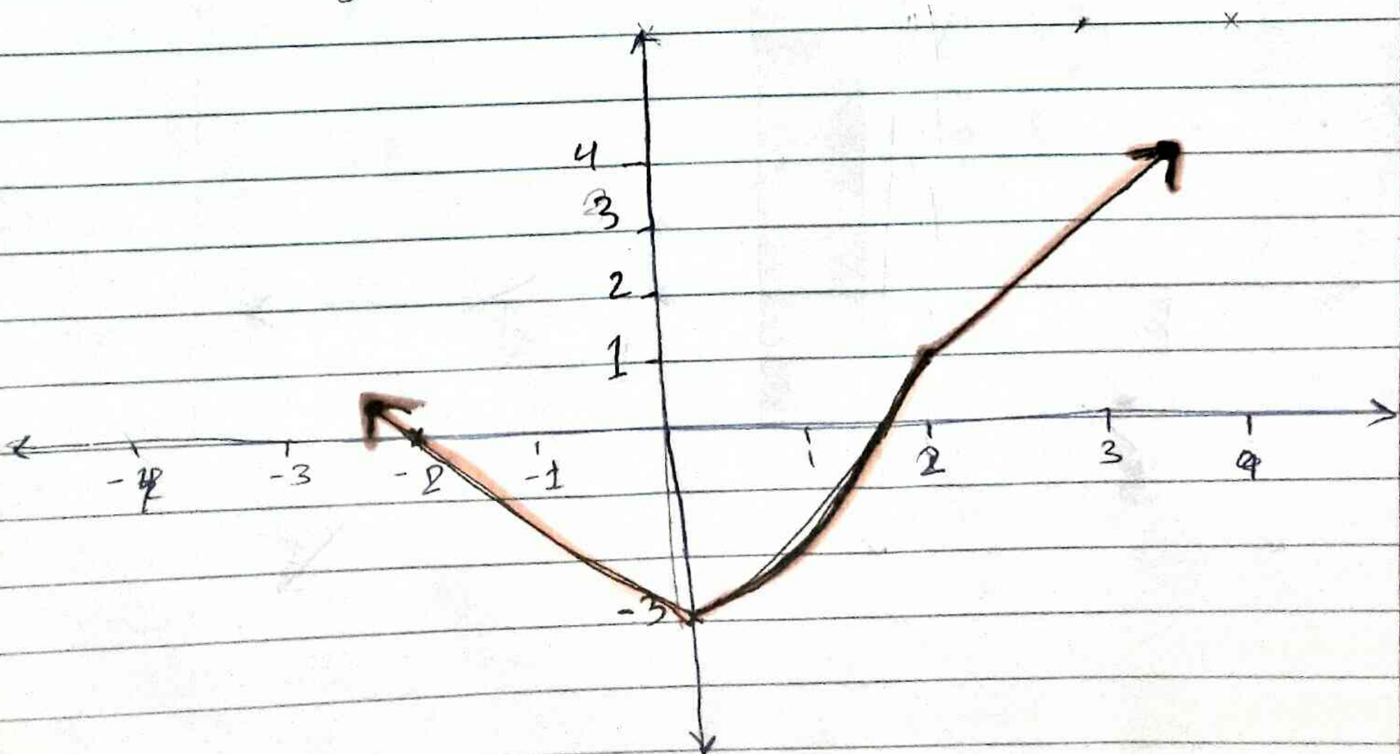
$$2a = 1 - 4$$

$$2a = 3 \Rightarrow a = \frac{-3}{2}$$

$$b = a = \frac{-3}{2}$$

$$g(x) = \begin{cases} \frac{3}{2}x + 3, & x \leq 0 \\ x^2 - 3, & 0 < x < 2 \\ 3x - 5, & x > 2 \end{cases}$$

$\frac{9}{2} - \frac{3}{2} = \frac{6}{2}$



Q4: Find the continuous extensions of the function $h(t) = \dfrac{t^2+3t-10}{t-2}$

$$\lim_{t \to 2} h(t) = \lim_{t \to 2} \frac{t^2+3t-10}{t-2}$$

$$= \lim_{t \to 2} \frac{\cancel{(t-2)}(t+5)}{\cancel{t-2}}$$

$$= \lim_{t \to 2} t+5 = 7$$

Extension function:-

$$H(t) = \begin{cases} \dfrac{t^2+3t-10}{t-2} & , t \neq 2 \\ 7 & , t = 2 \end{cases}$$

Q5: use the intermediate Value theorem to show that the function $f(x) = x^3 - 2x^2 + 2$ has a root.

→ $f(x)$ is continuous on $[-1, 0]$, "ploynomial function"

→ $f(-1) * f(0) < 0$

⇒ $\exists c \in [-1, 1]$ s.t $f(c) = 0$

⇒ $f(x)$ has a root

نعوضة

$f(-1) = -3-2+2 = -3$

$f(0) = 0 - 0 + 2 = 2$

Chapter 3:

1. find the derivatives of the following functions:

(a) $f(s) = \frac{\sqrt{s}-1}{\sqrt{s}+1} \Rightarrow f'(s) = \frac{\left((\sqrt{s}+1)(+\frac{1}{2\sqrt{s}})\right) - \left((\sqrt{s}-1)(\frac{1}{2\sqrt{s}})\right)}{(\sqrt{s}+1)^2}$

$$\frac{\cancel{\frac{\sqrt{s}}{2\sqrt{s}}} + \frac{1}{2\sqrt{s}} - \cancel{\frac{\sqrt{s}}{2\sqrt{s}}} + \frac{1}{2\sqrt{s}}}{(\sqrt{s})^2 + 2\sqrt{s} + 1}$$

$$= \frac{\frac{1}{\sqrt{s}}}{s^2 + 2\sqrt{s} + 1}$$

(b) $f(x) = \left(\frac{1}{x} - x\right)(x^2+1) \Rightarrow f(x) = x + \frac{1}{x} - x^3 - x$

$\Rightarrow f'(x) = \cancel{1} + \frac{-1}{x^2} - 3x^2 - \cancel{1}$

$\Rightarrow f'(x) = \frac{-1}{x^2} - 3x^2$

(c) $g(x) = \sec(2x+1)\cot(x^2)$

$\sec(2x+1) * (-\csc^2 x^2 * 2x) + \cot(x^2) * (\sec(2x+1)\tan(2x+1) * 2)$

(d) $S(t) = \frac{1 + \csc t}{1 - \csc t} = \frac{1 + \frac{1}{\sin t}}{1 - \frac{1}{\sin t}} = \frac{\frac{\sin + 1}{\cancel{\sin t}}}{\frac{\sin t - 1}{\cancel{\sin t}}}$

$= \left(\frac{\sin t + 1}{\sin t - 1}\right)$

$$\frac{(\sin t - 1)(\cos t) - (\sin t + 1)(\cos t)}{(\sin t - 1)^2}$$

$$\frac{\cancel{\sin t \cos t} - \cos t - \cancel{\sin t \cos t} - \cos t}{\sin t^2 - 2\sin t + 1}$$

$$\frac{-2\cos t}{\sin t^2 - 2\sin t + 1}$$

e) $f(x) = x^3 \sin x \cos x = x^3 \dfrac{\sin 2x}{2}$

$3x^2 \dfrac{\sin 2x}{2} + x^3 \cos 2x$

(f) $x^{1/2} + y^{1/2} = 1$

$$\frac{1}{2} x^{-\frac{1}{2}} + \frac{1}{2} y^{-\frac{1}{2}} y' = 0$$

$$\cancel{\tfrac{1}{2}} x^{-\frac{1}{2}} = -\cancel{\tfrac{1}{2}} y^{-\frac{1}{2}} y'$$

$$-\frac{x^{-\frac{1}{2}}}{y^{-\frac{1}{2}}} = y' \quad \Rightarrow \quad y' = -\left(\frac{y}{x}\right)^{\frac{1}{2}}$$

P_2 (i) $y = \cot^2 x$

$$\frac{dy}{dx} = 2 \cot x * -\csc^2 x$$

(ii) $x^2 + y^2 = x \Rightarrow y = \sqrt{x - x^2}$

$$\frac{\text{مشتقة ما بداخل الجذر}}{2 \times \text{الجذر}} \qquad \frac{dy}{dx} = \frac{1 - 2x}{2\sqrt{x - x^2}}$$

(iii) $y = \dfrac{\sin x}{1 - \cos x} \Rightarrow y' = \dfrac{(1 - \cos x)\cos x - (-\sin x)\sin x}{(1 - \cos x)^2}$

$$= \frac{\cos x - \cos^2 x - \sin^2 x}{(1 - \cos x)^2}$$

$$= \frac{\cos x - (\cos^2 x + \sin^2 x)}{(1 - \cos x)^2} = \frac{\cos x - 1}{(\cos x - 1)^2} = \frac{1}{\cos x - 1}$$

التبديل لا يؤثر لأنه مربع ✓

Q3 Find the points on the Curve $y = 2x^3 - 3x^2 - 12x + 20$, wher the tangent is parallel to the x-axis

$y' = 6x^2 - 6x - 12 \Rightarrow y' = 0$

$\frac{0 = 6x^2 - 6x - 12}{6} \Rightarrow 0 = x^2 - x - 2$

$(x+1)(x-2) = 0$

$x = -1 , 2$

$\downarrow \quad \downarrow$

$y = 27 , 0$

points ① $(-1, 27)$

② $(2, 0)$

Q4) for what values of the Constant a, if any, is

(i) continuous at $x = 0$?

(ii) differentiable at $x = 0$?

$$f(x) = \begin{cases} \sin(2x) & , x \leq 0 \\ ax & , x > 0 \end{cases}$$

(i) $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$

$\lim_{x \to 0} \sin 2x = \lim_{x \to 0} ax \Rightarrow a = \mathbb{R}$

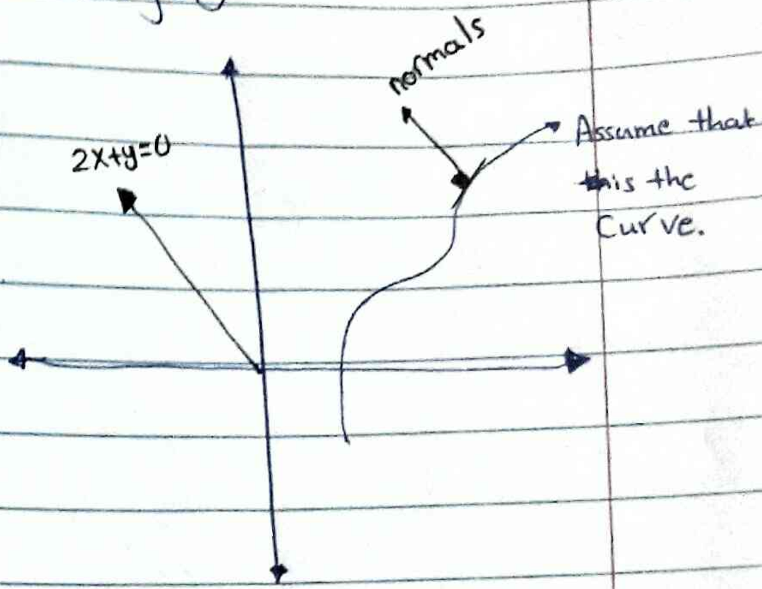
(ii) $(\sin(2x))' = 2\cos 2x$

$(ax)' = a$

$D^-\big|_{x=0} = D^+\big|_{x=0} \Rightarrow 2\cos 2x\big|_{x=0} = a\big|_{x=0} \Rightarrow 2 * \cos 0 = a$

$\boxed{2 = a}$

Q5) Find the normals to the curve $xy+2x-y=0$ that are parallel to the line $2x+y=0$



1. the normals // $2x+y=0$

$m(\text{normals}) = m(-2x+y)$

$m(\text{normals}) = -2$

2. $m(\text{normals}) * y'(\text{curve}) = -1$

$-2 * y' = -1$

$y' = \frac{1}{2}$

3. $xy+2x-y=0 \rightarrow y = \frac{-2x}{x-1} \rightarrow y' = \frac{(x-1)*-2 - -2x}{(x-1)^2}$

$y' = \frac{+2}{(x-1)^2} \Rightarrow * \; y' = \frac{1}{2}$

$\Rightarrow \frac{1}{2} = \frac{+2}{(x-1)^2}$

$\Rightarrow \underset{-4}{+4} = x^2 - 2x + \underset{-4}{1}$

$\Rightarrow x^2 - 2x + 3 = 0 \Rightarrow (x+1)(x-3) = 0$

$x = -1, \; 3$

$y = +1 \qquad -3$

two normals $\Rightarrow$ ① $y+3 = -2(x-3)$ ② $y+1 = -2(x+1)$

Q6 :- (a) $f(x) = \tan x$, $x = \pi/4$

$a = \boxed{\frac{\pi}{4}}$ / $f(\frac{\pi}{4}) = \tan\frac{\pi}{4} = \boxed{1}$ / $f'(\frac{\pi}{4}) = \sec^2\frac{\pi}{4} = \boxed{2}$

$L(x) = f(a) + f'(a)(x - \frac{\pi}{4})$

$L(x) = 1 + 2(x - \frac{\pi}{4})$

$= 1 + 2x - \frac{\pi}{2}$

(b) $g(x) = \frac{1}{x}$, $x = 1$

$a = \boxed{1}$ / $g(1) = \boxed{1}$ / $g'(1) = \frac{-1}{x^2} = \boxed{-1}$

$L(x) = 1 - 1(x - 1)$

$= 1 - x + 1$

$= 2 - x$

(c) $h(x) = \frac{x^2}{x^2+1}$, $x = 0$

$a = \boxed{0}$ / $h(0) = \boxed{0}$ / $h'(0) = \frac{(x^2+1)2x - (x^2 * 2x)}{(x^2+1)^2}$

$h'(0) = 0$

$L(x) = 0 + 0(x + 0)$

$= 0$

(d) $f(x) = 1 + \cos\theta$, $\theta = \frac{\pi}{3}$

$a = \frac{\pi}{3}$ / $f(\frac{\pi}{3}) = \frac{3}{2}$ / $f'(\frac{\pi}{3}) = -\sin\theta = \frac{-\sqrt{3}}{2}$

$L(x) = \frac{3}{2} + \frac{-\sqrt{3}}{2}\left(x - \frac{\pi}{3}\right)$

$= \frac{3}{2} - \frac{\sqrt{3}}{2}x + \frac{\pi\sqrt{3}}{6}$