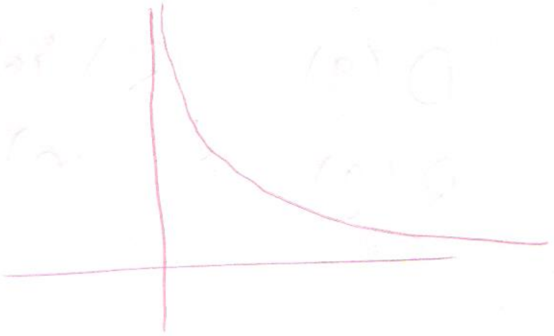
Chapter $\boxed{1}$

$\boxed{Q_1}$ • Find the Domain and Range

(a) $f(x) = \dfrac{1}{\sqrt{x}}$

$D(f) = (0, \infty)$ $\boxed{x > 0}$

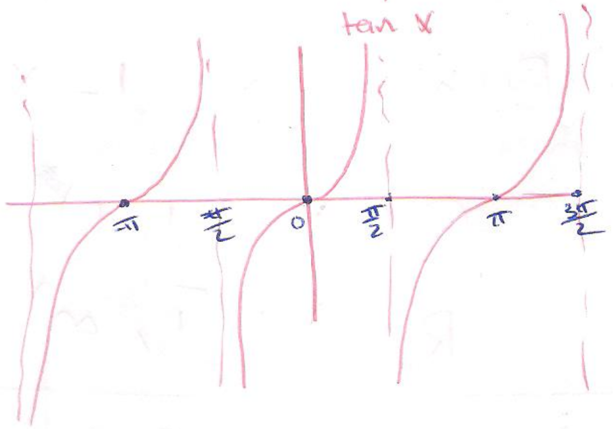
$R(f) = (0, \infty)$



(b) $f(x) = \tan(\pi x)$

$D(f) = \mathbb{R} \setminus \{ \frac{1}{2} + n, \; n = 0, \pm 1, \pm 2, \ldots \}$

$R(f) = \mathbb{R}$.

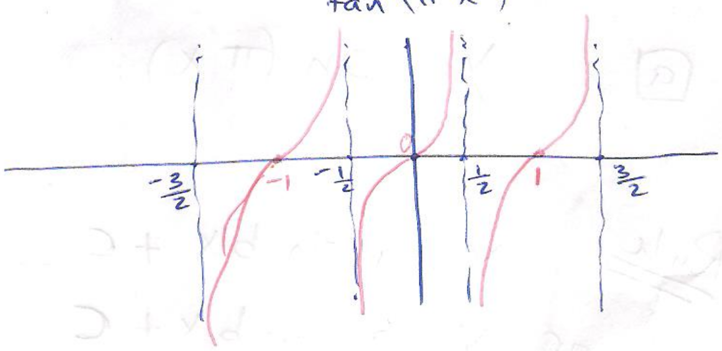


** $\tan(\pi x) = \dfrac{\sin \pi x}{\cos \pi x}$

$\cos \pi x \neq 0$

$\pi x \neq \frac{\pi}{2}, -\frac{\pi}{2} \ldots$

$x \neq \frac{1}{2} + n$



(c) $f(x) = 1 + |x|$

$D(f) = \mathbb{R}$.

$R(f) = [1, \infty)$

(d) $f(x) = \sec^2 x = \dfrac{1}{\cos^2 x}$

$\cos^2 x \neq 0$

$\cos x \neq 0 \longrightarrow x \neq \frac{\pi}{2} + n\pi$

$D(f) = \mathbb{R} \setminus \{ \frac{\pi}{2} + n\pi \}$

$n = 0, \pm 1, \pm 2, \ldots$

$R(f) = [1, \infty)$

(e) $g(x) = \frac{1}{x^2}$

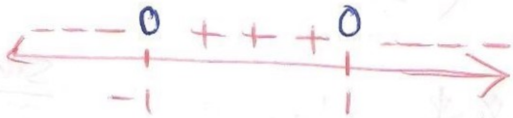
$D(g)$ $\boxed{x \neq 0}$

$D(g) = \mathbb{R} \setminus \{0\}$

$R(g) = (0, \infty)$

(f) $h(x) = \frac{1}{\sqrt{1-x^2}}$

$D(h) \longrightarrow 1 - x^2 > 0$



$D(h) = (-1, 1)$

$R(h) = [1, \infty)$

Q2 Sketch the following functions.

$\boxed{a}$ $y = \sin(\pi x)$

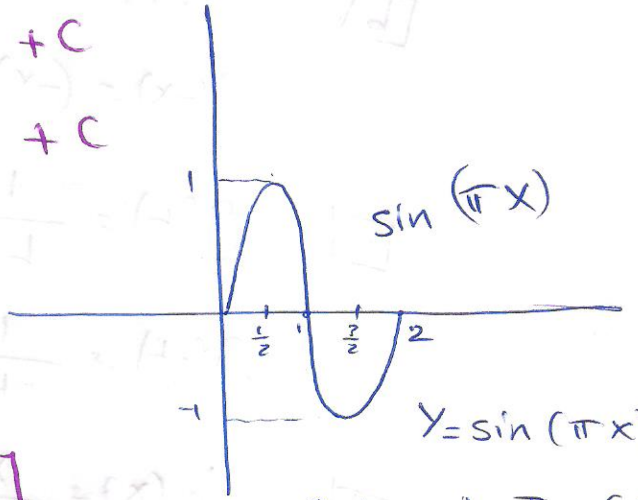
Rule $y = a\sin(bx) + c$

or $y = a\cos(bx) + c$

period $= \dfrac{2\pi}{|b|}$

Amplitude $= |a|$

Range $= [\underbrace{-|a| + c}_{Min}, \underbrace{|a| + c}_{Max}]$



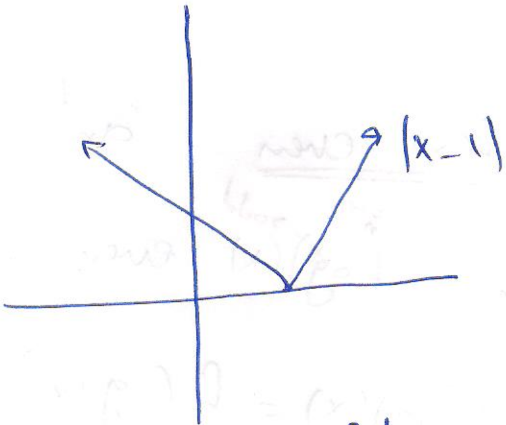
$y = \sin(\pi x)$

$a = 1$ $b = \pi$ $c = 0$

period $= 2$

Amplitude $= 1$

$\boxed{b}$ $y = |x - 1|$



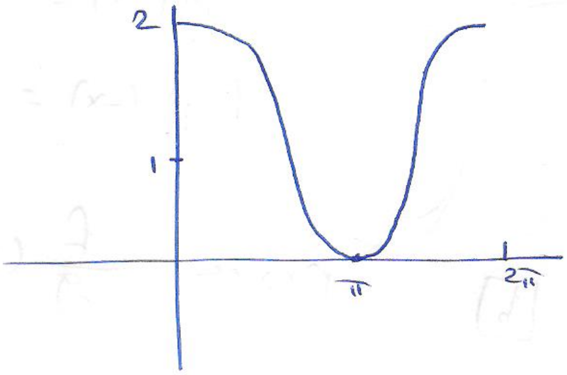
$\boxed{c}$ $y = \cos(x) + 1$

$\boxed{a = 1}$ $\boxed{b = 1}$ $\boxed{c = 1}$

Range $= [0, 2]$

period $= 2\pi$.

Amplitude $= 1$



$\boxed{Q3}$

$\boxed{a}$ $f(x) = x^2+1$

$f(-x) = (-x)^2+1 = x^2+1 = f(x)$ even.

$\boxed{b}$ $f(x) = x^3+x$

$f(-x) = (-x)^3 + -x = -x^3 - x = -(f(x))$ odd

$\boxed{c}$ $g(t) = \frac{1}{t-1}$

$g(-t) = \frac{1}{-t+1}$ neither odd nor even.

$\boxed{d}$ $h(x) = \frac{x}{x^2-1}$

$h(-x) = \frac{-x}{(-x)^2-1} = \frac{-x}{x^2-1} = -h(x)$ odd

$\boxed{Q4}$ If $f(x)$ even and $g(x)$ odd

$\boxed{a}$ prove $(\overset{\text{even}}{f}\circ\overset{\text{odd}}{g})(x)$ even

$(f\circ g)(x) = f(g(x))$

$(f\circ g)(-x) = f(g(-x)) = \underbrace{f(-g(x))}_{\text{even}} = f(g(x)) = f\circ g(x)$ even

$\boxed{b}$ prove $\frac{f}{g}(x)$ odd

$\frac{f}{g}(x) = \frac{f(x)}{g(x)}$

$\frac{f}{g}(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -\left(\frac{f(x)}{g(x)}\right) = -\frac{f}{g}(x)$ odd

Chapter 2

$\boxed{Q_1}$ (a) $\lim_{t\to -1} \dfrac{t^2+3t+2}{t^2-t-2} = \dfrac{(-1)^2+-3+2}{(-1)^2+1-2} = \dfrac{0}{0}$

$\lim_{t\to -1} \dfrac{\cancel{(t+1)}(t+2)}{(t-2)\cancel{(t+1)}} = \boxed{\dfrac{1}{-3}}$

(b) $\lim_{x\to 1} \dfrac{1-\sqrt{x}}{1-(\sqrt{x})^2} = \dfrac{1-\sqrt{1}}{1-1} = \dfrac{0}{0}$

$\lim_{x\to 1} \dfrac{\cancel{1-\sqrt{x}}}{\cancel{(1-\sqrt{x})}(1+\sqrt{x})} = \dfrac{1}{1+\sqrt{1}} = \boxed{\dfrac{1}{2}}$

(c) $\lim_{\theta\to 1} \dfrac{\theta^4-1}{\theta^3-1}$

$\lim_{\theta\to 1} \dfrac{(\theta^2-1)(\theta^2+1)}{(\theta-1)(\theta^2+\theta+1)} = \lim_{\theta\to 1} \dfrac{\cancel{(\theta-1)}(\theta+1)(\theta^2+1)}{\cancel{(\theta-1)}(\theta^2+\theta+1)}$

$= \dfrac{2\cdot 2}{3} = \boxed{\dfrac{4}{3}}$

(d) $\lim_{\theta\to 0} \dfrac{\sin 2\theta}{3\theta} = \boxed{\dfrac{2}{3}}$

$\boxed{\text{Rule}}$ $\lim_{\theta\to 0} \dfrac{\sin a\theta}{b\theta} = \dfrac{a}{b}$

OR $\lim_{\theta\to 0} \dfrac{\tan a\theta}{b\theta} = \dfrac{a}{b}$

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$\lim_{\theta\to 0} \dfrac{\frac{\sin 2\theta}{\theta}}{\frac{3\theta}{\theta}} = \dfrac{\lim_{\theta\to 0} \frac{\sin 2\theta}{\theta}}{\lim_{\theta\to 0} 3} = \dfrac{2}{3}$