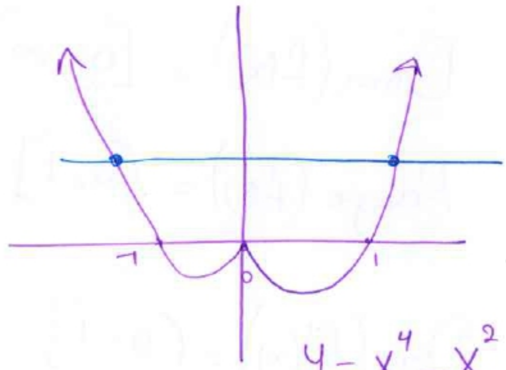
Section 7.1 Inverse Functions & Their Derivative

2



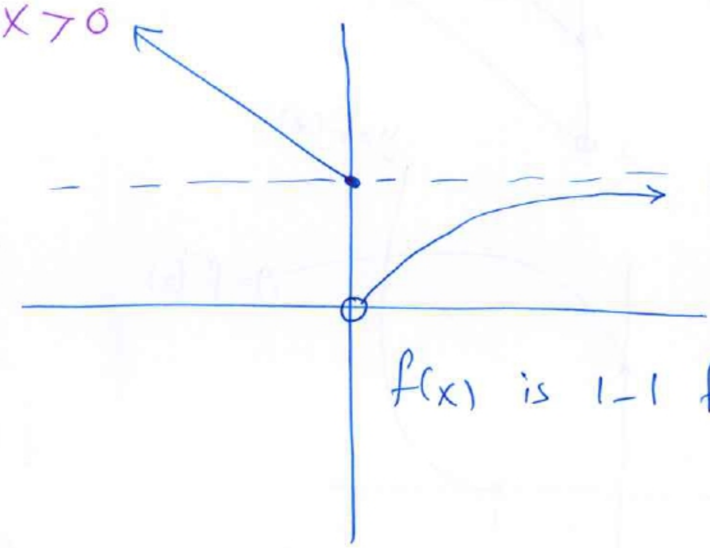
$y = x^4 - x^2$

y is not 1-1

"Since there is a horizontal line cuts the graph of the function twice".

4 Determine from its graph if the function is 1-1 ??

$$f(x) = \begin{cases} 1 - \frac{x}{2} & x \le 0 \\ \frac{x}{x+2} & x > 0 \end{cases}$$



$f(x)$ is 1-1 function

→ when $x \le 0$

$y = 1 - \frac{x}{2}$

x-intercept $y=0 \longrightarrow x=2$ $(2,0)$

y-intercept $x=0 \longrightarrow y=1$ $(0,1)$

when $x > 0$

$y = \frac{x}{x+2}$

$x=0 \longrightarrow y=0$

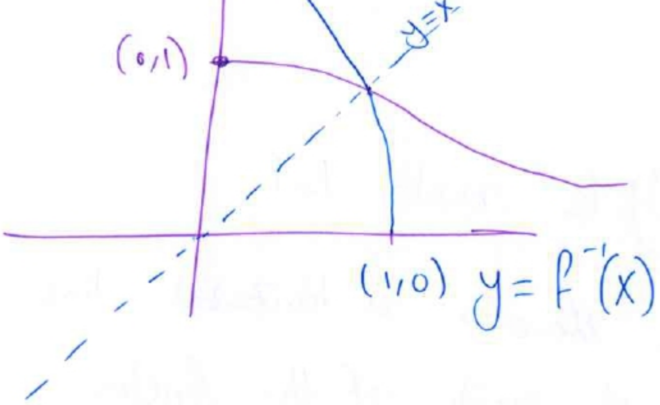
$y=0 \longrightarrow x=0$

H. Asy $y=1$

V. Asy $x=-2$

11

$y = f(x) = \dfrac{1}{x^2+1}, \ x \geq 0$



$Dom(f(x)) = [0, \infty)$

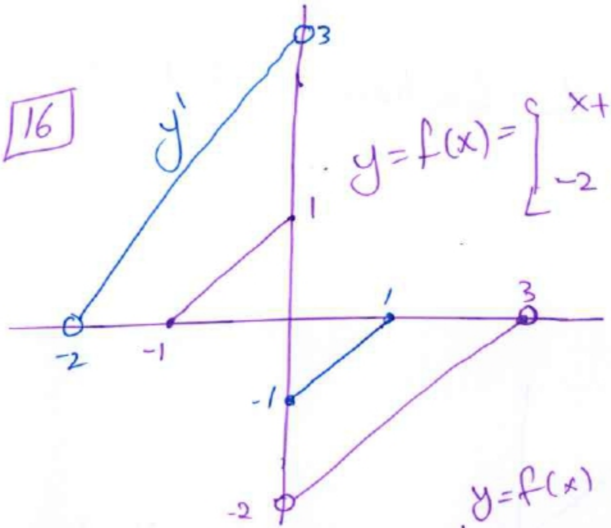
$Range(f(x)) = (0, 1]$

$Dom(f^{-1}(x)) = (0, 1]$

$Range(f^{-1}(x)) = [0, \infty)$

16

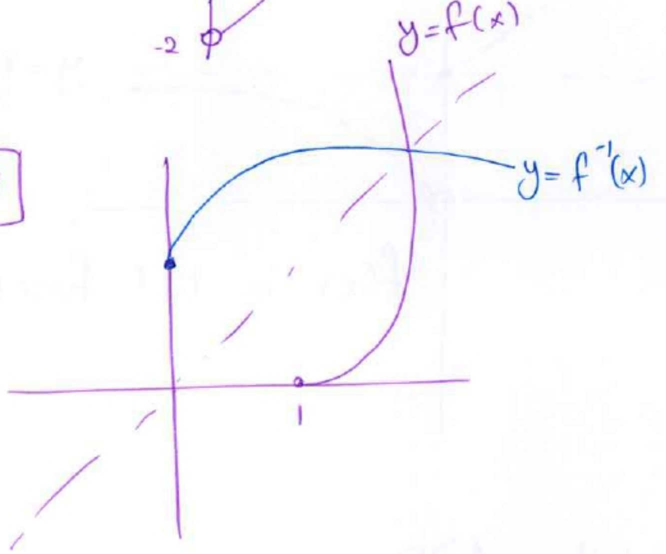
$y = f(x) = \begin{cases} x+1 & -1 \leq x \leq 0 \\ -2 + \frac{2}{3}x & 0 < x < 3 \end{cases}$



$Dom(f^{-1}) = [-2, 1]$

$Range(f^{-1}) = [-1, 3)$

22



$f(x) = x^2 - 2x + 1, \ x \geq 1$

$y = x^2 - 2x + 1$

$\sqrt{y} = \sqrt{(x-1)^2}$

$\sqrt{y} = |x-1|$

$\sqrt{y} = x - 1$

$x = \sqrt{y} + 1$

$\boxed{f^{-1}(x) = \sqrt{x} + 1}$

$\boxed{32}$ $f(x) = \frac{\sqrt{x}}{\sqrt{x}-3}$ Find $f^{-1}(x)$, $D_{f^{-1}}$, $R_{f^{-1}}$

- Since $f(x)$ is 1-1 then $f^{-1}(x)$ exist [f(x) dee on D_f]

- D_f : $x \geq 0$ & $\sqrt{x}-3 \neq 0$

$x \geq 0$ & $x \neq 9$

$D_f = [0,9) \cup (9,\infty)$

- To Find $f^{-1}(x)$:-

$$y = \frac{\sqrt{x}}{\sqrt{x}-3} \longrightarrow y\sqrt{x} - 3y = \sqrt{x}$$

$$y\sqrt{x} - \sqrt{x} = 3y$$

$$\sqrt{x}\,(y-1) = 3y$$

$$\sqrt{x} = \frac{3y}{y-1} \geq 0$$

++ $\quad$ 0 $\quad$ - - $\quad$ 1 $\quad$ ++

$$x = \left(\frac{3y}{y-1}\right)^2$$

$R_f = (-\infty, 0] \cup (1,\infty)$

$$x = \frac{9y^2}{(y-1)^2}$$

$D_{f^{-1}} = (-\infty,0] \cup (1,\infty)$

$$\therefore \boxed{f^{-1}(x) = \frac{9x^2}{(x-1)^2}}$$

$D_{f^{-1}} : (-\infty,0] \cup (1,\infty)$ $\quad$ $R_{f^{-1}} = [0,9) \cup (9,\infty)$

To show $\dfrac{9x^2}{(x-1)^2}$ is the inverse fonction of $\dfrac{\sqrt{x}}{\sqrt{x}-3}$

(فقط للتحقق)

If $x \in D_{f^{-1}}$

$$(f \circ f^{-1})(x) = f\left(\frac{9x^2}{(x-1)^2}\right) = \frac{\sqrt{\dfrac{9x^2}{(x-1)^2}}}{\sqrt{\dfrac{9x^2}{(x-1)^2}} - 3} = \frac{\dfrac{3x}{x-1}}{\dfrac{3x}{x-1} - 3}$$

$$= \frac{\dfrac{3x}{x-1}}{\dfrac{3x - 3x + 3}{x-1}}$$

$$= \frac{3x}{3} = x$$

<u>And</u>

If $x \in D_f$.

$$(f^{-1} \circ f)(x) = f^{-1}\left(\frac{\sqrt{x}}{\sqrt{x}-3}\right) = \frac{9\left(\dfrac{\sqrt{x}}{\sqrt{x}-3}\right)^2}{\left(\dfrac{\sqrt{x}}{\sqrt{x}-3} - 1\right)^2} = \frac{\dfrac{9x}{x-6\sqrt{x}+9}}{\dfrac{9}{x-6\sqrt{x}+9}}$$

$$= \frac{9x}{9} = x$$

$\boxed{33}$ $f(x) = x^2 - 2x \quad , \quad x \leq 1$

- $D_f \; : \; (-\infty, 1] = R_{f^{-1}}$

Note: Complete the square:-

Find $\left(\frac{b}{2a}\right)^2$.

$\left(\frac{b}{2a}\right)^2 = \left(\frac{-2}{2}\right)^2 = 1$

- $y + 1 = x^2 - 2x + 1$

$y + 1 = (x-1)^2$

$\sqrt{y+1} = |x - 1|$

$\sqrt{y+1} = 1 - x$

$x = 1 - \sqrt{y+1}$

$f^{-1}(x) = 1 - \sqrt{x+1}$

$y + 1 \geq 0$

$y \geq -1$

$R_f = [-1, \infty)$

$D_{f^{-1}} = [-1, \infty)$

$\therefore \quad f^{-1}(x) = 1 - \sqrt{x+1}$

$D_{f^{-1}} = [-1, \infty)$

$R_{f^{-1}} = (-\infty, 1]$

$f(x) = x^2 - 2x$

$D_f = (-\infty, 1]$

$R_f = [-1, \infty)$

Now, To show $1-\sqrt{x+1}$ is the inverse of $x^2-2x, x\leq 1$

If $x \in D_{f^{-1}}$

$$(f\circ f^{-1})(x) = f\left(1-\sqrt{x+1}\right)$$

$$= \left(1-\sqrt{x+1}\right)^2 - 2\left(1-\sqrt{x+1}\right)$$

$$= \cancel{1} - \cancel{2\sqrt{x+1}} + x + \cancel{1} \cancel{-2} + \cancel{2\sqrt{x+1}}$$

$(f\circ f^{-1})(x) = x$, for all $x \in [-1, \infty)$

if $x \in D_f$

$$(f^{-1}\circ f)(x) = f^{-1}\left(x^2-2x\right)$$

$$= 1 - \sqrt{x^2-2x+1}$$

$$= 1 - \sqrt{(x-1)^2}$$

$$= 1 - |x-1|$$

$$= 1 - (1-x) = 1-1+x = x$$

$(f^{-1}\circ f)(x) = x$, $\forall$ $x \in (-\infty, 1]$

$\boxed{42}$ Let $f(x) = x^2 - 4x - 5$, $x > 2$

Find the value of $\frac{df^{-1}}{dx}$ at the point $x = 0$

By using the formula:-

$$\left(\frac{df^{-1}}{dx}\right)\Bigg|_{x=0} = \frac{1}{\left(\frac{df}{dx}\right)\Big|_{x=f^{-1}(0)}}$$

$$= \frac{1}{f'(5)}$$

$$= \frac{1}{6}$$

$f^{-1}(0) = ??$

$0 = x^2 - 4x - 5$

$x^2 - 4x - 5 = 0$

$(x+1)(x-5) = 0$

$\boxed{\cancel{x = -1}}$ $\boxed{x = 5}$

reject since $-1 \notin D_f$

- $f(x) = x^2 - 4x - 5$
- $f'(x) = 2x - 4$

$\boxed{44}$ $y = g(x)$ is diff function

g pass through the origin with slope 2.

Find the slope of $g^{-1}(x)$ at the origin

$$\left(\frac{dg^{-1}}{dx}\right)\Bigg|_{x=0} = \frac{1}{\left(\frac{dg}{dx}\right)\Big|_{x=f^{-1}(0)}} = \frac{1}{g'(0)} = \frac{1}{2}$$