

7.2] Natural Logarithms

2] Express the following logarithms in term of $\ln 5$ & $\ln 7$

$$\text{a) } \ln\left(\frac{1}{125}\right) = \ln\left(\frac{1}{5^3}\right) = \ln(1) - 3\ln 5 \\ = -3\ln 5$$

$$\text{b) } \ln 9.8 = \ln \frac{98}{10} = \ln \frac{49}{5} = \ln \frac{7^2}{5} \\ = 2\ln 7 - \ln 5 \\ = 2\ln 7 - \ln 5$$

$$\text{c) } \ln 7\sqrt{7} = \ln (7)(7)^{1/2} = \ln 7^{3/2} = \frac{3}{2}\ln 7$$

$$\text{d) } \ln 1225 = \ln (7 \times 5)^2 = 2[\ln 7 + \ln 5]$$

$$\text{e) } \ln 0.056 = \ln \frac{56}{1000} = \ln \frac{8 \cdot 7}{8 \cdot 125} = \ln \frac{7}{125} \\ = \ln 7 - 3\ln 5$$

$$\text{f) } \frac{\left(\ln 35 + \ln\left(\frac{1}{7}\right)\right)}{\ln 25} = \frac{\ln \frac{35}{7}}{\ln 25} = \frac{\ln 5}{\ln 5^2} = \frac{\ln 5}{2\ln 5} \\ = \boxed{\frac{1}{2}}$$

4 Use the properties of logarithms to simplify expression

$$a) \ln \sec x + \ln \cos x$$

$$= \ln (\sec x)(\cos x)$$

$$= \ln 1 = 0$$

$$b) \ln (8x+4) - 2 \ln 2$$

$$= \ln (8x+4) - \ln 2^2$$

$$= \ln \frac{8x+4}{4} = \ln (2x+1)$$

$$c) 3 \ln (t^2-1)^{1/3} - \ln (t+1)$$

$$= \ln (t^2-1) - \ln (t+1)$$

$$= \ln \frac{(t^2-1)}{(t+1)} = \ln \frac{(t-1)(t+1)}{t+1} = \ln (t-1)$$

Find the derivative of y with respect to x , t or θ

$$\boxed{21} \quad y = \frac{\ln x}{1 + \ln x}$$

$$y' = \frac{(1 + \ln x)(\ln x)' - (\ln x)(1 + \ln x)'}{(1 + \ln x)^2}$$

$$= \frac{(1 + \ln x) \cdot \frac{1}{x} - \ln x \left(\frac{1}{x}\right)}{(1 + \ln x)^2}$$

$$= \frac{1}{x(1 + \ln x)^2}$$

$$\boxed{24} \quad y = \ln(\ln(\ln x))$$

$$y' = \frac{1}{\ln(\ln x)} \cdot (\ln(\ln x))'$$

$$= \frac{1}{\ln(\ln x)} \cdot \frac{1}{\ln x} (\ln x)'$$

$$= \frac{1}{\ln(\ln x)} \cdot \frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x \ln x \ln(\ln x)}$$

[34]

$$y = \ln \left(\frac{(x+1)^5}{(x+2)^{10}} \right)^{\frac{1}{2}}$$

$$= \ln \frac{(x+1)^{\frac{5}{2}}}{(x+2)^{10}}$$

$$y = \frac{5}{2} \ln(x+1) - 10 \ln(x+2)$$

$$y' = \frac{5}{2} \cdot \frac{1}{x+1} - 10 \cdot \frac{1}{x+2}$$

$$y' = \frac{5}{2(x+1)} - \frac{10}{x+2}$$

[44]

$$\int_2^4 \frac{dx}{x \ln x}$$

$$\int_{\ln 2}^{\ln 4} \frac{x \, du}{x \cdot u} = \int_{\ln 2}^{\ln 4} \frac{1}{u} \, du$$

$$= \ln |u| \Big|_{\ln 2}^{\ln 4}$$

$$= \ln \ln 4 - \ln \ln 2$$

$$= \ln \left(\frac{\ln 4}{\ln 2} \right)$$

$$= \ln(2)$$

By substitution: -

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$x=2 \longrightarrow u = \ln 2$$

$$x=4 \longrightarrow u = \ln 4$$

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$$\int \frac{dx}{2\sqrt{x} + 2x}$$

$$= \int \frac{dx}{2\sqrt{x}(1+\sqrt{x})}$$

By substitution: -

$$u = 1 + \sqrt{x}$$

$$du = \frac{1}{2\sqrt{x}} dx$$

$$= \int \frac{\cancel{2\sqrt{x}} du}{\cancel{2\sqrt{x}}(u)}$$

$$= \int \frac{1}{u} du = \ln |u| + C$$

$$= \ln |1 + \sqrt{x}| + C$$

$$= \ln (1 + \sqrt{x}) + C$$

since $1 + \sqrt{x} > 0$

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$$\int \frac{\sec x dx}{\sqrt{\ln(\sec x + \tan x)}}$$

$$\text{let } u = \ln(\sec x + \tan x)$$

$$du = \frac{\sec x (\sec x + \tan x) dx}{\sec x + \tan x}$$

$$du = \sec x dx$$

$$\int \frac{\sec x du}{\sqrt{u} \sec x} = \int \frac{1}{\sqrt{u}} du$$

$$= \frac{u^{1/2}}{1/2} + C$$

$$= 2 \left[\ln(\sec x + \tan x) \right]^{1/2} + C$$

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$$y = \frac{x \sqrt{x^2 + 1}}{(x+1)^{2/3}}$$

$$\ln y = \ln x + \frac{1}{2} \ln(x^2 + 1) - \frac{2}{3} \ln(x+1)$$

$$\frac{y'}{y} = \frac{1}{x} + \frac{1}{2(x^2+1)} \cdot 2x - \frac{2}{3} \frac{1}{x+1}$$

$$y' = \left[\frac{1}{x} + \frac{x}{x^2+1} - \frac{2}{3(x+1)} \right] \frac{x \sqrt{x^2+1}}{(x+1)^{2/3}}$$

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$$y = \sqrt[3]{\frac{x(x+1)(x-2)}{(x^2+1)(2x+3)}}$$

$$\ln y = \frac{1}{3} \left[\ln x + \ln(x+1) + \ln(x-2) - \ln(x^2+1) - \ln(2x+3) \right]$$

$$\frac{y'}{y} = \frac{1}{3} \left[\frac{1}{x} + \frac{1}{x+1} + \frac{1}{x-2} - \frac{1}{x^2+1} (2x) - \frac{2}{2x+3} \right]$$

$$y' = \frac{1}{3} \left[\frac{1}{x} + \frac{1}{x+1} + \frac{1}{x-2} - \frac{2x}{x^2+1} - \frac{2}{2x+3} \right] \left(\frac{x(x+1)(x-2)}{(x^2+1)(2x+3)} \right)^{1/3}$$

78 (a) Prove that $f(x) = x - \ln x$ is increasing for $x > 1$

(b) Using part (a), show that $\ln x < x$ if $x > 1$

(a) $f'(x) = 1 - \frac{1}{x} = \frac{x-1}{x} > 0$ for $x > 1$

→ since $f'(x) > 0$ on $(1, \infty)$

→ $f(x)$ is increasing on $(1, \infty)$

(b) f is increasing when $x > 1$ $\left[x_1 > x_2 \longrightarrow f(x_1) > f(x_2) \right]$

Then $x > 1$
 $f(x) > f(1)$

$$x - \ln x > 1 - \ln(1)$$

$$x - \ln x > 1 > 0$$

$$x - \ln x > 0 \longrightarrow$$

$x > \ln x$ if $x > 1$

71 Find the area between the curve $y = \ln x$ & $y = \ln 2x$

from $x=1$ - $x=5$

$$\text{Area} = \int_1^5 \ln(2x) - \ln(x) dx$$

$$= \int_1^5 \ln\left(\frac{2x}{x}\right) dx$$

$$= \int_1^5 \ln(2) dx = \ln(2) [5-1]$$

$$= 4 \ln(2)$$

$$\text{Area} = \ln 16$$

