7.3 Exponential Function:

4 $e^{x^2} e^{2x+1} = e^t$ Solve for t

$e^{x^2+2x+1} = e^t$

$x^2+2x+1 = t$

$\boxed{(x+1)^2 = t}$

Find the derivative with respect to x, t, or θ

14 $y = \ln(3\theta e^{-\theta})$

$y = \ln 3 + \ln\theta + \ln e^{-\theta}$

$y = \ln 3 + \ln\theta - \theta$

$y' = \frac{1}{\theta} - 1 = \frac{1-\theta}{\theta}$

18 $y = \ln(2e^{-\theta}\sin\theta)$

$y = \ln 2 + \ln e^{-\theta} + \ln\sin\theta$

$y = \ln 2 - \theta + \ln\sin\theta$

$y' = -1 + \frac{1}{\sin\theta}\cdot\cos\theta$

$y' = -1 + \cot\theta = \cot\theta - 1$

[10] Find $\frac{dy}{dx}$ for $y = \ln\left(\frac{10}{x}\right)$

$y = \ln 10 - \ln x$

$y' = -\frac{1}{x}$

[21] $y = e^{\cos t + \ln t}$

$y = e^{\cos t} \cdot e^{\ln t}$

$y = t \cdot e^{\cos t}$

$y' = t \cdot e^{\cos t}(-\sin t) + e^{\cos t}$

$y' = e^{\cos t}\left[1 - t\sin t\right].$

[23] $y = \int_0^{\ln x} \sin(e^t)\, dt$ By Using F.T.C

$y' = \sin(e^{\ln x}) \cdot \frac{1}{x} - \cancel{\sin(e^0) \cdot (0)}$

$y' = \frac{\sin x}{x}$

$\boxed{28}$ $\tan y = e^x + \ln x$

$$y' \sec^2(y) = e^x + \frac{1}{x}$$

$$y' = \left[e^x + \frac{1}{x}\right] \sec^{-2}(y)$$

$$y' = \left[e^x + \frac{1}{x}\right] \sec^{-2}\left[\tan^{-1}(e^x + \ln x)\right]$$

Evalute the integral 29 - 50 :-

$\boxed{38}$ $$\int \frac{e^{-\sqrt{r}}}{\sqrt{r}}\, dr$$

$u = -\sqrt{r}$

$du = \frac{-1}{2\sqrt{r}}\, dr$

$$\int \frac{e^u}{\sqrt{r}} \cdot -2\sqrt{r}\, du = -2 \int e^u\, du$$

$$= -2 e^u + C$$

$$= -2 e^{-\sqrt{r}} + C$$

$\boxed{44}$ $$\int_{\pi/4}^{\pi/2} \left(1 + e^{\cot\theta}\right) \csc^2\theta\, d\theta$$

$u = \cot\theta$

$du = -\csc^2\theta\, d\theta$

$x = \frac{\pi}{4} \longrightarrow u = 1$

$x = \frac{\pi}{2} \longrightarrow u = 0$

$$\int_1^0 (1 + e^u) \cancel{\csc^2\theta} \frac{du}{\cancel{-\csc^2\theta}}$$

$$\int_0^1 (1 + e^u)\, du = u + e^u \Big]_0^1 = 1 + e - 1 = \boxed{e}$$

$\boxed{48}$ $I = \int_0^{\sqrt{\ln\pi}} 2x\, e^{x^2} \cos\left(e^{x^2}\right) dx$

$U = e^{x^2}$

$dU = e^{x^2} \cdot 2x \, dx$

$\int \cancel{2x e^{x^2}} \cos(U) \frac{dU}{\cancel{2x e^{x^2}}}$

$x = 0 \longrightarrow U = 1$

$x = \sqrt{\ln\pi} \longrightarrow U = \pi$

$\int_1^\pi \cos(U)\, dU = \sin(U)\Big]_1^\pi$

$= 0 - \sin(1) = -\sin(1)$.

$\boxed{50}$ $I = \int \frac{1}{1+e^x} dx$

let $U = e^{-x} + 1$

$dU = -e^{-x} \, dx$

$= \int \frac{1}{e^x(e^{-x}+1)} dx$

$\int \frac{1}{e^x(U)} \frac{dU}{-e^{-x}} = -\int \frac{1}{U} dU$

$= -\ln|U| + C$

$= -\ln\left|e^{-x}+1\right| + C$

I.V.P

52 $\frac{dy}{dx} = e^{-t} \sec^2(\pi e^{-t})$, $\underbrace{y(\ln 4) = \frac{2}{\pi}}_{\text{Initial Condition given}}$

Find $y = ??$

$$y = \int e^{-t} \sec^2(\pi e^{-t})\, dt$$

$U = \pi e^{-t}$

$dU = -\pi e^{-t}\, dt$

$$\int \cancel{e^{-t}} \sec^2(U) \frac{dU}{-\pi \cancel{e^{-t}}}$$

$$y = \frac{-1}{\pi}\left[\tan(U)\right] + C = \frac{-1}{\pi} \tan(\pi e^{-t}) + C$$

To Find $C = ??$

$$y(\ln 4) = \frac{2}{\pi}$$

$$\frac{2}{\pi} = y(\ln 4) = \frac{-1}{\pi} \tan(\pi e^{-\ln 4}) + C$$

$e^{-\ln 4} = e^{\ln 4^{-1}}$

$= 4^{-1} = \frac{1}{4}$

$$\frac{2}{\pi} = \frac{-1}{\pi} \tan\left(\frac{\pi}{4}\right) + C$$

$$\frac{2}{\pi} = \frac{-1}{\pi} + C \longrightarrow C = \frac{3}{\pi}$$

$$\boxed{y = \frac{-1}{\pi} \tan(\pi e^{-t}) + \frac{3}{\pi}}$$

58 Find the derivative of y with respect to the given independent variable

$$y = 2^{(s^2)}$$

$$y' = 2^{s^2} \cdot (2s)(\ln 2)$$

63 $y = 7^{\sec\theta} \ln 7$

$$y' = (\ln 7)(7)^{\sec\theta} \cdot (\ln 7)(\sec\theta \tan\theta)$$

$$y' = (\ln 7)^2 \; 7^{\sec\theta}(\sec\theta \tan\theta)$$

65 $y = 2^{\sin(3t)}$

$$y' = 2^{\sin(3t)} \cdot (3\cos(3t))(\ln 2)$$

$$= 3 \ln 2 \; \cos(3t) \; 2^{\sin(3t)}$$

$$= (\ln 8) \cos(3t) \; 2^{\sin(3t)}$$

73 $y = \log_3 \left[\frac{x+1}{x-1}\right]^{\ln 3}$

$$y = \ln 3 \left[\log_3(x+1) - \log_3(x-1)\right]$$

$$= \ln 3 \left[\frac{\ln(x+1)}{\ln 3} - \frac{\ln(x-1)}{\ln 3}\right]$$

$$y = \ln(x+1) - \ln(x-1)$$

$$y = \ln(x+1) - \ln(x-1)$$

$$y' = \frac{1}{x+1} - \frac{1}{x-1}$$

$$y' = \frac{(x-1)-(x+1)}{x^2-1} = \frac{-2}{x^2-1}$$

$\boxed{75}$ $$y = \theta \sin(\log_7 \theta)$$

$$y' = \cancel{\theta} \cos(\log_7 \theta) \cdot \frac{1}{\cancel{\theta} \ln 7} + \sin(\log_7 \theta) \cdot (1)$$

$$y' = \frac{\cos(\log_7 \theta)}{\ln 7} + \sin(\log_7 \theta)$$

$\boxed{91}$ $$\int_2^4 x^{2x}(1+\ln x)\,dx$$

$$U = x^x$$

$$\ln U = x \ln x$$

$$\frac{1}{U} \cdot \frac{dU}{dx} = x \frac{1}{x} + \ln x$$

$$\frac{1}{U} \frac{dU}{dx} = (1 + \ln x)$$

$$\int (U)^2 \frac{\cancel{(1+\ln x)}\, dU}{\cancel{U}\,\cancel{(1+\ln x)}}$$

$x = 2 \longrightarrow U = 2^2 = 4$

$x = 4 \longrightarrow U = 4^4 = 256$

$$\int_4^{256} U\,dU = \left.\frac{U^2}{2}\right]_4^{256}$$

$$= \frac{1}{2}\left[(256)^2 - 16\right]$$

$\boxed{98}$ $\displaystyle\int_1^4 \frac{\log_2 x}{x}\,dx$

$U = \log_2 x$

$U = \dfrac{\ln x}{\ln 2}$

$dU = \dfrac{1}{\ln 2}\cdot\dfrac{1}{x}\,dx$

$x = 1 \longrightarrow U = \log_2 1 = 0$

$x = 4 \longrightarrow U = \log_2 4 = 2$

$$\int_0^2 \frac{U \cdot \cancel{x} \cdot \ln(2)}{\cancel{x}}\,dU$$

$$= \ln 2 \int_0^2 U\,dU$$

$$= \ln 2 \left.\frac{U^2}{2}\right]_0^2$$

$$= \ln 2\left[\frac{4}{2} - 0\right] = 2\ln 2 = \ln 4$$

$\boxed{106}$ $\displaystyle\int \frac{dx}{x(\log_8 x)^2}$

$U = \log_8 x = \dfrac{\ln x}{\ln 8}$

$dU = \dfrac{1}{\ln 8}\cdot\dfrac{1}{x}\,dx$

$$\int \frac{1}{\cancel{x}\; U^2}\,\cancel{x}\cdot\ln(8)\,dU = \ln 8 \int \frac{1}{U^2}\,dU$$

$$= \ln(8)\left[\frac{-1}{U}\right] + C$$

$$= \frac{-\ln(8)}{\log_8 x} + C$$

$$= \frac{-[\ln(8)]^2}{\ln x} + C$$

108 $\displaystyle\int_1^{e^x} \frac{1}{t}\,dt$

$$= \ln|t|\Big]_1^{e^x} = \ln|e^x| - \ln(1)$$

$$= \ln e^x - 0 = x$$

116 $\ln y = \ln x^{\sin x}$

Note.

$y = (\text{Variable})^{\text{variable}}$

Use logarithmic differentiation to find the derivation of y

$$\ln y = \sin x \ln x$$

$$\frac{y'}{y} = \frac{\sin x}{x} + \ln(x)\cos(x)$$

$$y' = \left[\frac{\sin x}{x} + (\ln x)(\cos x)\right] x^{\sin x}$$

118 $\cancel{\displaystyle\int_1^{e^x}}$ $y = (\ln x)^{\ln x}$

$$\ln y = \ln (\ln x)^{\ln x}$$

$$\ln y = (\ln x)(\ln \ln x)$$

$$\frac{y'}{y} = \cancel{(\ln x)}\frac{1}{\cancel{\ln x}} \cdot \frac{1}{x} + \ln(\ln x) \cdot \frac{1}{x}$$

$$y' = \left[\frac{1 + \ln(\ln x)}{x}\right] (\ln x)^{\ln x}$$