Mathematics Department

Math 1411 – Worksheet #7

Rasha Shadid

Name:

Q_1 Are the functions $f(x) = (x+1)^3 + 2$ and $g(x) = x + \frac{1}{x}$ one-to-one?

Q_2 let $f(x) = 8x^3 + 3$. Show that

(i) f^{-1} exist

(ii) Show that $f^{-1}(x) = \frac{1}{2}\sqrt[3]{x-3}$

(iii) Find $\frac{df^{-1}}{dx}$ at $x = 2$

Q_3 if $g(x)$ is the inverse function of $f(x)$

$f(4) = 5$, $f'(4) = \frac{2}{3}$, find $g'(5)$

Q_4 Find y' for each of the following:-

① $y = x\sqrt{\ln x}$

② $y = \ln\left(\frac{\sqrt{x+1}}{3+x^2}\right)$

③ $y = \int_{x^2}^{x^3} \ln t \, dt$

Q_5 Find the following integrals :-

① $\int \frac{x^3}{2x^4+3} \, dx$

② $\int \frac{1}{x\cos^2(8+\ln x)} \, dx$

Short Answers: Worksheet # 7

Q1 $f(x)$ is 1-1

$g(x)$ is not 1-1 $\left(\text{Since } g(2) = g\left(\frac{1}{2}\right)\right)$

Q2: (i) f is 1-1 $\longrightarrow f^{-1}$ exist

(ii) $f^{-1}(f(x)) = f^{-1}(8x^3+3) = x$

And $f(f^{-1}(x)) = f\left(\frac{1}{2}\sqrt[3]{x-3}\right) = x$

$\longrightarrow$ Then $\frac{1}{2}\sqrt[3]{x-3}$ is the inverse function of $8x^3+3$

(iii) $\left.\left(\frac{df^{-1}}{dx}\right)\right|_{x=2=f\left(\frac{-1}{2}\right)} = \frac{1}{6}$

Q3 $g'(5) = \frac{3}{2}$

Q4 ① $y' = \frac{1+\ln x}{2\sqrt{\ln x}}$ ② $y' = \frac{1}{2x+2} - \frac{1}{3+x^2}(2x)$

③ $y' = 3x^2 \ln x^3 - 2x \ln x^2$

Q5. ① $\frac{1}{8} \ln|2x^4+3| + C$

② $\tan(8+\ln x) + C$

Mathematics Department

Math 141 – Worksheet #8

Ch. 7.3 – 7.5

Rasha Shadid

Name: ______

Q1 Simplify the following expression :-

(a) $e^{2\ln 4} + \ln\sqrt{e^6}$

(b) $\log_9 27 + \log_2 \sqrt{8} - \ln e^3$

Q2: Find the derivative of the following

① $y = 3\log_8(\log_2 x)$

② $y = \displaystyle\int_{e^{4\sqrt{x}}}^{e^{2x}} \ln t \, dt$

Q3 Evaluate the following integral :–

① $\displaystyle\int \frac{e^{-1/x^2}}{x^3}\, dx$

② $\displaystyle\int \frac{e^{2x}}{e^x - 1}\, dx$

Q_4 : Find the limit of the following :-

① $\lim\limits_{x \to 0} \dfrac{x\, 2^x}{2^x - 1}$

② $\lim\limits_{x \to 0} \dfrac{\sin x - x}{x^3}$

③ $\lim\limits_{x \to (\frac{\pi}{2})^-} \left(\frac{\pi}{2} - x\right) \tan x$

④ $\lim\limits_{x \to 0} \left(e^x + x\right)^{1/x}$

⑤ $\lim\limits_{x \to \infty} \left(x^3 + e\right)^{\frac{1}{\ln x}}$

⑥ $\lim\limits_{x \to 0^+} \sin x \, \ln x$

Short Answers:-

Q1 (a) 19

(b) Zero.

Q2 (1) $y' = \frac{1}{\ln 2}\left(\frac{1}{x \ln x}\right)$

(2) $y' = 4x e^{2x} - 8 e^{4\sqrt{x}}$

Q3 (1) $I = 1/2\, e^{-1/x^2} + C$

(2) $I = e^x - 1 + \ln|e^x - 1| + C$

$= e^x + \ln|e^x - 1| + C_1$, since $C_1 = C - 1$

Q4 (1) $\frac{1}{\ln 2}$

(2) $\frac{-1}{6}$

(3) 1

(4) e^2

(5) e^3

(6) 0

Q4 $\boxed{6}$ $\lim_{x \to 0^+} \sin x \ln x$ $0(-\infty)$

$$\lim_{x \to 0^+} \frac{\ln x}{\csc x}$$

$$\lim_{x \to 0^+} \frac{\frac{1}{x}}{-\csc x \cot x}$$

$$\lim_{x \to 0^+} -\frac{\sin x \tan x}{x}$$ OR l'Hopital Rule

$$-\lim_{x \to 0^+} \frac{\sin x}{x} \quad \lim_{x \to 0^+} \tan x$$

$$= \quad -1 \cdot 0 = 0$$

Mathematics Department

Math 1411 _ Worksheet #9

"7.6, 7.7, 7.8"

. Rasha. Shadid

. Name :- - - - -

$\boxed{Q_1}$: Find the exact value of the following expression if its exist

ⓐ $\sin\left(\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)\right)$

ⓑ $\sin^{-1}\left(\sin\frac{7\pi}{3}\right)$

ⓒ $\cot\left(\sin^{-1}\left(-\frac{1}{2}\right) - \sec^{-1}(2)\right)$

$\boxed{Q_2}$ Find the solution of the following equation

$$\ln(x-e) = x^2 - \tan^2\left(\sec^{-1}x\right)$$

$\boxed{Q_3}$ Evalute the following integral

ⓐ $\displaystyle\int \frac{\sec^2 x \, dx}{\sqrt{4-\tan^2 x}}$

ⓑ $\displaystyle\int \frac{x^3+3x}{x^4+9}$

$\boxed{Q_4}$ Simplify the following expression

$\sinh(2 \ln x)$

$\boxed{Q_5}$ Evalute the following integral

ⓐ $\int e^{-x} \cosh x \, dx$

ⓑ $\int \operatorname{sech}^2 x \operatorname{sech} x \tanh x \, dx$

$\boxed{Q_6}$ Discuss the growth of following pair

x^2 and $\sqrt{4x^4+3x^2+1}$

Short Answers:

Worksheet # 9

Q1 (a) $\frac{1}{\sqrt{2}}$ (b) $\frac{\pi}{3}$ (c) 0

Q2 $\boxed{x = 2e}$

Q3 (a) $I = \sin^{-1}\left(\frac{\tan x}{2}\right) + C$

(b) $I = \frac{1}{4} \ln |x^4 + 9| + \frac{1}{2} \tan^{-1}\left(\frac{x^2}{3}\right) + C$

Q4 $\frac{x^4 - 1}{2x}$

Q5 (a) $I = \frac{1}{2} x - \frac{1}{4} e^{-2x} + C$

(b) $I = \frac{-1}{3} \operatorname{sech}^3 x + C$

Q6 x^2 & $\sqrt{4x^4+3x^2+1}$ grows at the same rate as $x \to \infty$

$$\lim_{x \to \infty} \frac{x^2}{\sqrt{4x^4+3x^2+1}} = \frac{1}{2}$$