...ditional exercises for the Lecture.

8.1) 14 $\int 4x \sec^2 2x \, dx$

Let $u = 2x \Rightarrow du = 2dx$ } No need for this step.

$\Rightarrow \int u \sec^2 u \, du$. Now by parts:

let $u = 4x$, $dv = \sec^2 x$
$du = 4dx$, $v = \tan x$

Let $w = u$ $\quad dv = \sec^2 u \, du$
$dw = du$ $\quad v = \tan u$

$\int \frac{\sin u}{\cos u} = \ln|\cos|$

$\Rightarrow \int u \sec^2 u \, du = u \tan u - \int \tan u \, du$

$= u \tan u - \ln|\sec u| + C$

$= 2x \tan 2x - \ln|\sec(2x)| + C$

.1) 50 $\int_0^{\frac{1}{\sqrt{2}}} 2x \sin^{-1}(x^2) \, dx$

Let $u = \sin^{-1}(x^2)$, $dv = 2x \, dx$

$du = \frac{2x \, dx}{\sqrt{1-x^4}}$, $v = x^2$

$\int_0^{\frac{1}{\sqrt{2}}} 2x \sin^{-1}(x^2) \, dx = x^2 \sin^{-1}(x^2) \Big]_0^{\frac{1}{\sqrt{2}}} - \int \frac{2x^3}{\sqrt{1-x^4}} dx.$

$x^2 \sin^{-1}(x^2) \Big]_0^{\frac{1}{\sqrt{2}}} - \int \frac{du}{-2\sqrt{u}} = \frac{\pi}{12} + \left(\sqrt{1-x^4}\right)\Big|_0^{\frac{1}{\sqrt{2}}}$

Let $u = 1 - x^4$
$du = -4x^3 dx$
$\Rightarrow \frac{du}{-2} = 2x^3 dx$

$= \frac{\pi}{12} + \sqrt{\frac{3}{4}} - 1.$

(32) $\displaystyle\int_{-\pi}^{\pi}(1-\cos^2 t)^{3/2}\,dt = \int_{-\pi}^{\pi}(\sin^2 t)^{3/2}\,dt$

$\displaystyle = \int_{-\pi}^{\pi}|\sin^3 t|\,dt = -\int_{-\pi}^{0}\sin^{3} t\,dt + \int_{0}^{\pi}\sin^3 t\,dt$ $2k+1$

$\displaystyle = -\int_{-\pi}^{0}(\underbrace{1-\cos^2 t})\sin t\,dt + \int_{0}^{\pi}(\underbrace{1-\cos^2 t})\sin t\,dt$

Let $u = \cos t$
$du = -\sin t\,dt$

$\displaystyle = \int_{-1}^{1}1-u^2\,du - \int_{1}^{-1}1-u^2\,du = 2\int_{-1}^{1}(1-u^2)\,du$

$\displaystyle = 2\left[u-\frac{u^3}{3}\right]_{-1}^{1} = 2\left[1-\frac{1}{3}-\left[(-1)-\frac{(-1)^3}{3}\right]\right] =$

$\displaystyle 2\left(\frac{3\times 2}{3}-\frac{2}{3}\right) = 2\left(\frac{4}{3}\right) = \boxed{\frac{8}{3}}$

2/
(47) $\displaystyle\int \tan^5 x\,dx = \int \tan^4 x\tan x\,dx = \int(\sec^2 x-1)^2\tan x\,dx$

$\displaystyle = \int(\sec^4 x - 2\sec^2 x + 1)\tan x\,dx$

$\displaystyle = \int \sec^4 x\tan x\,dx - 2\int \sec^2 x\,\underbrace{\tan x}\,dx + \int \tan x\,dx$

$u = \tan x$
$du = \sec^2 x\,dx$

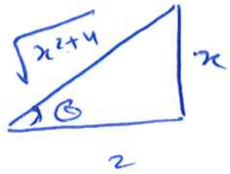
$\displaystyle = \int \sec^3 x\,\underbrace{\sec x\tan x}\,dx - 2\left[\frac{\tan^2 x}{2}\right] + \ln|\sec x| + C$

$u = \sec x$
$du = \sec x\tan x\,dx$

$\displaystyle = \frac{\sec^4 x}{4} - \tan^2 x + \ln|\sec x| + C.$

8.3) 17 $\displaystyle\int \frac{x^3\,dx}{\sqrt{x^2+4}}$ Trigonometric substitution.

Let $x = 2\tan\theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$



$dx = 2\sec^2\theta\, d\theta$

$\sqrt{x^2+4} = \sqrt{4\tan^2\theta+4} = 2|\sec\theta| = 2\sec\theta.$

$$\Rightarrow \int \frac{x^3\,dx}{\sqrt{x^2+4}} = \int \frac{8\tan^3\theta\cdot(\cancel{2}\sec^{\cancel{2}}\theta)\,d\theta}{\cancel{2}\cancel{\sec}\theta} = \int 8\tan^3\theta\sec\theta\, d\theta$$

$$= 8\int \frac{\sin^3\theta}{\cos^3\theta}\cdot\frac{1}{\cos\theta}\,d\theta = 8\int \frac{\sin^2\theta\sin\theta}{\cos^4\theta}\,d\theta$$

$$= 8\int \frac{(1-\cos^2\theta)\sin\theta}{\cos^4\theta}\,d\theta$$

Let $u = \cos\theta$

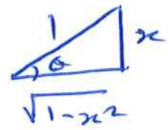
$du = -\sin\theta\, d\theta$

$$= 8\int -\frac{(1-u^2)}{u^4}\,du = 8\int \frac{u^2-1}{u^4}\,du = 8\left[\int \frac{1}{u^2} - \frac{1}{u^4}\,du\right]$$

$$= 8\left[-\frac{1}{u} + \frac{u^{-3}}{3}\right] + C = 8\left[\frac{-1}{\cos\theta} + \frac{1}{3\cos^3\theta}\right] + C$$

$$= 8\left[-\frac{\sqrt{x^2+4}}{2} + \frac{1}{3}\left(\frac{\sqrt{x^2+4}}{2}\right)^3\right] + C.$$

8.3) 23 $\displaystyle\int_0^{\sqrt{3}/2} \frac{4x^2\,dx}{(1-x^2)^{3/2}}$



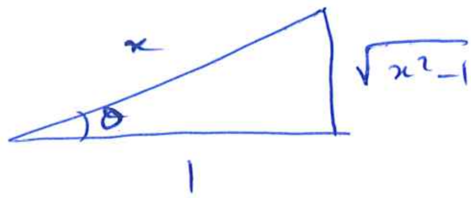
Let $x = \sin\theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$\sqrt{1-x^2} = \sqrt{1-\sin^2\theta} = \cos\theta$

$$\Rightarrow \int_0^{\pi/3} \frac{4\sin^2\theta\cos\theta\,d\theta}{\cos^3\theta} = 4\int_0^{\pi/3} \frac{(1-\cos^2\theta)\cancel{\cos\theta}}{\cos^{\cancel{3}\,2}\theta}\,d\theta.$$

$$= 4\int_0^{\pi/3} \sec^2\theta\,d\theta - 4\int_0^{\pi/3} 1\,d\theta = 4\left[\tan\theta - \theta\right]_0^{\pi/3} = 4\sqrt{3} - \frac{4\pi}{3}$$

8.3) 25 $\displaystyle\int \frac{dx}{(x^2-1)^{3/2}}$, $\frac{x}{a} \geq 1$



Let $x = \sec\theta$, $0 < \theta < \frac{\pi}{2}$

$dx = \sec\theta\tan\theta\,d\theta$

$\sqrt{(x^2-1)} = \sqrt{\sec^2\theta - 1} = \tan\theta$

$$\int \frac{\sec\theta\,\cancel{\tan\theta}\,d\theta}{\tan^{\cancel{3}\,2}\theta} = \int \frac{\cos\theta}{\sin^2\theta}\,d\theta = -\frac{1}{\sin\theta} + C$$

$$= -\frac{x}{\sqrt{x^2-1}} + C$$

8.4) (16) $\displaystyle\int \frac{x+3}{2x^3-8x}\,dx = \int \frac{x+3}{2x(x+2)(x-2)}$

$$= \int\left(\frac{A}{2x} + \frac{B}{x+2} + \frac{C}{x-2}\right)dx \quad --- \quad (*)$$

$$\Rightarrow \quad x+3 = A(x+2)(x-2) + B(2x)(x-2) + C(2x)(x+2)$$

$$\Rightarrow \quad x+3 = Ax^2 - 4A + 2x^2B - 4xB + 2x^2C + 4xC$$

$$\Rightarrow \quad x+3 = (A+2B+2C)x^2 + (4C-4B)x - 4A$$

$$\Rightarrow \quad \left.\begin{array}{l} A+2B+2C = 0 \\ 4C-4B = 1 \\ -4A = 3 \end{array}\right\} \Rightarrow \quad \begin{array}{l} A = -\frac{3}{4} \\ B = \frac{1}{16} \\ C = 5/16 \end{array}$$

$$(*) \Rightarrow \int\left(\frac{-3/4}{2x} + \frac{1/16}{x+2} + \frac{5/16}{x-2}\right)dx$$

$$= -\frac{3}{8}\ln|x| + \frac{1}{16}\ln|x+2| + \frac{5}{16}\ln|x-2| + C_1$$

8.4) 29 $\displaystyle\int \frac{x^2}{x^4-1}\,dx = \int \frac{x^2}{(x^2+1)(x-1)(x+1)}$

$$= \int \left(\frac{A}{x+1} + \frac{B}{x-1} + \frac{Cx+D}{x^2+1} \right) dx \quad \cdots \quad (*)$$

$$\Rightarrow \quad x^2 = A(x-1)(x^2+1) + B(x+1)(x^2+1) + (Cx+D)(x^2-1)$$

$$= (A+B+C)x^3 + (-A+B+D)x^2 +$$

$$(A+B-C)\,x \; - A+B-D.$$

$$\Rightarrow \quad \left.\begin{aligned} A+B+C &= 0 \\ -A+B+D &= 1 \\ A+B-C &= 0 \\ -A+B-D &= 0 \end{aligned}\right\} \Rightarrow \quad \begin{aligned} A &= -\tfrac{1}{4} \\ B &= \tfrac{1}{4} \\ C &= 0 \\ D &= \tfrac{1}{2} \end{aligned}$$

$$(*) \Rightarrow \int \frac{-1/4}{x+1} + \frac{1/4}{x-1} + \frac{\frac{1}{2}}{x^2+1}\,dx$$

$$= -\frac{1}{4}\ln|x+1| + \frac{1}{4}\ln|x-1| + \frac{1}{2}\tan^{-1}x + C.$$