Chapter 1 : 1.1 problems 1 1

3 + 4 + 5 + 10 + 17 + 26 + 27 + 36 + 57 + 61 + 63 + 66 + 71 + 72 + 82
+ 84 + 85 + 90 + 95 + 97 + 101

3 Solve the following equations:

a $|2x-4|=6$

$2x-4=6$ or $2x-4=-6$

$2x=10$ $\quad$ $2x=-2$

$x=5$ $\quad$ $x=-1$

solution $=\{-1, 5\}$

please distinguish between set $\{\ \}$ and interval $(\ )$ $[\ ]$ $(\ ]$ $[\ )$

b $|x-3|=2$

$x-3=2$ or $x-3=-2$

$x=5$ or $x=1$

solution $=\{1, 5\}$

(2)

(2)

ⓒ $|2x+3| = 5$

$2x+3 = 5$ or $2x+3 = -5$

$2x = +2$ $\quad$ $2x = -8$

$x = 1$ $\quad$ $x = -4$

solution $= \{1, -4\}$

ⓓ $|7-3x| = -2$

No solution

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(4) Solve the following equations

ⓐ $|2x+4| = |5x-2|$

$2x+4 = 5x-2$ or $2x+4 = -(5x-2)$

$-3x = -6$ $\quad$ $2x+4 = -5x+2$

$\boxed{x = 2}$ $\quad$ $7x = -2$

$\boxed{x = \frac{-2}{7}}$

solution $= \left\{2, \frac{-2}{7}\right\}$

(2)

(3)

(b) $|5-3u| = |3+2u|$

$5-3u = 3+2u$ or $5-3u = -(3+2u)$

$5-3u = -3-2u$

$2 = 5u$

$-u = -8$

$\boxed{u = \frac{2}{5}}$

$\boxed{u = 8}$

solution $= \{8, \frac{2}{5}\}$

(c) $\left|4 + \frac{t}{2}\right| = \left|\frac{3}{2}t - 2\right|$

$4 + \frac{t}{2} = \frac{3}{2}t - 2$ or $4 + \frac{t}{2} = -\left(\frac{3}{2}t - 2\right)$

$\boxed{6 = t}$

$4 + \frac{t}{2} = -\frac{3}{2}t + 2$

$2t = -2$

$\boxed{t = -1}$

(d) $|2s-3| = |7-s|$

$2s-3 = 7-s$ or $2s-3 = -(7-s)$

$2s-3 = s-7$

$3s = 10$

$s = -4$

$\boxed{s = \frac{10}{3}}$

$\boxed{s = -4}$

solutia $= \{-4, \frac{10}{3}\}$

5 Solve the following inequalities

4 4

$|a| < b \Rightarrow -b < a < b$

$|a| > b \Rightarrow a > b$ or $a < -b$

a $|5x-2| \leq 4$

$-4 \leq 5x-2 \leq 4$

$-2 \leq 5x \leq 6$

$\frac{-2}{5} \leq x \leq \frac{6}{5}$

So solution is $= \{x : \frac{-2}{5} \leq x \leq \frac{6}{5}\}$

$= [\frac{-2}{5}, \frac{6}{5}]$

b $|1-3x| > 8$

$1-3x > 8$ or $1-3x < -8$

$-3x > 7$ or $-3x < -9$

$\boxed{x < \frac{-7}{3}}$ $\boxed{x > 3}$

solution $= \{x : x < \frac{-7}{3}$ or $x > 3\}$

$= (-\infty, \frac{-7}{3}) \cup (3, \infty)$

5

(c) $|7x+4| \geq 3$

$7x+4 \geq 3$ or $7x+4 \leq -3$

$7x \leq -7$

$7x \geq -1$

$\boxed{x \leq -1}$

$\boxed{x \geq -\frac{1}{7}}$

solution $= \left\{ x : x \leq -1 \text{ or } x \geq -\frac{1}{7} \right\}$

$= (-\infty, -1] \cup [-\frac{1}{7}, \infty)$

(d) $|6 - 5x| < 7$

$-7 < 6 - 5x < 7$

$-13 < -5x < 1$

$\frac{13}{5} > x > -\frac{1}{5}$

solution $= (-\frac{1}{5}, \frac{13}{5})$

(10) Determine the equation of the line that satisfies the stated requirements and put it in standard Form. (6)

$Ax+By+C=0$ standard form

$y-y_1=m(x-x_1)$ point - slope form

$y=mx+b$ slope - y - intercept form

(10) The line passing through $(-3,5)$ with slope $\frac{1}{2}$

$y-y_1=m(x-x_1)$

$y-5=\frac{1}{2}(x--3)$

$y-5=\frac{1}{2}x+\frac{3}{2}$

$y=\frac{1}{2}x+\frac{3}{2}+5$

$y=\frac{1}{2}x+\frac{13}{2}$

$$\boxed{\frac{1}{2}x-y+\frac{13}{2}=0}$$

One of m

17 The vertical line through $(-1, \frac{7}{2})$

$x = -1$

$x + 1 = 0$

vertical line

$x =$ x-coordinate.

Horizontal line

$y =$ y-coordinate.

26 The line with slope $\frac{1}{5}$ and x-intercept $(3, 0)$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{1}{5}(x - 3)$$

$$y = \frac{x}{5} - \frac{3}{5}$$

$$\frac{x}{5} - y - \frac{3}{5} = 0$$

or $\boxed{x - 5y - 3 = 0}$

27 The line passing through $(2,-3)$ and parallel to

$$x+2y-4=0$$

Solution:

parallel to $x+2y-4=0$
so the two lines have same slope which is $m=\frac{-1}{2}$

$$y-y_1=m(x-x_1)$$

$$y-3=\frac{-1}{2}(x-2)$$

$$y+3=-\frac{1}{2}x+1$$

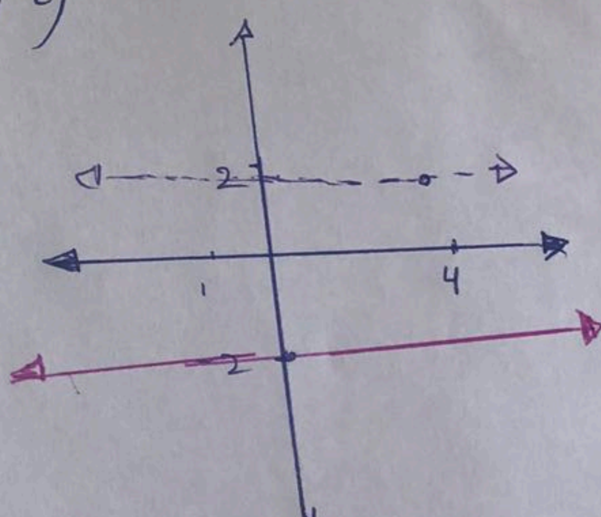
$$\cancel{y=-1}$$

$$y+\frac{1}{2}x+2=0 \Longrightarrow \boxed{2y+x+4=0}$$

35 35. The line passing through $(4,2)$ and parallel to the horizontal line passing through $(1,-2)$

solution: point $=(4,2)$

slope $=0$



$$y-2=0(x-4)$$

$$y-2=0$$

$$\boxed{y=2}$$

$$\boxed{y-2=0}$$

(36) The line passing through $(-1,5)$ and parallel to the horizontal line through $(2,-1)$ (9)

solution: point $(-1,5)$

slope $=0$

$\boxed{y=5}$

$\boxed{y-5=0}$ ✓

(57) (a) Find the equation of the circle with center $(2,5)$ and radius $=3$

~~$y-5=3($~~

$(x-2)^2+(y-5)^2=9$

(b) where does the circle intersect the y-axis

$y\text{-int} \to x=0$

$4+(y-5)^2=9 \to (y-5)=\pm\sqrt{5}$

$\Rightarrow y=5+\sqrt{5}$ or $y=5-\sqrt{5}$

$=5+2.236$ $\quad$ $=5-2.236$

$=7.236$ $\quad$ $=2.7639$

(0.8) $\quad$ $(0,2)$

(5) Does the circle intersect the x-axis? Explain (4)

10

$$(x-2)^2+(y-5)^2=9$$

$$(x-2)^2+5^2=9$$

$$(x-2)^2=9-25=-16$$

so the circle doesn't intersect the x-axis

(61) Find the center and the circle given by the equation

$$0=x^2+y^2+2y-4x-11$$

Solution:

$$x^2-4x+y^2+2y=11$$

x^2	y^2
-4	2
$\frac{-4}{2}=-2$	$\frac{2}{2}=1$
$(-2)^2=4$	$(1)^2=1$

$$x^2-4x+4+y^2+2y+1=11+4+1$$

$$(x-2)(x-2)+(y+1)(y+1)=16$$

$$(x-2)^2+(y+1)^2=16$$

center $=(2,-1)$

radius $=4$

63 a. Convert 75° to radian measure

$$75 * \frac{\pi}{180} = \frac{75\pi}{180} = \frac{15\pi}{36} = \frac{5\pi}{12}$$

Or

$$\frac{75^\circ}{360^\circ} = \frac{??}{2\pi}$$

$$?? = 2\pi * \frac{75}{360}$$ ✓

b. Convert $\frac{17}{12}\pi$ to degree measure

$$\frac{17\pi}{12} \longrightarrow {}^\circ$$

$$\frac{17\pi}{12} * \frac{180}{\cancel{\pi}} = \frac{17 * 180}{12} = \frac{17 * 30}{2} = 17 * 15 = 255^\circ$$

66 Evaluate the following expressions without using calculator

$\sin -x = -\sin x$ ~~$-\cos x$~~

$\cos -x = \cos x$

a. $\sin\left(-\frac{5\pi}{4}\right)$

$$\sin\left(-\frac{5\pi}{4}\right) = -\sin\left(\frac{5\pi}{4}\right)$$

$$= - \; -\sin\frac{\pi}{4}$$

$$= \frac{1}{\sqrt{2}}$$

$\frac{5\pi}{4}$ in Q_3

Reference angle.

$$\frac{5\pi}{4} - \pi = \frac{\pi}{4}$$

Definition:

Let $f: A \longrightarrow B$ be one to one function with range $f(A)$. The inverse function f^{-1} has domain $f(A)$ and range A and is defined by

$$f^{-1}(y) = x \quad \text{iff} \quad y = f(x)$$

for all $y \in f(A)$

(b) $\cos \frac{5\pi}{6}$

$\frac{5\pi}{6}$ in QII

$\frac{2}{\sqrt{2}}$

cos (−)

Reference angle is $\pi - \frac{5\pi}{6} = \frac{\pi}{6}$

$$\cos \frac{5\pi}{6} = -\cos\frac{\pi}{6}$$

$$= -\frac{\sqrt{3}}{2}$$

(c) $\tan \frac{\pi}{3}$

$$\tan\frac{\pi}{3} = \frac{\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

66 (a) $\sin\frac{3\pi}{4}$

$\frac{3\pi}{4}$ in Q_2, sin in Q2 is +ve

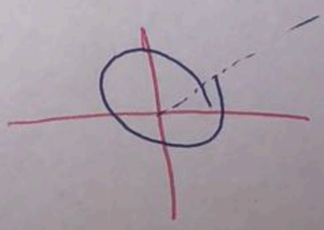
Reference angle is $\pi - \frac{3\pi}{4} = \frac{\pi}{4}$

$$\sin\frac{3\pi}{4} = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$\sin -x = -\sin x$

$\cos -x = \cos x$

(b) $\cos\left(-\frac{13\pi}{6}\right) =$



$$\cos\frac{13\pi}{6} = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

(13)

(c) $\tan\left(\frac{4\pi}{3}\right)$

$\frac{4\pi}{3}$ in Q3 , tan +.

Reference angle is $\frac{4\pi}{3} - \pi = \frac{\pi}{3}$

$\tan\frac{4\pi}{3} = \tan\frac{\pi}{3} = \sqrt{3}$

(71) Solve $2\cos\theta \cdot \sin\theta = \sin\theta$ on $[0, 2\pi)$

$2\cos\theta \sin\theta - \sin\theta = 0$

$\sin\theta\,[2\cos\theta - 1] = 0$

QII	QI
sin +	all
$\pi - \theta$	θ
QIII	QIV
tan +	cos +
$\theta - \pi$	$360 - \pi$

$\sin\theta = 0 \longrightarrow \theta = 0 , \pi$

$2\cos\theta - 1 = 0 \longrightarrow \cos\theta = \frac{1}{2} \rightarrow$

$\theta = 60 = \frac{\pi}{3}$

$\theta = 300 = 300 \times \frac{\pi}{180} = \frac{30\pi}{18} = \frac{5\pi}{3}$

(72) Solve $\sec^2 x = \sqrt{3} \tan x + 1$ on $[0, \pi)$ (14)

$$1 + \tan^2 x = \sqrt{3} \tan x + 1$$

$$\tan^2 x - \sqrt{3} \tan x = 0$$

$$\tan x [\tan x - \sqrt{3}] = 0$$

$\tan x = 0$ or $\tan x = \sqrt{3}$

$x = 0$ $x = \frac{\pi}{3}$

82 Solve for x

a $3^x = 81$

$3^x = 3^4$

$\boxed{x=4}$

we can also solve:

$3^x = 81$

take ln to both sides

$\ln 3^x = \ln 81$

$x \ln 3 = \ln 81$

$x = \frac{\ln 81}{\ln 3} = \frac{\ln 3^4}{\ln 3} = \frac{4 \ln 3}{\ln 3} = 4$

So $\boxed{x=4}$

b $9^{2x+1} = 27$

$(3^2)^{2x+1} = 3^3$

$3^{2.(2x+1)} = 3^3$

$3^{4x+2} = 3^3 \Longrightarrow$

$4x+2=3$

$4x=1$

$\boxed{x=\frac{1}{4}}$

b

Qf

$\ln 9^{2x+1} = \ln 27$

$(2x+1)\ln 9 = \ln 27$

$2x+1 = \frac{\ln 27}{\ln 9} = \frac{\ln 3^3}{\ln 3^2} = \frac{3\ln 3}{2\ln 2} = \frac{3}{2}$

So $2x+1 = \frac{3}{2}$

$2x = \frac{1}{2}$

$\boxed{x = \frac{1}{4}}$

(c) $10^{5x} = 1000$

$10^{5x} = 10^3$

$5x = 3$

$\boxed{x = \frac{3}{5}}$

Or $10^{5x} = 1000$

take ln $\ln 10^{5x} = \ln 1000$

$5x \ln 10 = \ln 1000$

$x = \frac{\ln 1000}{5 \cdot \ln 10} = \frac{\ln 10^3}{5 \cdot \ln 10}$

$= \frac{3}{5}$

C

$\boxed{84}$ Solve for x

$$\boxed{a^{\log_a x} = x}$$

$$\boxed{\log_a a^x = x}$$

(a) $\ln(2x-3) = 0$

take e to both sides

$$e^{\ln(2x-3)} = e^0$$

$$2x-3 = 1$$

$$2x = 4$$

$$x = \cancel{\tfrac{4}{2}}\ 2$$

You have to check:

$$\ln(2\cdot 2 - 3) = \ln(4-3) = \ln 1 = 0 \checkmark$$

<u>or</u> transform the question from logarithmic form to exponential form

$$\boxed{\begin{array}{c} a^b = c \\ \hline \log_a c = b \end{array}}$$

$$\ln(2x-3) = 0$$

$$e^0 = 2x-3$$

$$1 = 2x-3$$

$$2x = 4$$

$$x = 2$$

Check!

d

(b) $\log_2(1-x) = 3$

$2^3 = 1-x$

$8 = 1-x$

$x = -7$

Check!

$\log_2(1-7) = \log_2 8 = 3$ ✓

Or $\log_2(1-x) = 3$

take 2

$2^{\log_2(1-x)} = 2^3$

$1-x = 8$

$x = -7$

Check.

(c) $\ln x^3 - 2\ln x = 1$

$\ln a - \ln b = \ln \frac{a}{b}$

$\ln x^3 - \ln x^2 = 1$

$\ln \frac{x^3}{x^2} = 1$

$\boxed{\ln x = 1}$

$x = e^1$

Check

$\ln e^3 - 2\ln e = \ln e^3 - \ln e^2$

$= \ln \frac{e^3}{e^2} = \ln e = 1$ ✓

85 Simplify each expression and write in the standard form $a+bi$

- $(3-2i)-(-2+5i)$

$3-2i+2-5i$

$=5-7i$

90 $(2-3i)(5+2i)$

$= 2.5 + 2.2i - 3i.5 - 3i.2i$ $\qquad i.i$ $\quad \sqrt{-1}.\sqrt{-1} = -1$

$= 10 + 4i - 15i + 6$

$= 16 - 11i$

95 $Z = 3-2i$

$u = -4+3i$

$v = 3+5i$

$w = 1-i$

$\overline{Z+v} = \overline{3-2i+3+5i} = \overline{6+3i} = 6-3i$

97 $\overline{vw}$

$\overline{(3+5i)(1-i)} = \overline{3-3i+5i+5}$

$= \overline{8+2i} = 8-2i$

101 $2x^2-3x+2=0$ C

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-{-3}\pm\sqrt{(-3)^2-4\cdot2\cdot2}}{2\cdot2}$$

$$=\frac{3\pm\sqrt{9-16}}{4}$$

$$=\frac{3\pm\sqrt{-7}}{4}=\frac{3\pm\sqrt{7}i}{4}$$

Solution $=\left\{\frac{3+\sqrt{7}i}{4},\ \frac{3-\sqrt{7}i}{4}\right\}$