$4+7+9+18+21+33+36+58+74+81+82+83+84+85+100$

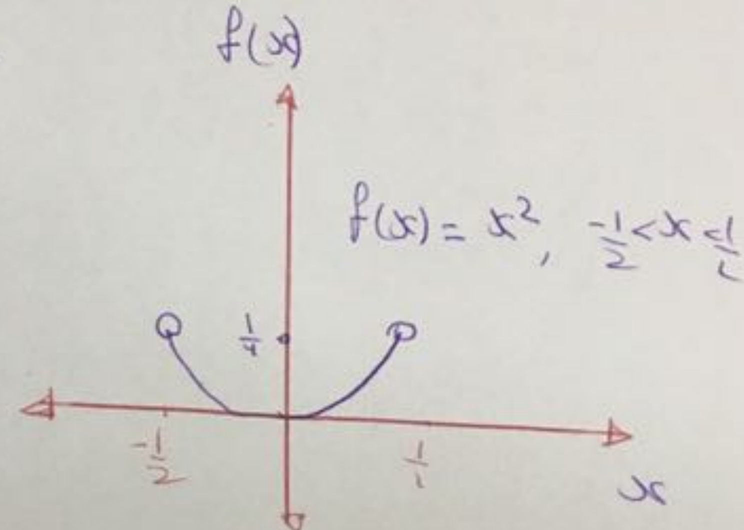
(4) State the range of the given function. Graph each function

$f(x) = x^2, \quad -\frac{1}{2} < x < \frac{1}{2}$

Solution:

Domain $= (-\frac{1}{2}, \frac{1}{2})$

Range $= [0, \frac{1}{4}]$



(7) Sketch the graph of each function and decide in each case whether the function is even, odd or doesn't show any obvious symmetry. Then use the criterion in subsection 1.2.1 to check your answer.

$f(x) = 2x$

solution

symmetric about the origin

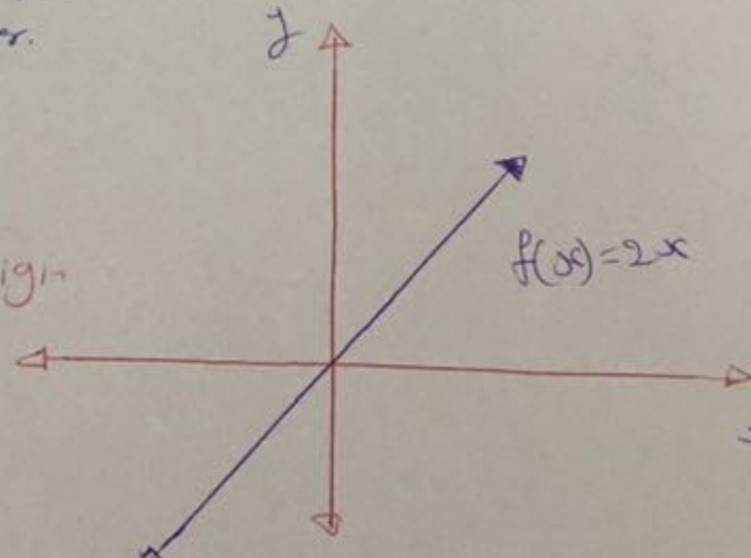
→ odd

or

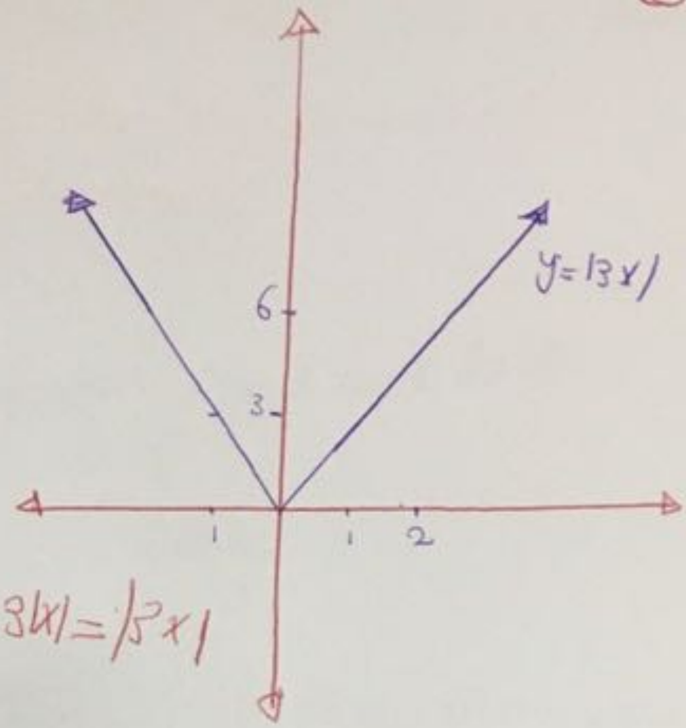
$f(-x) = 2(-x) = -2x$

$-f(x) = -(2x) = -2x$

$f(-x) = -f(x)$ so odd



9 $f(x) = |3x|$



Solution:
Symmetric about the y-axis
So its even.

Or

$f(x) = |3x|$

$f(-x) = |3.-x| = |-3x| = |-3|.|x| = 3|x| = |3x|$

$|-3x| = |3x|$

So its even

---

12 suppose that $f(x) = x^4, \; x \geq 3$

$g(x) = \sqrt{x+1}, \; x \geq 3$

Find $(f \circ g)(x)$ together with its domain

Solution:

$(f \circ g)(x) = f(g) = f(\sqrt{x+1})$

$= (\sqrt{x+1})^4 = (x+1)^2$

Domain of fog is we chose values of x such that $g(x)$ belong to domain of $f(x)$

$\sqrt{x+1} \geq 3$

$\boxed{x \geq 8}$

21 Use the graphing calculator to graph $f(x) = x^2, x \geq 0$ and $g(x) = x^4, x \geq 0$, together, for which values of x is $f(x) > g(x)$ and for which is $f(x) < g(x)$

Solution:

$f(x) = x^2 . \; g(x) = x^4$

$f(x) > g(x)$

$\sqrt{x^2} > \sqrt{x^4}$

$|x| > x^2$

x is positive so $|x| = x$

$x > x^2$

$x^2 - x < 0$

$x(x-1) < 0$

$x = 0, \; x = 1$

$+++ \; --- \; +++$ sign of $\boxed{x^2 - x}$

$0 \quad 1$

so $f(x) > g(x)$ if $x \in (0,1)$

and $f(x) < g(x)$

$\sqrt{x^2} < \sqrt{x^4}$

$|x| < x^2$

$x < x^2$

$\rightarrow x^2 - x > 0$

$x(x-1) > 0$ if $\boxed{x > 1}$ $(1, \infty)$

or $\cancel{x < 0}$

$++ \; -- \; ++$

$0 \quad 1$

$$\boxed{\begin{array}{l} f(x) > g(x) \text{ for } 0 < x < 1 \\ f(x) < g(x) \text{ for } x > 1 \end{array}}$$

33 find the largest possible domain and determine its range

$$f(x) = \frac{1}{1-x}$$

solution :-

Domain is $1-x \neq 0$

$x \neq 1$

so $D : \mathbb{R} - \{1\}$

Range $\mathbb{R} - \{0\}$

---

36 $f(x) = \frac{1}{1+x^2}$

Domain $= \mathbb{R}$

Range $= (0, 1]$

Note that

$1 \leq 1 + x^2$

$\frac{1}{1+x^2} \leq 1$

$\frac{1}{1+x^2}$ can't be zero

since

$\frac{a}{b} = 0$ if $a = 0$

58 Assume that the population size at time $t = N(t)$ is that $N(t) = 40 \cdot 2^t$, $t \geq 0$

a) Find population size at time $t=0$

b) Show that $N(t) = 40\, e^{t \ln 2}$, $t \geq 0$

c) How long will it take the population size reaches 1000?

Solution:

a) $N(t) = 40 \cdot 2^t$, $t \geq 0$

$N(0) = 40(2^0) = 40$

b) $a^b = c^{\log_c (a^b)}$

or $\boxed{x = \log_a (a^x)}$ and $\boxed{x = a^{\log_a x}}$

$\boxed{\log_e = \ln}$

so $40 \cdot 2^t = 40 \cdot e^{\log_e (2^t)} = 40 \cdot e^{\ln 2^t}$

$= \cancel{40\, e^{t \ln 2}} = 40\, e^{t \ln 2}$, $t \geq 0$

c) $N(t) = 40 \cdot 2^t$

$\frac{1000}{40} = \frac{40}{40} \cdot 2^t$

$25 = 2^t \longrightarrow$

$\ln 25 = \ln 2^t$

$t = \frac{\ln 25}{\ln 2} = 4.64$ years

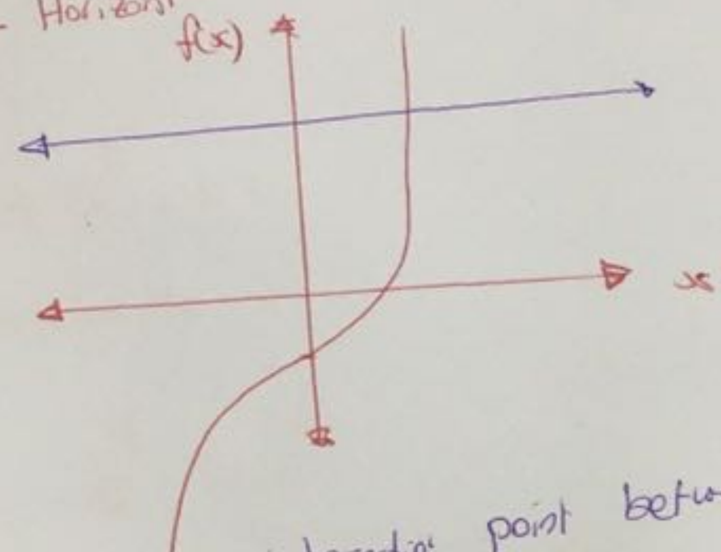
7. (a) Show that x^3-1 is one to one and find its inverse

(b) Graph $f(x)$ and $f^{-1}(x)$ together. with line $y=x$

solution

(a) To show that a function is one to one we have two ways

(1) Graph + Horizontal line test



always there is one intersection point between the horizontal line and $y=x^3-1$

so its one to one

one to one
$x_1 \neq x_2$
so $f(x_1) \neq f(x_2)$

Or

(2) assume that $f(x_1)=f(x_2)$

$$x_1^3-1 = x_2^3-1$$

$$x_1^3 = x_2^3$$

$$\boxed{x_1 = x_2}$$

So $f(x)$ is one to one.

$f(x) = x^3 - 1$

$y = x^3 - 1$

$x = y^3 - 1$

$y^3 = x + 1$

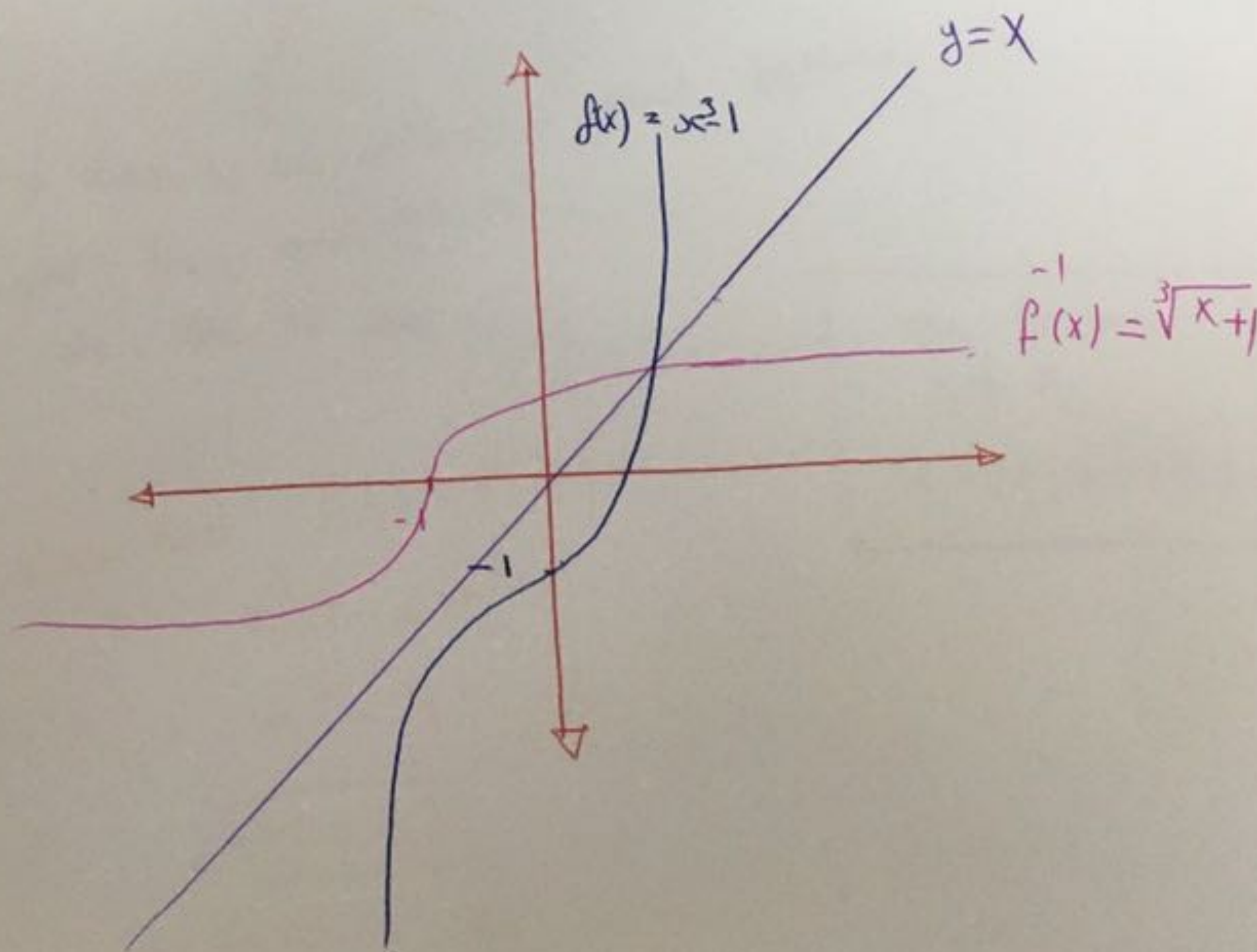
$y = \sqrt[3]{x+1}$

$f^{-1}(x) = \sqrt[3]{x+1}$

So $f(x) = x^3 - 1$ and $f^{-1}(x) = \sqrt[3]{x+1}$

x^3 } 1 unit down

$\sqrt[3]{x}$ — one unit left



8) Simplify the following expressions

a) $2^{5\log_2 x} = 2^{\log_2 x^5} = x^5$

Using the Rule $a^{\log_a x} = x$

b) $3^{4\log_3 x} = 3^{\log_3 x^4} = x^4$

c) $5^{5\log_{\frac{1}{5}} x} = 5^{\log_{\frac{1}{5}} x^5}$

$= \left(5^{-1 \cdot -1}\right)^{\log_{\frac{1}{5}} x^5}$

$= \left(\left(5^{-1}\right)^{-1}\right)^{\log_{\frac{1}{5}} x^5}$

$= \left(5^{-1}\right)^{-\log_{\frac{1}{5}} x^5} = \left(5^{-1}\right)^{\log_{\frac{1}{5}} \left(x^5\right)^{-1}}$

$= \left(\frac{1}{5}\right)^{\log_{\frac{1}{5}} x^{-5}}$

$= \boxed{x^{-5}}$

(d) $\log_{\frac{1}{2}} 4^x = \log_{\frac{1}{2}} (2^2)^x$

we used $\boxed{\log_a a^x = x}$

$= \log_{\frac{1}{2}} (2^{-1})^{-2x}$

$= \log_{\frac{1}{2}} \left(\frac{1}{2}\right)^{-2x}$

This is part d from Ex. 8c

$= -2x$

(d/81) $4^{-2\log_2 x} = 4^{\log_2 x^{-2}} = (2^2)^{\log_2 x^{-2}} = 2^{2\log_2 x^{-2}}$

$= 2^{\log_2 (x^{-2})^2}$

$= 2^{\log_2 x^{-4}} = x^{-4}$

(e) $2^{3\log_{\frac{1}{2}} x} = 2^{\log_{\frac{1}{2}} x^3} = \left((2^{-1})^{-1}\right)^{\log_{\frac{1}{2}} x^3}$

$= \left(\frac{1}{2}\right)^{-\log_{\frac{1}{2}} x^3} = \left(\frac{1}{2}\right)^{\log_{\frac{1}{2}} (x^3)^{-1}}$

$= \frac{1}{2}^{\log_{\frac{1}{2}} x^{-3}} = x^{-3}$

$$4^{-\log_{\frac{1}{2}} x}$$

$$\left(2^{2}\right)^{-\log_{\frac{1}{2}} x}$$

$$= 2^{-2\log_{\frac{1}{2}} x}$$

$$= \left(\left(2^{-1}\right)^{-1}\right)^{-2\log_{\frac{1}{2}} x}$$

$$= \left(\tfrac{1}{2}\right)^{(-1)\cdot\left(-2\log_{\frac{1}{2}} x\right)}$$

$$= \left(\tfrac{1}{2}\right)^{2\log_{\frac{1}{2}} x} = \tfrac{1}{2}^{\log_{\frac{1}{2}} x^2} = \boxed{x^2}$$

---

82 simplify the follows

a $\log_4 16^x = \log_4\left(4^2\right)^x = \log_4 4^{2x} = \boxed{2x}$

---

b $\log_2 16^x = \log_2\left(2^4\right)^x = \log_2 2^{4x} = \boxed{4x}$

c) $\log_3 27^x$

$= \log_3 \left(3^3\right)^x$

$= \log_3 3^{3x} = 3x$

d) Done in page 9

e) $\log_{\frac{1}{2}} 8^{-x}$

$= \log_{\frac{1}{2}} \left(2^3\right)^{-x} = \log_{\frac{1}{2}} \left(\left(2^{-1}\right)^{-1}\right)^{-3x}$

$= \log_{\frac{1}{2}} \left(\frac{1}{2}\right)^{3x}$

$= 3x$

f) $\log_3 9^{-x} = \log_3 \left(3^2\right)^{-x} = \cancel{\log_3 \left(\left(3^{-1}\right)^{-2}\right)^{-x}}$

$\cancel{= \log_3 \left(3^{-1}\right)^{2x}}$

$= \log_3 3^{-2x}$

$= \boxed{-2x}$

83 Simplify the following

a) $\ln x^2 + \ln x^3$

$= \ln x^5$

$= \boxed{5 \ln x}$

$\ln ab = \ln a + \ln b$

$\ln \frac{a}{b} = \ln a - \ln b$

$\ln a^k = k \ln a$

b) $\ln x^4 - \ln x^{-2}$

$= \ln \frac{x^4}{x^{-2}}$

$= \ln x^6 = \boxed{6 \ln x}$

c) $\ln (x^2 - 1) - \ln (x+1)$

$= \ln \frac{x^2 - 1}{x+1}$

$= \ln \frac{(x-1)(x+1)}{x+1}$

$= \boxed{\ln (x-1)}$

d) $\ln x^{-1} + \ln x^{-3}$

$= \ln x^{-1} \cdot x^{-3}$

$= \ln x^{-4} = \boxed{-4 \ln x}$

(84) Simplify the following expressions

$$a^{\log_a x} = x$$

and

$$e^{\ln x} = x \quad \text{since } \ln = \log_e$$

(a) $e^{3\ln x}$

$= e^{\ln x^3} = \boxed{x^3}$

(b) $e^{-\ln(x^2+1)}$

$= e^{\ln(x^2+1)^{-1}}$

$= (x^2+1)^{-1} = \boxed{\dfrac{1}{x^2+1}}$

(c) $e^{-2\ln \frac{1}{x}}$

$= e^{\ln\left(\frac{1}{x}\right)^{-2}}$

$= e^{\ln x^2}$

$= x^2$

$$\left(\frac{1}{x}\right)^{-2} = \left(\frac{x}{1}\right)^{2} = x^2$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$$

(d) $e^{-2\ln x}$

$= e^{\ln x^{-2}} = \boxed{x^{-2}} = \boxed{\dfrac{1}{x^2}}$

85) Write the following in base e and simplify

$$\boxed{a^b = \log_c c^{a^b}}$$

and

$$\boxed{a^b = c^{\log_c a^b}}$$

a) $3^x = e^{\ln 3^x}$

$= \boxed{e^{x\ln 3}}$

b) 4^{x^2-1}

$= e^{\ln 4^{x^2-1}}$

$= \boxed{e^{(x^2-1)\ln 4}}$

c) 2^{-x-1}

$= e^{\ln 2^{(-x-1)}} = \boxed{\cancel{(-x-1)\ln 2}\ e}$

$= e^{(-x-1)\ln 2}$

d) $3^{-4x+1} = e^{\ln 3^{-4x+1}}$

$= e^{(-4x+1)\ln 3}$

Let $f(x) = -\frac{3}{2} \sin\left(\frac{\pi}{3} x\right)$, $x \in R$

find the amplitude and period

Recall

$f(x) = a \sin(kx)$

amplitude $= |a|$

period $= \frac{2\pi}{|k|}$

amplitude $= \left|\frac{-3}{2}\right| = \frac{3}{2}$

period $= \frac{2\pi}{\frac{\pi}{3}} = 6$