Appendix 7 :- Complex numbers

Real part ← $Z = x + iy$ → Imaginary part

Natural ← $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}$ → Complex

Integers, Rational, Real

Conjugate $\bar{Z} = x - iy$ $\quad \overline{Z_1 + Z_2} = \bar{Z}_1 + \bar{Z}_2$ $\quad \overline{Z_1 Z_2} = \bar{Z}_1 \cdot \bar{Z}_2$

if $Z_1 = Z_2$
$Re(1) = Re(2)$
$Im(1) = Im(2)$

Modulus $|Z| = \sqrt{x^2 + y^2}$
$|Z| = |\bar{Z}| = |-\bar{Z}|$

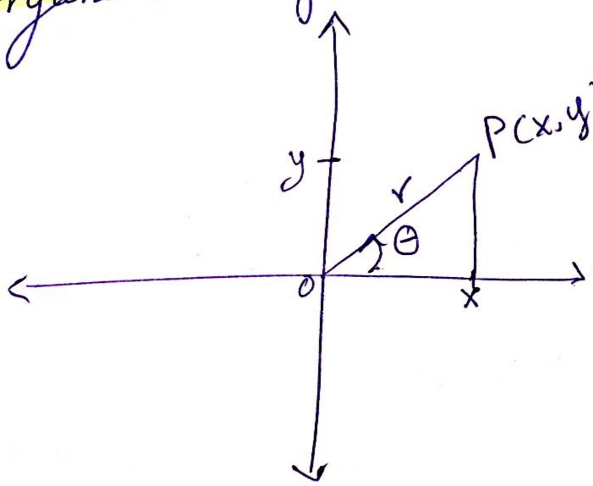
Proprieties :-

1- Addition : $(Z_1 + Z_2) = (a + c) + (b + d)i$

2- Multiplication : $Z_1 Z_2 = (a + bi)(c + di)$

3- Division $\frac{Z_1}{Z_2} = \frac{a + bi}{c + di} \times \frac{(c - di)}{(c - di)}$

Argand Diagram :-



length: $r = \sqrt{x^2 + y^2}$: $r \geq 0$

θ : Argument of Z

Euler formula:

$e^{i\theta} = \cos\theta + i\sin\theta$ } we use it to find Z^n

$Z = re^{i\theta}$

Hence: $|w||\bar{w}| = r^2 = w \cdot \bar{w}$

$w = x + iy$, $\bar{w} = x - iy$

De Moivre's Theorem

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

Alaa Eltaimi

Th: Every polynomial of degree n has exactly n roots

How to solve problems :-

to find The Argument of Z:

S1 :- we find the length.

S2 : نجيب قاعدة [illegible]

S3 : we use The Euler formula $\left(e^{i\theta} = \cos\theta + i\sin\theta \right)^n$

to find Power of Z

The same way up

and Then we us $Z = \left(re^{i\theta}\right)^n$

to Draw Argand Diagram That satisfies a Given Conditions

$|Z+1| = |Z-1|$

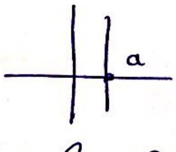
or $|Z+i| \neq Z+1$ } we solve The equation

Notes: if we cancel y → it mean y takes $(-\infty, \infty)$ values

→ $y \in (-\infty, \infty)$

If $X = a$ or $X \geq a$ or $X > a$ or $X < a$

→ we draw it like This



→ If $|Z| = 2$ it's a circle with a 2 radius

→ $|Z-1| = 2$ its a circle goes ~~up right~~ left

To solve $X^n - 1$ or $X^n - 2$ or $X^n - i$ or ...

we move a to The right and $X = (a)^{1/n}$

we say $a = a + 0y$ Then Euler formula