Chap 4: Increasing and decreasing functions

Basic Def:- f is a fonction defind on interval I:-

→ if wherever $x_2 > x_1$ we have $f(x_2) > f(x_1)$ $\forall x \in I$ then f is increasing on I

→ if wherever $x_2 > x_1$ we have $f(x_2) < f(x_1)$ $\forall x \in I$ then f is decreasing on I

Th (that we use):- f is cont on $[a,b]$ & diff on (a,b) $\xrightarrow{\text{Then}}$

- if $f'(x) > 0$ $\forall x \in (a,b)$ → f is increasing on $[a,b]$
- if $f'(x) < 0$ $\forall x \in (a,b)$ → f is decreasing on $[a,b]$

Extreme values

They exist: when $f'(x) = 0$, $f'(x)$ DNE

and may occurs when for $y = f(x)$
① end points
② Interior points

Absolute

- M is an Abs. Max at $c \in I$ if $M = f(c) \geq f(x)$ and its a local Max on small interval around c
- M is an Abs Min m at c if $m = f(c) \leq f(x)$ and its a local min on small interval around c

local

- M is a local Max on I if $M = f(c) \geq f(x)$ $\forall x \in I$
- m is a local min on I if $M = f(c) \leq f(x)$ $\forall x \in I$

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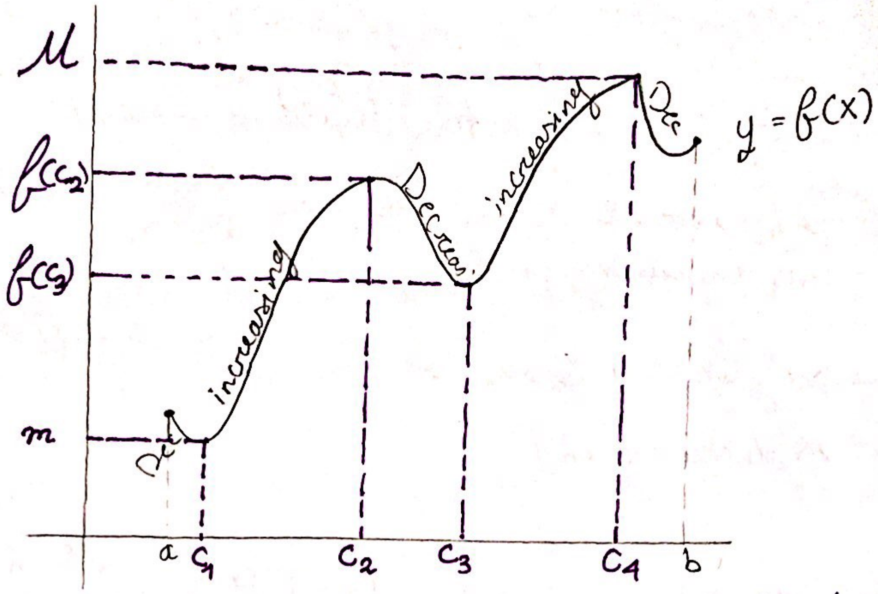
Sketch

- Analysing -

- a, b End points
- c_1, c_2, c_3, c_4 } Interior points

critical points:-

$(c_1, f(c_1))$
$(c_2, f(c_2))$
$(c_3, f(c_3))$
$(c_4, f(c_4))$



- f has Abs. Max of M at C_4 → + also local Max
- f has Abs. Min of m at C_1 → + also local Min
- f has local Max of $f(c_2)$ at C_2
- f has local Min of $f(c_3)$ at C_3

R! : Abs ⇒ local

R! : if f = X Then (اقتران خطي) Then f has Abs. Max & Abs Min at all I's

Theory: if f is cont on [a,b] then f has Abs. Max & Abs Min

VIN

the extreme values May occure for $y = f(x)$ at

- end points
- Interior points ∈ D:
  - $f'(x) = 0$
  - f' is not defined

* To find them we check the ~~ex~~ critical points and the end points

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Critical points

Interior
S.t:-
$f'(0) = 0$ or f' is undefined at these points

Th: If f is diff on I and has extreme values at $x = c \in I$ Then $f'(c) = 0$

But $f'(Z) = 0$ Then f may not have extreme values at $x = Z$

to classify the critical points we use either

The FDT
First Derivative Test

- suppose that f has critical point at $x = c$ and f' exists on open I containing c

Then:-

[1] if f' changes sign from + to − at $x = c$ then $f(c)$ is local Max

[2] if f' changes sign from − to + at $x = c$ then $f(c)$ is local Min

[3] if f' does not change sign at $x = c$ then f does not have extreme values at $x = c$

The SDT
Second derivative Test

- suppose $f'(c) = 0$ and f'' is Cont on an open I containing c. Then:-

[1] if $f''(c) > 0$ then $f(c)$ is local min

[2] if $f''(c) < 0$ then $f(c)$ is local Max

[3] if $f''(c) = 0$ Then the test fails

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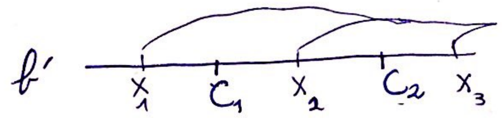
Remark :- $f''(c)$ → ≥ 0 → Con Cave up
→ ≤ 0 → Con Cave down

inflection point : f has an inflection point at $x = c$
if : $\boxed{1}$ f has tangent at $x = c$ and to find it we use: $f''(x) = 0$
$\boxed{2}$ f changes concavity

Remarks for Solving Questions:-

- when we find the critical points and we want to check when is f increasing or decresing we use points $x_1, x_2 \ldots$ that lies in the I and we calculate f' at these points

Exp



we find f'
and when $f' > 0$ Then increase
and when $f' < 0$ Then decreasing

Th (Rolls Th)

$f(x)$ has to be:-
1- $f(x)$ is cont on $[a, b]$
2- $f(x)$ is diff on (a, b)
3- $f(a) = f(b)$

→ $\exists$ at least one point $c \in (a, b)$ s.t $f'(c) = 0$

Th (Mean value Th)

if :-
1- $f(x)$ is Cont on $[a, b]$
2- $f(x)$ is diff on (a, b)

→ $\exists$ one point $c \in (a, b)$ s.t

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

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Sketching functions

• steps :-

1. find the Domain $D(f) =$
2. find the Asymptotes

   How to find them?
   Go back to Chap: 2

3. find the critical points
4. find th decreasing & increasing Intervals of $f(x)$

   How to Know a point is critical or not
   Go back to Chap: 4

5. find the Intervals of concavity for $f(x)$
6. find inflection points

   How to Know a point is an inflection points or not?
   Go back to chap: 4

7. find local Max and Min points

Th: Rolls Th:- if $f(x)$ is ① cont on $[a,b]$ and diff on (a,b)
② $f(a) = f(b)$ Then
→ $\exists$ at least one point $c \in (a,b)$
s.t $f'(c) = 0$

Th: Mean Th: if $f(x)$ is ① cont on $[a,b]$ & diff on (a,b) then
→ $\exists$ one point $c \in (a,b)$ s.t
$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Haa Etaiwi