Chap 6 :-

6.1 • Volumes using Cross sections

Volumes Rules

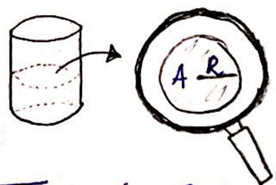
1- cylindrical solid:-
V=(Base Area) Height

2-

There is 3 methods used to find a volume of a given solid

Disk Method.

• It's a special case of Washer Method. [$r(x)=0$]



$A = \pi R^2$

• There is 2 cases :-

$Cs \perp x$-axis
$A(x) = \pi R^2(x)$

$Cs \perp y$-axis
$A(y) = \pi R^2(y)$

Washer Method.

• 2 cases
we use it when the solid does not border on or cross the axis of revolution

→ If the $Cs \perp x$-axis which results by Rotation about x-axis with outer Radius $R(x)$ and Inner Radius $r(x)$ then

$$V = \int_a^b A(x)\,dx = \int_a^b \pi\left[R^2(x) - r^2(x)\right]dx$$

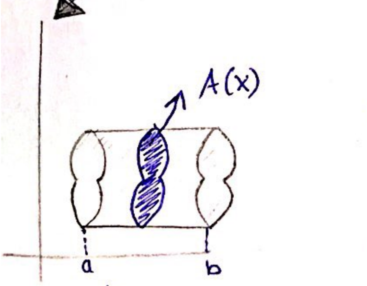
→ If the $Cs \perp y$-axis which results by Rotation about y-axis with outer Radius $R(x)$ and Inner Radius $r(x)$ then

$$V = \int_c^d \pi\left[R^2(y) - r^2(y)\right]dy$$

Shell Method

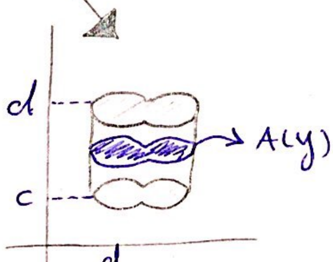
Slicing by parallel Planes:-

* How to find a volume given
- Graph the solid.
- Determine the cross section
- Then if $Cs \perp x$-axis : 1st Case
  if $Cs \perp y$-axis : 2nd Case



• $V = \int_a^b A(x)\,dx$

• Rotation about x-axis or any line parallel to it



• $V = \int_c^d A(y)\,dy$

• Rotatation about y-axis or any line parallel to it

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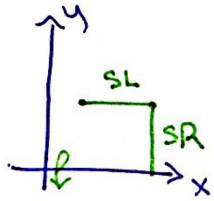
6.2 Shell method:

We use this method to find the volume of a solid generated by revolving a given region about:-

X-axis $\quad d \to$ c,d على محور الصادات

$$V = 2\pi \int_c^d (\text{shell Radius})(\text{shell length})\, dy$$

- distance between $\boxed{\text{shell length}}$ and the axis of Revolution (x)
  SR
- the segment's length That is Parallel to the axis of revolution (x)
  SL

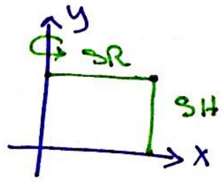


y-axis

$$V = 2\pi \int_a^b (\text{shell Radius})(\text{shell hight})\, dx$$

a,b على محور السينات

- distance between the shell's hight and the axis of Revolution (y)
- sigment's hight that is parallel to the axis of Revolution (y)



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6.3 Arc Length

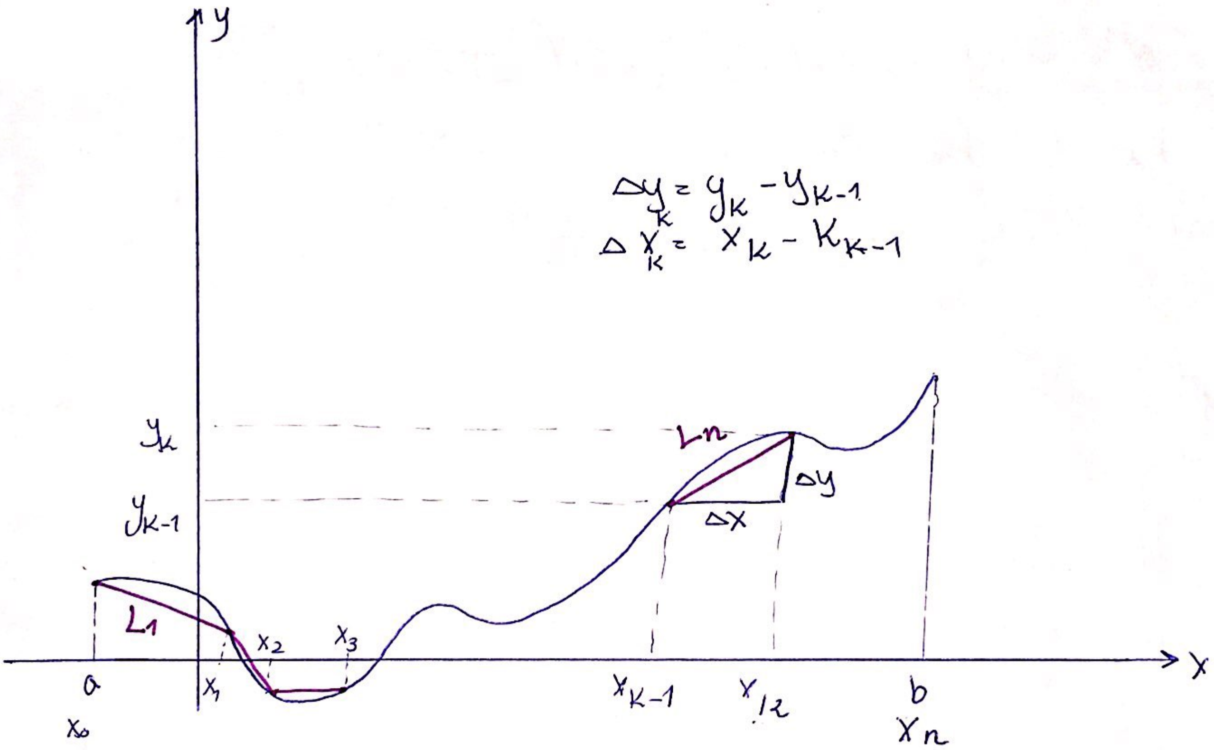
If $f'(x)$ is cont on $[a,b]$ then the Arc length of the curve $y=f(x)$ is given by:-

$$L = \int_a^b \sqrt{1+(f'(x))^2}\,dx = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

Explanation:-

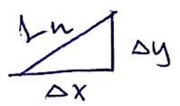
How?

- Let's say you have the curve $f(x)$, wich happenels to be continuous and diff at $[a,b]$ WoW!



L: is the true length
$\tilde{L}$: Approximated length

$$\tilde{L} = L_1 + L_2 + \cdots + L_n = \sum_{k=1}^{n} L_k = \sum_{k=1}^{n} \sqrt{(\Delta x)^2 + (\Delta y)^2}$$



Remember MVT
Mean Value Theorem

$$f'(c) = \frac{f(b)-f(a)}{b-a} = \frac{\Delta y}{\Delta x}$$

$\Longrightarrow$ By MVT there is $c_k \in (x_{k-1}, x_k)$

$$f'(c_k) = \frac{\Delta y_k}{\Delta x_k} \Rightarrow \Delta y_k = f'(c_k)\,\Delta x_k$$

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But $\tilde{L} = \sum_{k=0}^{n} \sqrt{(\Delta x_k)^2 + (f'(c_k)\,\Delta x_k)^2}$

$\tilde{L} = \sum_{k=0}^{n} \sqrt{1 + f'(c_k)}\;\Delta x_k$

→ to ~~mo~~ improve $\tilde{L}$ ⇒ we use n numbers of sub intervals as n gets large

$L = \lim_{n\to\infty} \tilde{L}$

$= \lim_{n\to\infty} \sum_{k=1}^{n} \sqrt{1+f'(c_k)^2}\;\Delta x_k$

$= \int_a^b \sqrt{1 + f'(x)^2}\,dx$ Finally

Remark: very important and Tricky ⚠

- f' has to be Continuous on $[a,b]$

If not :- Try ⇒ $L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$

$\boxed{6.4}$

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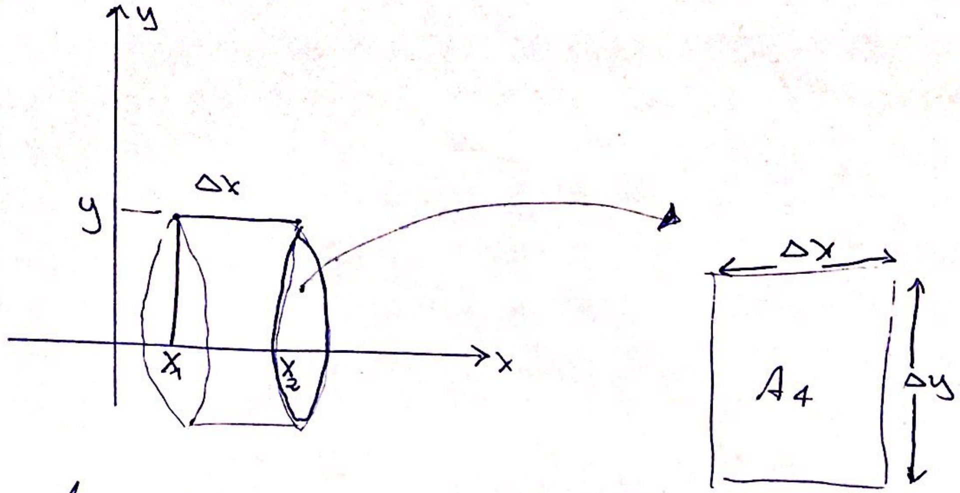
64 Area of surface using Revolution



Area : $(\Delta x)(2y\pi$

Area = $(\Delta x)(\Delta y)$

⊛ X axis :-

The surface area of the region ~~bounded~~ generated by Revolving Δx about X-axis is :-

$(2\pi y \, \Delta x)$

Definitions :-

X axis

If $y = f(x) \geq 0$ is continuously differentiable on interval $[a,b]$ then the surface Area of the region generated by revolving the curve $y = f(x)$ about X-axis is

$$\boxed{S = \int_a^b 2\pi y \sqrt{1 + \underbrace{f'(x)^2}_{\left(\frac{dy}{dx}\right)^2}} \, dx}$$

y-axis

If $x = g(y) \geq 0$ is cont diff on $I = [c,d]$ then the surface Area of the region Generated by Revolving the curve $x = g(y)$ about y-axis is :-

$$\boxed{S = \int_c^d 2\pi \, g(y) \sqrt{1 + \underbrace{(g'(y))^2}_{\left(\frac{dx}{dy}\right)^2}} \, dy}$$

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