

7.2: Natural logarithms

Def: defined by :- $\ln x = \int_1^x \frac{1}{t} dt$

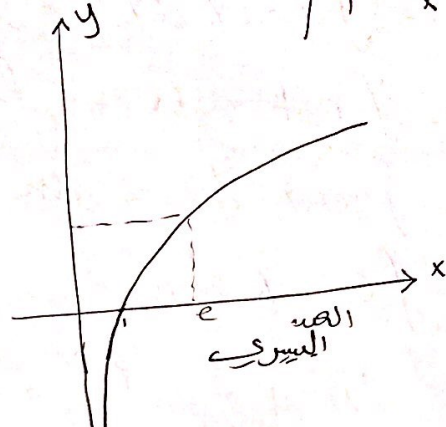
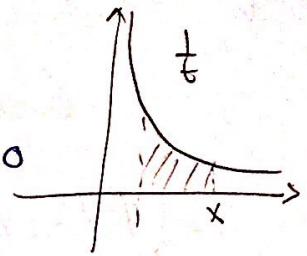
$$D: (0, \infty)$$

$$R: (-\infty, \infty)$$

Notice that :- $\lim_{x \rightarrow \infty} \ln x = 0$

$$\lim_{x \rightarrow 0^+} \ln x = -\infty$$

$x > 0$



• Differentiation of Natural logarithm

1- If $g(x)$ is diff and never zero and $y = \ln g(x)$

$$\text{then } y' = \frac{dx}{dx} = \frac{g'(x)}{g(x)}$$

2- and $\int \frac{g'(x)}{g(x)} = \ln |g(x)| + C$

↳ the number should always be positive
That's why we use Absolute value

• Properties of Natural logarithm

for $a > 0$ & $b > 0$:-

1- $\ln(a)(b) = \ln a + \ln b$

2- $\ln \frac{(a)}{(b)} = \ln a - \ln b$

3- $\ln a^r = r \ln a$

4- $\ln \frac{1}{a} = \ln 1 - \ln a = 0 - \ln a = -\ln a$

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* Natural logarithm and trigonometric functions :-

- 1- $\int \tan x \, dx = \ln |\sec x| + C = -\ln |\cos x| + C$
- 2- $\int \cot x \, dx = -\ln |\csc x| + C = \ln |\sin x| + C$
- 3- $\int \sec x \, dx = \ln |\sec x + \tan x| + C$
- 4- $\int \csc x \, dx = -\ln |\csc x + \cot x| + C$

Why? (good question :P)

• to prove it :-

$$1- \int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx = \int \frac{-(\cos x)'}{\cos x} \, dx$$

$(\cos x)' = \sin x$ \nearrow

$$= -\ln |\cos x| + C$$

$$= \ln |\sec x| + C$$

VI Remark :- In logarithms $-\ln x = \ln \frac{1}{x}$ (it is proved in the first page)

$$2- \int \cot x \, dx = \int \frac{\cos x}{\sin x} \, dx = \int \frac{(\sin x)'}{\sin x} \, dx$$

$(\sin x)' = \cos x$ \nearrow

$$= \ln |\sin x| + C$$

$$= -\ln |\csc x| + C$$

* Don't forget the Abs. value *

$$3- \int \sec x \, dx = \int \frac{\sec x (\sec x + \tan x) \, dx}{\tan x + \sec x} = \int \frac{\sec^2 x + \tan x \sec x}{\tan x + \sec x} \, dx$$

$(\tan x)' = \sec^2 x$
 $(\sec x)' = \tan x \sec x$ \nearrow

$$= \int \frac{(\tan x)' + (\sec x)'}{\tan x + \sec x} \, dx$$

$$= \ln |\tan x + \sec x| + C$$

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$$4- \int \csc x \, dx = \int \frac{\csc x (\csc x + \cot x)}{\csc x + \cot x} \, dx \quad (3)$$

$$= \int \frac{\csc^2 x + \csc x \cot x}{\csc x + \cot x} \, dx$$

$$\begin{aligned} (\cot x)' &= -\csc^2 x \\ (\csc x)' &= -\csc x \cot x \end{aligned} \quad \Rightarrow \int \frac{-(\cot x)' + (\csc x)'}{\csc x + \cot x}$$

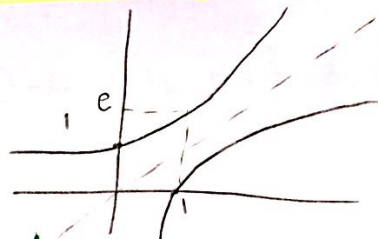
$$= -\ln |\csc x + \cot x| + C$$

7.3: Exponential function

$$f(x) = e^x = \ln^{-1} x$$

$$D: (-\infty, \infty)$$

$$R: (0, \infty)$$



• Remember:-

$$\ln(1) = 0$$

$$\ln^{-1} 1 = e$$

• Properties of e^x :-

$$1- e^{\ln x} = x \quad \forall x > 0$$

$$2- \ln e^x = x \quad \forall x$$

$$3- e^{x_1} e^{x_2} = e^{(x_1+x_2)}$$

$$4- \frac{e^{x_1}}{e^{x_2}} = e^{(x_1-x_2)}$$

$$5- (e^{x_1})^r = e^{rx_1}$$

$$6- e^{-x} = \frac{1}{e^x}$$

• Differentiation and integration of e^x

$$1- \text{if } y(x) = a^{u(x)}$$

→ where $a > 0$ and $u(x)$ is diff

$$\text{Then } y'(x) = a^{u(x)} \ln a \cdot u'(x)$$

$$2- \int a^{u(x)} \ln a \cdot u'(x) \, dx = a^{u(x)} + C$$

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- Natural logarithm is a special case of General logarithmic function (G.L.f)

General: $y(x) = \log_a u(x) = \frac{\ln u(x)}{\ln a}$ $a > 0$
 $u(x) > 0$ $a \neq 1$

Natural: a (the base) = e so $\ln e = 1 \Rightarrow \log_e u(x) = \frac{\ln u(x)}{1} = \ln u(x)$

- Diff + integration of 'G.L.f'

~~by diff~~

• If $y(x) = \log_a u(x) \Rightarrow y'(x) = \frac{1}{\ln a} \left(\frac{u'(x)}{u(x)} \right)$

and $\int \frac{u'(x)}{u(x)} \frac{1}{\ln a} dx = \log_a u(x) + C$

• Properties :-

1- $\log_a a^x = x \quad \forall x$ = why? $\because \log_a a^x = \frac{\ln a^x}{\ln a} = x \frac{\ln a}{\ln a} = x$

2- $a^{\log_a x} = x \quad \forall x > 0$

3- $\log_a xy = \log_a x + \log_a y$

4- $\log_a \frac{x}{y} = \log_a x - \log_a y$

5- $\log_a \frac{1}{y} = \log_a 1 - \log_a y = 0 - \log_a y = -\log_a y$

6- $\log_a x^y = y \log_a x = y \left(\frac{\ln x}{\ln a} \right)$

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