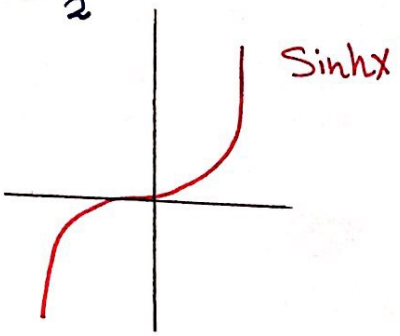
hyperbolic functions

- $\sinh x = \frac{e^x - e^{-x}}{2}$

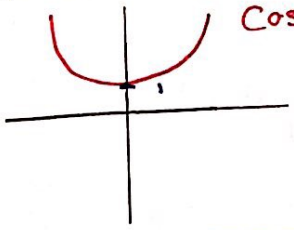
D : IR
R : IR
odd



- $\cosh x = \frac{e^x + e^{-x}}{2}$

• عبارة عن جمع بين اقترانين موجبين ولا يقطعها لأنه الاقتران الأول موجب ولا يقطع المحور

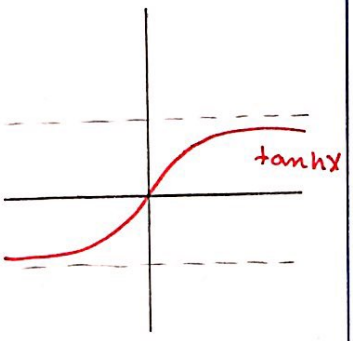
D : IR
R : $[1, \infty)$
Even



- $\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

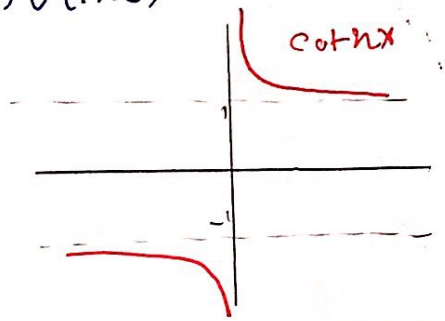
D : IR
R : $(-1, 1)$
odd
It has h. Asy
at $y = 1$ &
$y = -1$

If $e^x > e^{-x}$ => موجب
If $e^x < e^{-x}$ => سالب



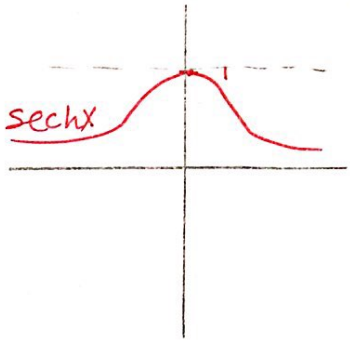
- $\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$

D : IR \ {0}
R : $(-\infty, -1) \cup (1, \infty)$
odd



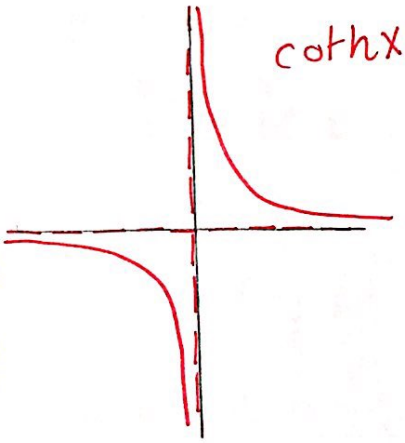
- $\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$

D : IR
R : (0, 1]
even
has a h. Asy
at $y = 0$



- $\operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}$

D : IR \ {0}
R : IR \ {0}
odd.
It has a V. Asy
at $x = 0$
and a h. Asy
at $y = 0$



Alaa Khaimi

Derivatives of hyperbolic functions

Assume $u(x)$ is differentiable

- If $y = \sinh u(x) \longrightarrow y' = \cosh u \frac{du}{dx}$
- If $y = \cosh u(x) \longrightarrow y' = \sinh u \frac{du}{dx}$
- If $y = \tanh u(x) \longrightarrow y' = \operatorname{sech}^2 u \frac{du}{dx}$
- If $y = \coth u(x) \longrightarrow y' = -\operatorname{csch}^2 u \frac{du}{dx}$
- If $y = \operatorname{sech} u(x) \longrightarrow y' = -\operatorname{sech} u \tanh u \frac{du}{dx}$
- If $y = \operatorname{csch} u \longrightarrow y' = -\operatorname{csch} u \coth u \frac{du}{dx}$

Integrals of hyperbolic functions

- $\int \sinh x \, dx = \cosh x + C$
- $\int \cosh x \, dx = \sinh x + C$
- $\int \operatorname{sech}^2 x \, dx = \tanh x + C$
- $\int \operatorname{csch}^2 x \, dx = -\coth x + C$
- $\int \operatorname{sech} x \tanh x \, dx = -\operatorname{sech} x + C$
- $\int \operatorname{csch} x \coth x \, dx = -\operatorname{csch} x + C$

Alaa Ettaimi

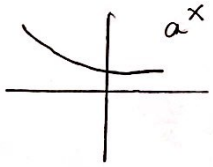
Identities of hyperbolic functions

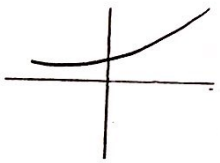
- $\cosh^2 x - \sinh^2 x = 1$
- $\sinh 2x = 2\sinh x \cosh x$
- $\cosh 2x = \cosh^2 x + \sinh^2 x$
  - $= 2\cosh^2 x - 1$
  - $= 1 + \sinh^2 x$
- $\tanh^2 x + \operatorname{sech}^2 x = 1$
- $\coth^2 x - \operatorname{csch}^2 x = 1$

7.8 : Relative Rates of Growth

- Important Notes

$$\lim_{x\to\infty} a^x = \begin{cases} 0 & \text{if } 0<a<1 \\ \infty & \text{if } a>1 \end{cases}$$





e^x is faster than $\ln x$

Definition

If $f(x)$ and $g(x)$ are positive functions
let $0<L<\infty$ be a positive constant

If $\lim_{x\to\infty} \frac{f(x)}{g(x)}$

- ∞, f grows faster than g as $x\to\infty$
- 0, f grows slower than g as $x\to\infty$
- L, f grows at the same rate as g as $x\to\infty$

Alaa Ftaiwi