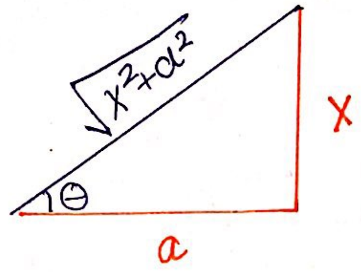
Trigonometric substitution

$X = a\tan\theta$

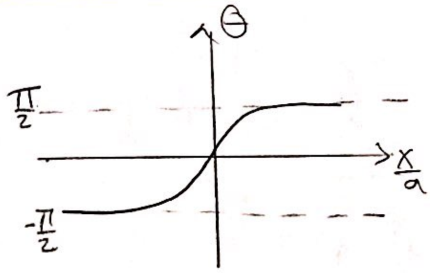


$dX = a\sec^2\theta \, d\theta$

$\theta = \tan^{-1}\left(\frac{X}{a}\right)$

$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

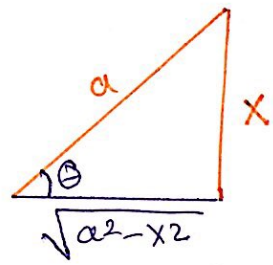
which means $\cos\theta$ is always positive



$\sqrt{X^2+a^2} = \sqrt{a^2\tan^2\theta + a^2} = \sqrt{a^2(\tan^2\theta+1)} = \sqrt{a^2\sec^2\theta} = a\,|\sec\theta|$

But $\sec\theta > 0$ Then $= a\sec\theta$

$X = a\sin\theta$

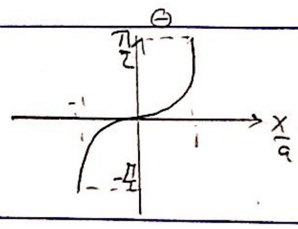


$d\theta = a\cos\theta \, d\theta$

$\theta = \sin^{-1}\left(\frac{X}{a}\right)$

$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

which means $\cos\theta$ is always positive



$\sqrt{a^2-X^2} = \sqrt{a^2-a^2\sin^2\theta} = \sqrt{a^2(1-\sin^2\theta)} = \sqrt{a^2\cos^2\theta} = a\,|\cos\theta|$

But $\cos\theta > 0 \Rightarrow = a\cos\theta$

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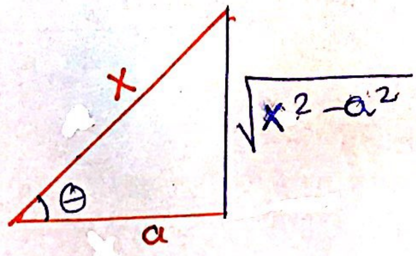
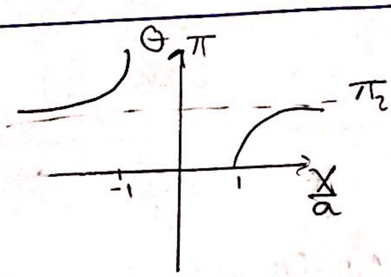
$X = a\sec\Theta$



$dx = a\sec\Theta\tan\Theta \, d\Theta$

$\Theta = \sec^{-1} = \left(\frac{X}{a}\right)$



· There is 2 cases

Case ①:- $\frac{X}{a} \geq 1 \implies 0 \leq \Theta < \frac{\pi}{2}$

Case ②:- $\frac{X}{a} \leq -1 \implies \frac{\pi}{2} < \Theta \leq \pi$

In our Book we only deal with Case ① :-

$\sqrt{x^2-a^2} = \sqrt{a^2\sec^2\Theta - a^2} = \sqrt{a^2(\sec^2\Theta - 1)} = \sqrt{a^2\tan^2\Theta} = a|\tan\Theta|$

since we are only dealing with Case 1①

Then $\implies = a\tan\Theta$

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