

Chap 2: limits & continuity

$$\lim_{x \rightarrow x_0} f(x) = L \iff \lim_{x \rightarrow x_0^+} f(x) = \lim_{x \rightarrow x_0^-} f(x) = L$$

Th:

- The sandwich Theorem: if $g(x) \leq f(x) \leq h(x)$
if $\lim_{x \rightarrow c} g(x) = \lim_{x \rightarrow c} h(x) = L$
Then $\lim_{x \rightarrow c} f(x) = L$

- $f(x)$ is continuous at x_0 iff: $\lim_{x \rightarrow x_0} f(x)$ exist and $f(x_0)$ exist
and $\lim_{x \rightarrow x_0} f(x) = f(x_0)$

$f(x)$ is continuous on $[a, b]$ iff f is cont at every point of $[a, b]$

Exp: $\sin(x)$ / $\cos(x)$ / $|x|$ / and any polynomials are cont on \mathbb{R}
- all Rational functions $\frac{f(x)}{g(x)}$ are cont on \mathbb{R} except the zeros of $g(x)$

Discontinuity: when $f(x)$ is Discontinuous in a specific point
we use a Removable Discontinuity point and we create a
Continuous extension: $f(x)$ is Discont at x_0 and $\lim_{x \rightarrow x_0} f(x) = a$ Then
$$F(x) = \begin{cases} f(x), & x \neq x_0 \\ a, & x = x_0 \end{cases}$$

The Intermediate Value Theorem IVT

* suppose that $f(x)$ is cont on $[a, b]$ and $f(a) < y_0 < f(b)$

Then \exists a number $x_0 \in [a, b]$ s.t. $f(x_0) = y_0$

Imp: If They ask if $f(x)$ has a zero at $[a, b]$
we find $f(a), f(b)$ and if $y_0 = 0$ is between
Then There is a zero

To draw a Rational function we use asymptotes

Asymptotes are 3 kinds: \rightarrow H-Asy: line $y = b$
s.t. $\lim_{x \rightarrow \infty} f(x) = b$ or $\lim_{x \rightarrow -\infty} f(x) = b$

\rightarrow V-Asy line: $x = a$
s.t. $\lim_{x \rightarrow a^+} f(x) = \pm \infty$ or $\lim_{x \rightarrow a^-} f(x) = \pm \infty$

\rightarrow oblique-Asy: when $\frac{g(x)}{h(x)}$
 $g(x)$ degree $>$ $h(x)$ degree

Notes

$$\lim_{x \rightarrow \infty} \frac{\sin x}{x} = 0$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$