Chap 4: Increasing and decreasing functions

Basic Def:- f is a fonction defind on interval I :-

→ if wherever $x_2 > x_1$ we have $f(x_2) > f(x_1)$ $\forall x \in I$ then f is increasing on I

→ if wherever $x_2 > x_1$ we have $f(x_2) < f(x_1)$ $\forall x \in I$ then f is decreasing on I

Th (that we use):- f is cont on $[a,b]$ & diff on (a,b) Then

→ if $f'(x) > 0$ $\forall x \in (a,b)$ → f is increasing on $[a,b]$

→ if $f'(x) < 0$ $\forall x \in (a,b)$ → f is decreasing on $[a,b]$

Extreme values

Absolute

- M is an Abs Max at $c \in I$ if $M = f(c) \geq f(x)$ and its a local Max on small interval around c
- M is an Abs Min m at c if $m = f(c) \leq f(x)$ and its a local min on small interval around c

local

- M is a local Max on I if $M = f(c) \geq f(x)$ $\forall x \in I$
- m is a local min on I if $M = f(c) \leq f(x)$ $\forall x \in I$

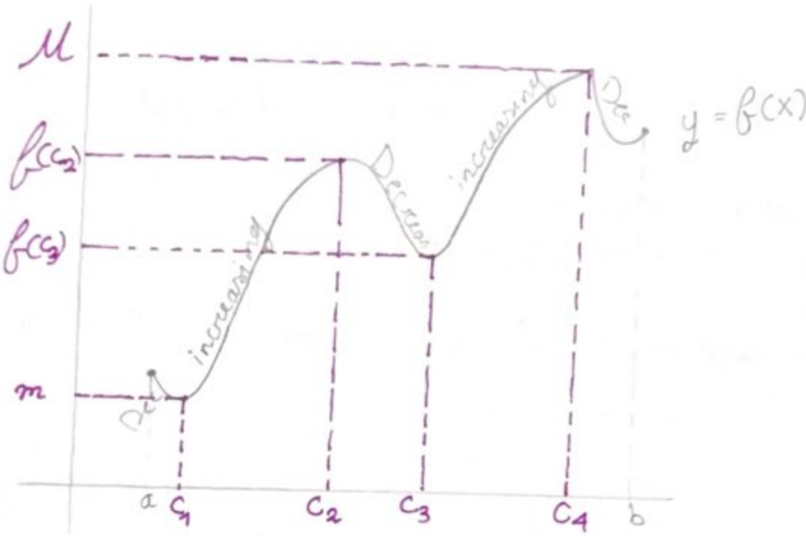
Sketch

-Analysing-

- a, b End points
- c_1, c_2, c_3, c_4 } Interior points

critical points:-

$(c_1, f(c_1))$
$(c_2, f(c_2))$
$(c_3, f(c_3))$
$(c_4, f(c_4))$



- f has Abs. Max of M at C_4 → + also local Max
- f has Abs. Min of m at C_1 → + also local Min
- f has local Max of $f(c_2)$ at C_2
- f has local Min of $f(c_3)$ at C_3

R! : Abs ⇒ local

R! : if f = X Then (اقترانات خطية) Then f has Abs. Max & Abs Min at all I

Theory: if f is on [a,b] then f has Abs. Max & Abs Min

VIN

the extreme values May occure for $y = f(x)$ at

- end points
- Interior points $\in D$:
  - $f'(x) = 0$
  - f' is not defined

critical points

Interior
s.t:-
$f'(0)=0$ or f' is un-defined
at these points

Th: if f is diff on I and has extreme values at $x = c \in I$ Then $f'(c) = 0$

But $f'(Z) = 0$ Then f may not have extreme values at $x = Z$

to classify the critical points
we use either

The FDT
First Derivative Test

- suppose that f has critical point at $x = c$ and f' exists on open I containing c

Then:-

$\boxed{1}$ if f' changes sign from + to − at $x = c$ then $f(c)$ is local Max

$\boxed{2}$ if f' changes sign from − to + at $x = c$ then $f(c)$ is local Min

$\boxed{3}$ if f' does not change sign at $x = c$ then f does not have extreme values at $x = c$

The SDT
Second derivative Test

- suppose $f''(c) = 0$ and f'' is cont on an open I containing c. Then:-

$\boxed{1}$ if $f''(c) > 0$ then $f(c)$ is local min

$\boxed{2}$ if $f''(c) < 0$ then $f(c)$ is local Max

$\boxed{3}$ if $f''(c) = 0$ Then the test fails

Remark :-

$f''(c) \geq 0 \rightarrow$ ConCave up

$f''(c) \leq 0 \rightarrow$ ConCave down

inflection point : f has an inflection point at $x = c$

if : [1] f has tangent at $x = c$

[2] f changes ConCavity

and to find it we use: $f''(x) = 0$

Remarks for Solving Questions :-

- when we find the critical points and we want to check when is f increasing or decresing we use points $x_1, x_2 \dots$ that lies in the I and we calculate f' at these points

Exp

f' —— x_1 —— C_1 —— x_2 —— C_2 —— x_3

we find f' and when $f' > 0$ Then increasing

and when $f' < 0$ Then decreasing

Sketching functions

- **Steps** :-

1. find the Domain D(f) =
2. find the Asymptotes — How to find them? Go back to Chap:2
3. find the critical points
4. find th decreasing & increasing Intervals of f(x) — How to Know a point is critical or not — Go back to Chap:4
5. find the Intervals of Concavity for f(x)
6. find inflection Points — How to Know a point's an inflection points or not? — Go back to Chap:4
7. find local Max and Min points

Th: Rolls Th:- if f(x) is ① cont on [a,b] and diff on (a,b) ② f(a) = f(b) Then
→ ∃ at least one point C ∈ (a,b) s.t $f'(c) = 0$

Th: Mean Th: if f(x) is ① cont on [a,b] & diff on (a,b) then
→ ∃ one point c ∈ (a,b) s.t

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$