Chap 6 :-

6.1 • Volumes using Cross sections

Volumes Rules

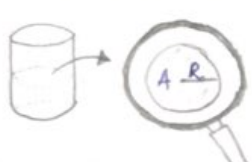
1- cylindrical solid:-
$V = (\text{Base Area})\text{ Height}$

2-

There is 3 methods used to find a volume of a given solid

Disk Method

• it's a special case of Washer Method. $[r(x)=0]$



$A = \pi R^2$

• There is 2 cases :-

$Cs \perp x$-axis
$A(x) = \pi R^2(x)$

$Cs \perp y$-axis
$A(y) = \pi R^2(y)$

Washer Method

• 2 cases
we use it when the solid does not border on or cross the axis of revolution

→ If the $Cs \perp x$-axis which results by Rotation about x-axis with outer Radius $R(x)$ and Inner Radius $r(x)$ then

$$V = \int_a^b A(x)\,dx = \int_a^b \pi\,[R^2(x) - r^2(x)]\,dx$$

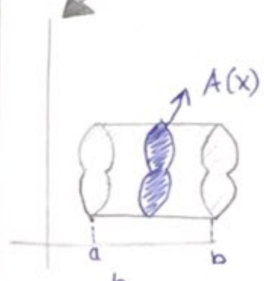
→ If the $Cs \perp y$-axis which results by Rotation about y-axis with outer Radius $R(x)$ and Inner Radius $r(x)$ then

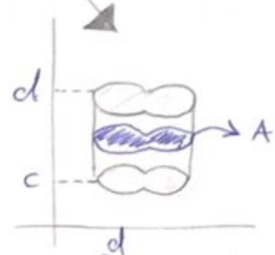
$$V = \int_c^d \pi\,[R^2(y) - r^2(y)]\,dy$$

Shell Method

Slicing by Parallel Planes:-

* How to find a volume given
- Graph the solid
- Determine the cross section
- Then if $Cs \perp x$-axis : 1st case
  if $Cs \perp y$-axis : 2nd case





• $V = \int_a^b A(x)\,dx$

• Rotation about x-axis or any line parallel to it

• $V = \int_c^d A(y)\,dy$

• Rotatation about y-axis or any line parallel to it

6.2 Shell method

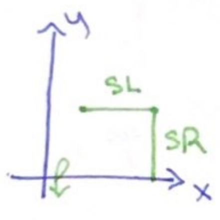
We use this method to find the volume of a solid generated by revolving a given region about:-

X-axis

$d \rightarrow c, d$ على محور الصادات

$$V = 2\pi \int_c^d \left(\text{shell Radius}\right)\left(\text{shell length}\right) dy$$

- distance between $\boxed{\text{shell length}}$ and the axis of Revolution (x) — SR
- the segment's length that is parallel to the axis of revolution (x) — SL

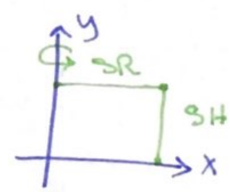


y-axis

$$V = 2\pi \int_a^b \left(\text{shell Radius}\right)\left(\text{shell hight}\right) dx$$

a, b على محور السينات

- distance between the shell's hight and the axis of Revolution (y)
- sigment's hight that is parallel to the axis of Revolution (y)



6.3 Arc length

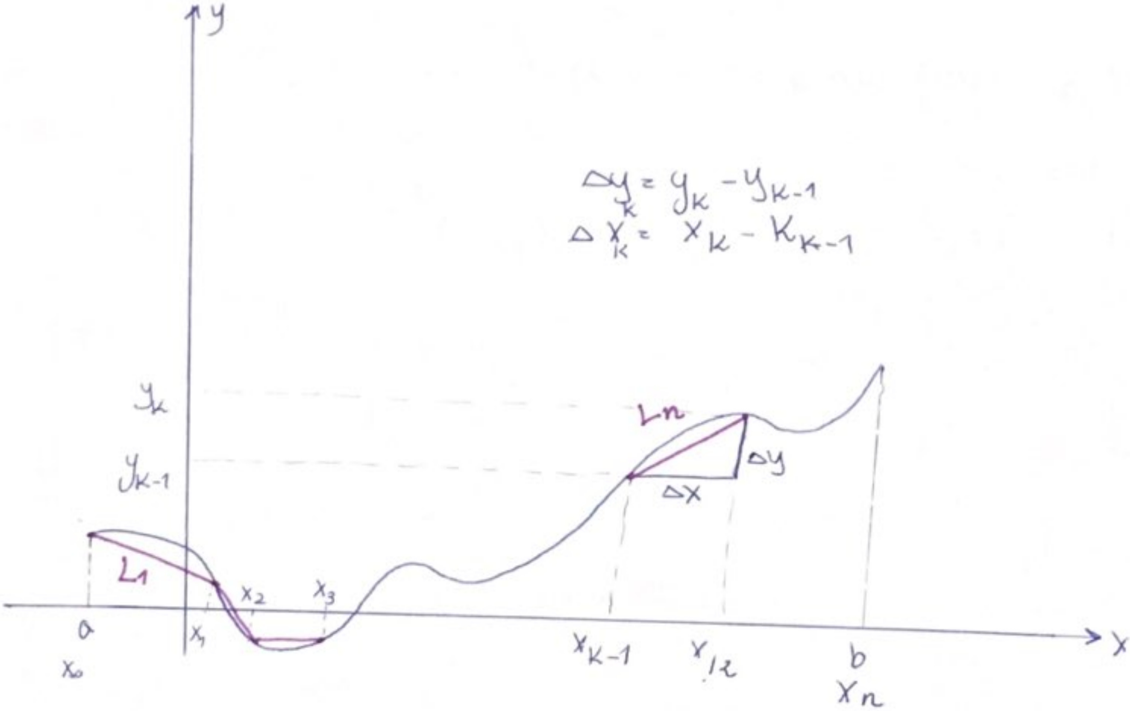
If $f'(x)$ is cont on $[a,b]$ then the Arc length of the curve $y=f(x)$ is given by:-

$$L = \int_a^b \sqrt{1+(f'(x))^2}\, dx = \int_a^b \sqrt{1+(\quad)^2}\, dx$$

Explanation:-

How?

- Let's say you have the curve $f(x)$, wich happeneds to be continuous and diff at $[a,b]$ WoW!



L: is the true lenefth

$\tilde{L}$: Approximated length

$\tilde{L} = L_1 + L_2 + \;—\; + L_n$

$= \sum_{k=1}^{n} L_k$

$= \sum_{k=1}^{n} \sqrt{(\Delta x)^2 + (\Delta y)^2}$

Ln, Δy, Δx

Remember MVT
Mean Value Theorm

$f'(c) = \frac{f(b)-f(a)}{b-a} = \frac{\Delta y}{\Delta x}$

$\Rightarrow$ By MVT there is $C_k \in (x_{k-1}, x_k)$ (a, b)

$f'(c_k) = \frac{\Delta y_k}{\Delta x_k} \Rightarrow \Delta y_k = f'(c_k)\, \Delta x_k$

But $\tilde{L} = \sum_{K=0}^{n} \sqrt{(\Delta X_K)^2 + (f'(C_K)\ \Delta X_K)^2}$

$\tilde{L} = \sum_{K=0}^{n} \sqrt{1 + f'(C_K)}\ \Delta X_K$

→ to ~~mo~~ improve $\tilde{L}$ ⇒ we use n numbers of sub intervals as n gets large

$$L = \lim_{n \to \infty} \tilde{L}$$

$$= \lim_{n \to \infty} \sum_{K=1}^{n} \sqrt{1 + f'(C_K)^2}\ \Delta X_K$$

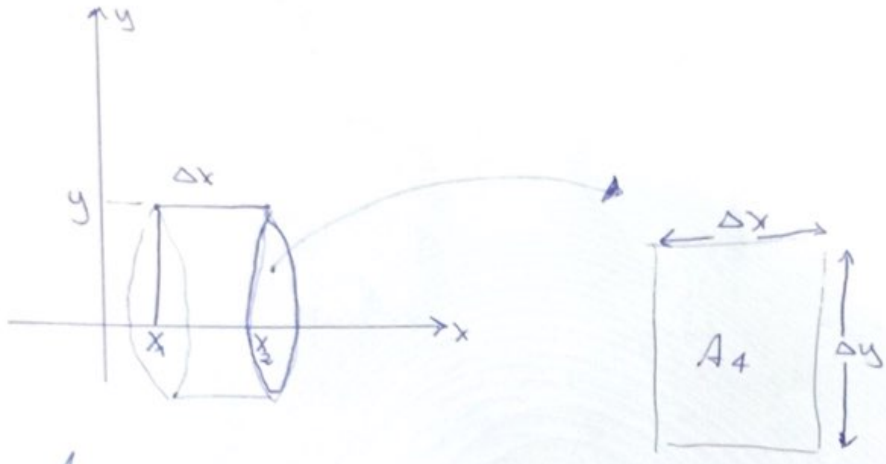
$$= \int_b^a \sqrt{1 + f'(x)^2}\ dx$$ Finally

Remark very important and Tricky ⚠

- f' has to be Continuous on $[a,b]$

If not :- Try ⇒ $L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$

6.4 Area of surface using Revolution



Area : $(\Delta x)(2y\pi$

Area = $(\Delta x)(\Delta y)$

X-axis :-

The surface area of the region ~~bounded~~ generated by Revolving Δx about X-axis is :-

$2\pi y \; \Delta x$

Definitions :-

X-axis

If $y = f(x) \geq 0$ is continuously differentiable on interval $[a,b]$ then the surface Area of the region generated by revolving the curve $y = f(x)$ about X-axis is

$$S = \int_a^b 2\pi y \sqrt{1 + f'(x)^2} \, dx$$

$$\left(\frac{dy}{dx}\right)^2$$

y-axis

If $x = g(y) \geq 0$ is cont diff on $I = [c,d]$ then the surface Area of the region Generated by Revolving the curve $x = g(y)$ about y-axis is:-

$$S = \int_c^d 2\pi \, g(y) \sqrt{1 + (g'(y))^2} \, dy$$

$$\left(\frac{dx}{dy}\right)^2$$