

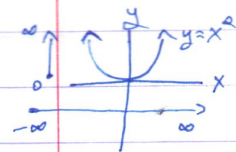
CH1: Functions الأقرانات

[1-1]

Definition: A function F is a rule that assigns to each point X in the domain a unique range of point $y = f(x)$ in the range of $f(x)$.

ex: $f(x) = x^2$, find D, R .

Sol: $D = (-\infty, \infty)$, $R = [0, \infty)$.

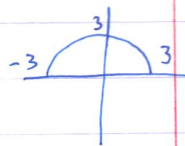


ex: $X = y^2$ is not a function
 $X = 2 \Rightarrow y^2 = 2, y = \pm\sqrt{2}$



ex: $y = \sqrt{9 - x^2}$, find D and R .

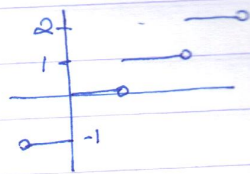
Sol: $y^2 = 9 - x^2 \Rightarrow x^2 + y^2 = 9$



$D = [-3, 3]$, $R = [0, 3]$.

ex: $y = \lfloor x \rfloor$ → greatest integer function.

$$= \begin{cases} -1, & -1 \leq x < 0 \\ 0, & 0 \leq x < 1 \\ 1, & 1 \leq x < 2 \\ 2, & 2 \leq x < 3 \end{cases}$$



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$$D = [-\infty, \infty], R = \text{Integers} = \{0, \pm 1, \pm 2, \dots\}.$$

ex: $y = |x|$

$$D = (-\infty, \infty), R = [0, \infty)$$

ex: $y = \lfloor |x| \rfloor$

$$D = (-\infty, \infty), R = \{0, 1, 2, \dots\}$$

[1-2]

Trigonometric Functions الاقترانات المثلثية

- ①: $y = \sin \theta$ L_P
 ②: $y = \cos \theta$ L_P
 ③: $y = \tan \theta$ L_P $\frac{\sin}{\cos}$
 ④: $y = \cot \theta$ L_P $\frac{\cos}{\sin}$
 ⑤: $y = \sec \theta$ L_Q $\frac{1}{\cos}$
 ⑥: $y = \csc \theta$ L_Q $\frac{1}{\sin}$

Domain and range:

$f \theta$	D	R
$\sin \theta$	$[-\infty, \infty]$	$(-1, 1)$
$\cos \theta$	$[-\infty, \infty]$	$(-1, 1)$
$\tan \theta$	$[-\infty, \infty] - \left\{ \frac{\pi}{2} + n\pi \right\}$ $n=0, \pm 1, \pm 2, \dots$	$(-\infty, \infty)$
$\sec \theta$	$= = =$	$y \in (-\infty, -1] \cup [1, \infty)$
$\cot \theta$	$(-\infty, \infty)$ $\neq 0$ $n\pi : n=0, \pm 1, \dots$	$= = =$
$\csc \theta$	$= = =$	$= = =$

Period :

* $\sin \theta, \cos \theta$, Period = 2π . $\Rightarrow \sin(\theta + 2k\pi)$
* $\sec \theta, \csc \theta$,

* $\cot \theta, \tan \theta$, Period = π . $\Rightarrow \tan(\theta + k\pi)$
 $k = 0, \pm 1, \pm 2, \dots$

Trigonometric Identities :
علاقات التفاضل

①: $\sin^2 X + \cos^2 X = 1$

②: $\sin 2X = 2 \sin X \cos X$

③: $\cos 2X \begin{cases} \rightarrow \cos^2 X - \sin^2 X \\ \rightarrow 1 - 2 \sin^2 X \\ \rightarrow 2 \cos^2 X - 1 \end{cases}$

④: $\sin^2 X = \frac{1 - \cos 2X}{2}$

⑤: $\cos^2 X = \frac{1 + \cos 2X}{2}$

⑥: $\sec^2 X = 1 + \tan^2 X$

⑦: $\csc^2 X = 1 + \cot^2 X$

⑧: $\sin(A+B) = \sin A \cos B + \cos A \sin B$

⑨: $\cos(A+B) = \cos A \cos B - \sin A \sin B$

Even function :

$$f(-x) = f(x)$$

Odd function :

$$f(-x) = -f(x)$$

ex: $f(x) = x^4 + x^2$

$$f(-x) = (-x)^4 + (-x)^2$$

$$f(x) = x^4 + x^2$$

$\therefore f$ is even.

ex: $f(x) = x^3 + x$

$$f(-x) = (-x)^3 - x$$

$$= -(x^3 + x) = -f(x)$$

$\therefore f$ is odd

ex: $f(x) = x^4 + x^3$

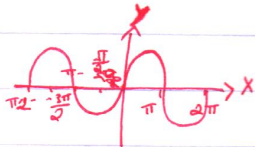
$$f(-x) = (-x)^4 + (-x)^3$$

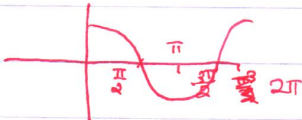
$$= x^4 - x^3$$

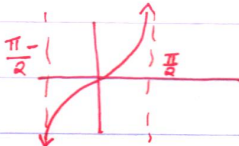
Neither


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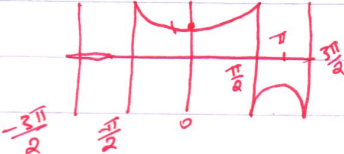
Graphs of the Trigonometric functions

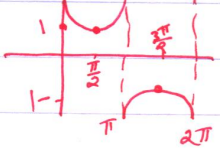
①: $y = \sin x$ A hand-drawn graph of the sine function y = sin x. The x-axis is labeled with 0, π/2, π, and 2π. The y-axis is labeled with 1 and -1. The curve starts at (0,0), reaches a peak at (π/2, 1), crosses the x-axis at (π, 0), reaches a trough at (3π/2, -1), and returns to the x-axis at (2π, 0).

②: $y = \cos x$ A hand-drawn graph of the cosine function y = cos x. The x-axis is labeled with 0, π/2, π, 3π/2, and 2π. The y-axis is labeled with 1 and -1. The curve starts at (0,1), crosses the x-axis at (π/2, 0), reaches a trough at (π, -1), crosses the x-axis again at (3π/2, 0), and returns to the y-axis at (2π, 1).

③: $y = \tan x$ A hand-drawn graph of the tangent function y = tan x. The x-axis is labeled with π/2 and 3π/2. The y-axis is labeled with 1 and -1. Vertical dashed lines represent asymptotes at x = π/2 and x = 3π/2. The curve passes through the origin (0,0) and has a period of π.

④: $y = \cot x$ A hand-drawn graph of the cotangent function y = cot x. The x-axis is labeled with 0 and π. The y-axis is labeled with 1 and -1. Vertical dashed lines represent asymptotes at x = 0 and x = π. The curve passes through the point (π/2, 0) and has a period of π.

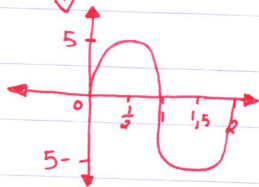
⑤: $y = \sec x$ A hand-drawn graph of the secant function y = sec x. The x-axis is labeled with -3π/2, π/2, and 3π/2. The y-axis is labeled with 1 and -1. Vertical dashed lines represent asymptotes at x = -3π/2, π/2, and 3π/2. The curve has branches that go to positive infinity and negative infinity between the asymptotes.

⑥: $y = \csc x$ A hand-drawn graph of the cosecant function y = csc x. The x-axis is labeled with π and 2π. The y-axis is labeled with 1 and -1. Vertical dashed lines represent asymptotes at x = π and x = 2π. The curve has branches that go to positive infinity and negative infinity between the asymptotes.

Q2 (a) Page 6 :

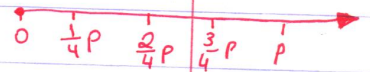
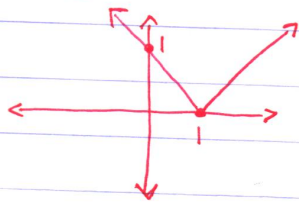
$$y = 5 \sin \pi x$$

$$\text{Period} = \frac{2\pi}{|x \text{ value}|} = \frac{2\pi}{|\pi|} = 2$$



if $y = a \sin(bx)$ or
 $y = a \cos(bx) \therefore \text{Period} = \frac{2\pi}{|b|}$
Range = ~~a~~ $[-|a|, |a|]$

example: sketch $y = |x-1|$



Q1 / page 6 find Domain and Range,

a:) $f(x) = \frac{1}{x}$, $D = (0, \infty)$.
 $R = (0, \infty)$.

c:) $f(x) = 1 + |x|$
 $D = (-\infty, \infty)$
 $R = [1, \infty)$.

Domain:
 $1 - x^2 > 0$
 $-x^2 > -1 \quad |x| < 1$
 $\sqrt{x^2} < 1 \quad -1 < x < 1$

f:) $f(x) = \frac{1}{\sqrt{1-x^2}}$ / $D = (-1, 1)$.
 $R = [1, \infty)$.

Q3:) ^{c:)} $g(t) = \frac{1}{t-1}$, is g even/odd or neither?

$$g(-t) = \frac{1}{-t-1}$$

$$= \frac{1}{-(t+1)} \therefore g \text{ is neither even nor odd.}$$

d:) $h(x) = \frac{x}{x^2-1}$?

$$h(-x) = \frac{-x}{(-x)^2-1}$$

$$= \frac{-x}{x^2-1} = -h(x)$$

$\Rightarrow h$ is odd function.

Q4:) b:) if f is even and $g(x)$ is odd, then $h(x)$ is odd.

$$\text{Sol: } h(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -h(x) \text{ (odd function).}$$

Find the domain and range of the following function. (D)

a: $f(x) = \frac{1}{\sqrt{x}}$ Domain $x > 0$

Range: $\frac{x \quad \sqrt{\quad}}{-1 \quad 0 \quad 1} (0, \infty)$

b: $f(x) = \tan(\pi x)$

Domain: $\pi x \neq \frac{\pi}{2}, \frac{3\pi}{2}, \dots$

$\pi x \neq \frac{(2n+1)\pi}{2}, n \in \mathbb{Z}$

\therefore Domain: $\mathbb{R} \setminus \left\{ \frac{2n+1}{2}, n \in \mathbb{Z} \right\}$

* Range: $(-\infty, \infty)$

c: $1 + |x|$

Domain: $(-\infty, \infty) \mathbb{R}$

Range: $[1, \infty)$

d: $f(x) = \sec^2 x$

Domain: $\mathbb{R} - \left\{ \frac{\pi}{2}, \frac{3\pi}{2}, \dots \right\}$ $\sec^2 x$

Range: $\infty > \frac{1}{\cos^2 x} \geq 1$