3.1 Definition of derivate.

The derivative of f at $\underline{X=a}$.

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Provided this limit exists

$$f'_{+}(a) = \lim_{h \to 0^{+}} \frac{f(a+h) - f(a)}{h}$$

Right hand derivative of f at a.

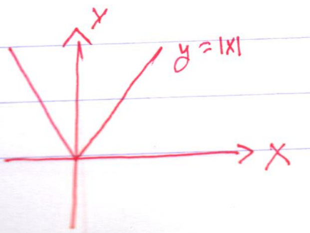
$$f'_{-}(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

قابل للإشتقاق

Remark: a function f is diffrentiable at $X=c$ if and only if $f'_{+}(c) = f'_{-}(c)$.

Rule: (Theorem), if f is differentiable at $X=c$, then it is continuous at $X=c$ the converse need to be true.

ex: $f(x) = |X| = \begin{cases} X, & X \geq 0 \\ -X, & X < 0 \end{cases}$



$\Rightarrow f(x) = |X|$ is continuous at $X=0$

$f'(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$ // $f'_{+}(0) = 1,\ f'_{-}(0) = -1$

Does not exist

$\Rightarrow f'(0)$ DNE

$\therefore$ f is not diffable at $x=0$.

Differentiation Rules :

* $[f(x) \pm g(x)]' = f'(x) \pm g'(x)$: قاعدة الجمع والطرح

* $[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$ قاعدة الضرب

* $\left[\frac{f(x)}{g(x)}\right]' = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$ قاعدة القسمة

* $(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$ قاعدة السلسلة.

Chain rule
قاعدة السلسلة

3.3 Derv of trigonometric functions

للتسهيل:
مشتقة أي
واحدة تبدأ بـ c
تكون سالبة

y	y'
sin	cos
cos	-sin
tan	$\sec^2$
cot	$-\csc^2$
sec	sec tan
csc	-csc cot

Excercises : Page 16 :

Q1 :) a :) $f(x) = \dfrac{\sqrt{x} - 1}{\sqrt{x} + 1}$

$$f'(x) = \frac{(\sqrt{x}+1)\left(\frac{1}{2\sqrt{x}}\right) - (\sqrt{x}-1)\left(\frac{1}{2\sqrt{x}}\right)}{(\sqrt{x}+1)^2}$$

$$= \frac{\cancel{\frac{1}{2}} + \frac{1}{2\sqrt{x}} - \cancel{\frac{1}{2}} + \frac{1}{2\sqrt{x}}}{(\sqrt{x}+1)^2}$$

$$= \frac{1}{\sqrt{x}\,(\sqrt{x}+1)^2}$$

c :) $f(x) = \sec(2x+1)\cot(x^2)$

قاعدة الضرب
~~والسلسلة~~

$$f'(x) = \left[2\sec(2x+1)\tan(2x+1)\right]\left[\cot(x^2)\right] + ?$$
$$\left[\sec(2x+1)\right]\left[-\csc^2(x^2) \cdot 2x\right].$$

3.3 continue,

ex, find the equation of the tangent line and the normal line of $f(x) = \sec x \tan x$ at $x = \frac{\pi}{4}$.

Sol: $f'(x) = \sec x \cdot \sec^2 x + (\tan x)(\sec x \tan x)$

ميل المماس ← slope of the tangent line $= f'(\frac{\pi}{4})$.

$$\Rightarrow m = f'(\tfrac{\pi}{4}) = (\sec\tfrac{\pi}{4})^3 + (\tan\tfrac{\pi}{4})^2 \sec\tfrac{\pi}{4}.$$

$$= (\sqrt{2})^3 + (1)^2(\sqrt{2})$$

$$= 2\sqrt{2} + \sqrt{2} = \underline{3\sqrt{2}}$$

$\Rightarrow m = 3\sqrt{2}$, slope of the tangent

$x_1 = \frac{\pi}{4} \Rightarrow y_1 = f(\frac{\pi}{4}) = \sec\frac{\pi}{4} \tan\frac{\pi}{4}$

$= \sqrt{2}(1) = \sqrt{2}$

Point $(\frac{\pi}{4}, \sqrt{2})$, $m = 3\sqrt{2}$

$y - y_1 = m(x - x_1)$

$y - \sqrt{2} = 3\sqrt{2}(x - \frac{\pi}{4})$

Normal line $y - \sqrt{2} = \frac{-1}{3\sqrt{2}}(x - \frac{\pi}{4})$.

Q3 , page 16 :

Find the Points on the curve
$y = 2x^3 - 3x^2 - 12x + 20$

Where the tangent is <u>Parallel</u> to the X-axis.
موازي

Sol: Slope of the tangent $= 0$

$y' = 6x^2 - 6x - 12 = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$\therefore x = (\underset{x_1}{2}, \underset{x_2}{-1})$

if $x_1 = 2$, $y_1 = f(2) = 0$

$(2, 0)$ ✓

if $\cancel{x}_2 = -1$, $y_2 = f(-1) = 27$

$(-1, 27)$ ✓

tangent lines, $y = 0$, $y = 27$

Normal lines $x = 2$, and $x = -1$

3.4: Implicit differentiation

$\cancel{y = x^2 + 1}$

ex: if $xy = \cot(xy)$, find y'? and y''

Sol: Differentiate both sides with respect to x.

$1(y) + xy' = -\csc^2(xy)[y + xy']$

$y + xy' = -y\csc^2(xy) - x\csc^2(xy) \cdot y'$

$xy' + x\csc^2(xy)y' = -y\csc^2(xy) - y$

$y'[x + x\csc^2(xy)] = -y\csc^2(xy) - y$

$$y' = -\frac{y\csc^2(xy) - y}{x + x\csc^2(xy)}$$

$$= \frac{y[\cancel{(\csc^2(xy))} - 1]}{x[1 + \cancel{\csc^2(xy)}]}$$

$$\boxed{y' = \frac{-y}{x}.}$$

$$y'' = -\frac{x \cdot y' - y(1)}{x^2}$$

$$= -\frac{x\cdot\left(\frac{-y}{x}\right) - y}{x^2} = \boxed{\frac{2y}{x^2}}.$$

3.5: linearization and differentials.

the lenearization of $y=f(x)$ about $x=a$, is:

$L(x) = f(a) + f'(a)(x-a)$.

ex: find the ~~tan~~ linearization of $f(x)=\sqrt{x}$ about $x=4$.

sol: $f(4)=\sqrt{4}=2$, $f'(x)=\frac{1}{2\sqrt{x}}$

$\therefore L(x)=f(4)+f'(4)(x-4)$. $f'(4)=\frac{1}{4}$

$L(x) = 2 + \frac{1}{4}(x-4)$.

$L(x) = \frac{1}{4}x + 1 \Rightarrow \sqrt{x} \approx \frac{1}{4}x+1$ about 4

notice $\sqrt{4.1} \approx 2.0248$

$L(4.1) = \frac{1}{4}(4.1)+1 = 2.025$

ناخذ أقرب قيمة لها جذر تكعيبي وهو هنا $\sqrt{8}$.

ex: Approximate $\sqrt[3]{8.01}$

Sol: $f(x) = \sqrt[3]{x}$, $a = 8$

* أقرب قيمة لها جذر تكعيبي.

We need $L(8.01)$

$f(8) = \sqrt[3]{8} = 2$

$f'(x) = \frac{1}{3} x^{-\frac{2}{3}}$

$f'(8) = \frac{1}{3} (8)^{-\frac{2}{3}} = \frac{1}{12}$.

$$L(x) = f(8) + f'(8)(x-8)$$

$$= 2 + \frac{1}{12}(x-8)$$

$$= \frac{4}{3} + \frac{1}{12}x \approx \sqrt[3]{x} \text{ about } x = 8.$$

$$\sqrt[3]{8.01} \approx L(8.01) = \frac{4}{3} + \frac{1}{12}(8.01)$$

$$= \frac{4}{3} + \frac{801}{1000} = \frac{2401}{1200}.$$

* Differential of $y = f(x)$ is dy

$dy = f'(x)dx$

ex: $f(x) = \sin x$ find df.

$df = (\cos x)\, dx$

ex: $y = x^2$, find differential of y.

$dy = 2x\,dx$.

ex: find the differential in the area of the circle.

Sol:

$A = \pi r^2$

$dA = 2\pi r\,dr$.

ex: the radius r of a circle increased from 10 to 10.1, find dA

Sol: $\boxed{r = 10}$, $dr = 0.1$, $dA = 2\pi(10)(0.1)$

$= 2\pi$

* [3.5] Differentials :

ex: if $f(x) = x^2$, find df.

Sol: $df = 2x\,dx$

ex: area of the circle

$A = \pi r^2$

$A' = 2\pi r\,dr$

ex: The radius "r" of a circle increases from 10 to 10.1 estimate the area and compare your estimation to the true Value.

$\boxed{r = 10}$ $\boxed{dr = 0.1}$ → الزيادة أو النقصان *

$A = \pi r^2$

$dA = 2\pi r\,dr$

$= 2\pi \times 10 \times 0.1$

$= 2\pi$ ← هذه تمثل الزيادة وليست القيمة النهائية

$\therefore$ estimated area $= A + dA$

$= \pi(10)^2 + 2\pi$

$\approx 102\pi$.

True Area $= \pi(10.1)^2 = (102.01)\pi$

Error: = | True - estimate |.

= $\underline{0.01\pi}$

* Q7/a Page 17:

The radius of a circle increased from 2 to 2.02 ?.

a: estimate the resulting change in Area.

Sol: $\boxed{r=2}$ $\boxed{dr=0.02}$

$A = 2\pi r^2$

$dA = 2\pi r dr = 2\pi(2)(\frac{2}{100})$

$= \frac{8}{100}\pi$

*estimated area = $A + dA$

$= \pi 2^2 + 0.08\pi$

$= 4.08\pi$.

5.6

D

CH3: d) H.W

Q#1 : $f(s) = \frac{\sqrt{s}-1}{\sqrt{s}+1}$

$f'(s) = \frac{(\sqrt{s}+1)\frac{1}{2\sqrt{s}} - (\sqrt{s}-1)\cdot\frac{1}{2\sqrt{s}}}{(\sqrt{s}+1)^2}$

$= \frac{1}{\sqrt{s}\,(\sqrt{s}+1)}$

b:) H.W

c:) $g(x) = \sec(2x+1)\,\cot(x^2)$

$g'(x) = \sec(2x+1)\,-\csc^2(x^2)\,2x$

$+\cot(x^2)\cdot\sec(2x+1)\tan(2x+1)\cdot 2$

e:) $f(x) = x^3\sin x\cos x$

$f'(x) = x^3\sin x(-\sin x) + \cos(x)\cdot(x^3\cos x + 3x^2\sin x)$

$= -x^3\sin^2 x + x^3\cos^2 x + 3x^2\sin x\cos x$

f: $x^{\frac{1}{2}} + y^{\frac{1}{2}} = 1$

$\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}y^{\frac{1}{2}}y' = 0 \Rightarrow \frac{\frac{1}{2}y^{-\frac{1}{2}}y'}{\frac{1}{2}y^{-\frac{1}{2}}} = \frac{-\frac{1}{2}x^{-\frac{1}{2}}}{\frac{1}{2}y^{-\frac{1}{2}}}$

$= \frac{-x^{-\frac{1}{2}}}{y^{-\frac{1}{2}}}$

$y' = -\sqrt{\frac{y}{x}}$

Q#2: $\frac{dy}{dx}$? D,

①: $y = \cot^2(x)$

$$\frac{dy}{dx} = 2\cot(x) - \csc^2(x)$$

$$= -2\cot(x)\csc^2(x)$$

②: $x^2 + y^2 = x$

$2x + 2y \cdot y' = 1$

$2x + 2y \cdot y' = 1$

$$y' = \frac{1 - 2x}{2y}$$

③: $y = \frac{\sin x}{1 - \cos x}$

$$y' = \frac{(1-\cos x)(\cos x - \sin x) \cdot \sin x}{(1-\cos x)^2}$$

$$\frac{\cos x - \cos^2 x - \sin^2 x}{(1 - \cos^2 x)}$$

$$\frac{-1(1-\cancel{\cos x})}{(1-\cancel{\cos x})} \quad - \quad \frac{\cos x - 1}{(1-\cos x)^2} = \frac{-1}{(1-\cos x)}$$

Q_3*:

D

find the points in the curve
$y = 2x^3 - 3x^2 - 12x + 20$

where the tangent is parallel to X-axis
$y' = 0$ (the slope of the tangent).

$y' = 6x^2 - 6x - 12 = 0$
$x^2 - x - 2 = 0$
$(x+1)(x-2) = 0$
either $x = -1$ or 2

when $x = 2$
$y = 0$
when $x = -1$
$y = 27$
$(-1, 27)$.

Q4: $f(x) = \begin{cases} \sin 2x, & x \le 0 \\ ax, & x > 0 \end{cases}$

(i) continuous at $x = 0$
$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} ax = 0$

$\lim_{x \to 0^-} \sin 2x = 0$. , $0 = 0$

So for any value at the function f is cont at $x = 0$.

$f'(x) = \begin{cases} 2\cos 2x, & x < 0 \\ a, & x > 0 \end{cases}$

f is diff'able at $x = 0$

→

then $f'_{+}(0) = f'_{-}(0)$, $a = 2\cos(0)$

$\underline{\underline{a = 2}}$

العمودي

Q5: find the normal to the curve $xy + 2x - y = 0$ that parallel to $2x + y = 0$

$y = \underline{\underline{-2}}x$, the slope of the normal $= \underline{\underline{-2}}$

slope

the slope of the tangent $= \underline{\underline{\frac{1}{2}}}$.

$xy' + y + 2 - y' = 0$

$(x-1)y' = -2 - y$

$y' = \dfrac{-2-y}{x-1}$

$\dfrac{-2-y}{x-1} = \dfrac{1}{2}$

$2(-2-y) = x$

$y = -\frac{1}{2}x - \frac{3}{2}$

Q#6

a:) $f(x) = \tan x \quad , \quad c = \frac{\pi}{4}$

$L(x) = f(c) + f'(c)(x - c)$

$= f(\frac{\pi}{4}) + f(\frac{\pi}{4})(x - \frac{\pi}{4})$

$f'(x) = \sec^2 x = \frac{1}{\cot^2 x}$

$L(x) = 1 + 2(x - \frac{\pi}{4})$

b:) $g(x) = \frac{1}{x} \quad , c = 1$

$L(x) = g(c) + g'(c)(x - c)$

$g(1) + g'(1)(x-1)$

$1 + -1(x-1)$

$g'(x) = \frac{-1}{x^2} = 2 - x$

C, D H.W