Chap 1: Review of functions

Domain
Range
f(x)

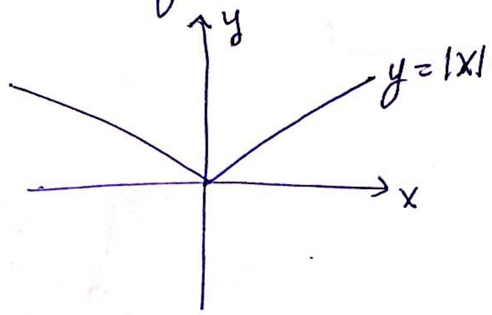
A function f is a Rule That assignes to each point x in The Domain a unique point "image" y f(x) in the Range R

VL test : To know weither The diagram is a function or not

How to know even or odd?

Known functions

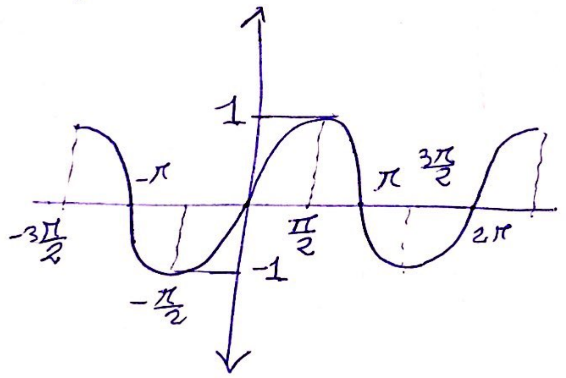
Absolute Value



$D = (-\infty, \infty)$
$R = [0, \infty)$
even

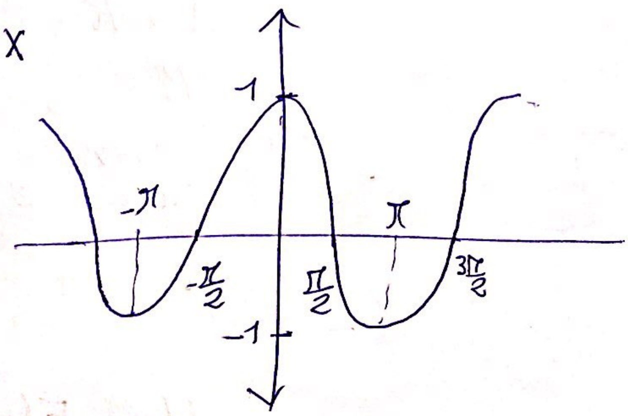
Trigonometric functions

$y = \sin x$



$D = (-\infty, \infty) = \mathbb{R}$
$R = [-1, 1]$
odd $\sin(-x) = -\sin(x)$
period 2π

$y = \cos x$

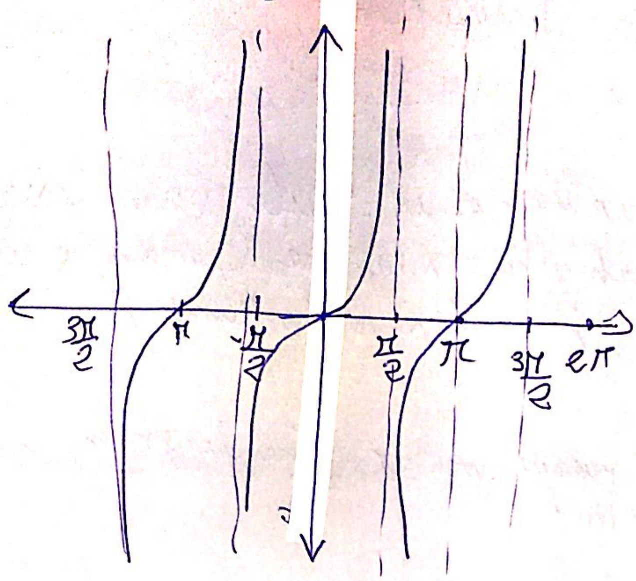


D: R
Range: $[-1, 1]$
even $\cos(-x) = \cos(x)$
period 2π

Maa Elaimi

$y = \tan x = \frac{\sin x}{\cos x}$

$n = 0, \pm 1$



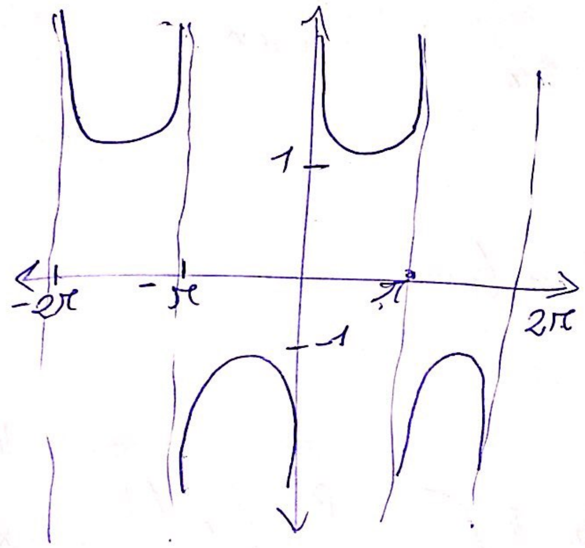
$D = (-\infty, \infty) \setminus \{\frac{\pi}{2} + n\pi\}$

Range $= (-\infty, \infty)$

odd

period $= \pi$

$y = \csc x = \frac{1}{\sin x}$ co-secant



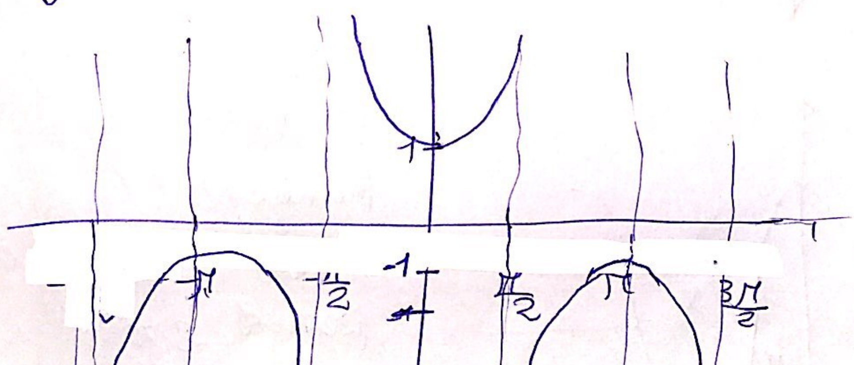
$D = \mathbb{R} \setminus \{\pm\pi, \pm 2\pi \dots\}$

$R = (-\infty, -1] \cup [1, \infty)$

odd

period 2π

$y = \sec x = \frac{1}{\cos x}$ secant



$D = \mathbb{R} \setminus \{\pm\frac{\pi}{2}, \pm\frac{3\pi}{2} \dots\}$

$R = (-\infty, -1] \cup [1, \infty)$

even

period 2π

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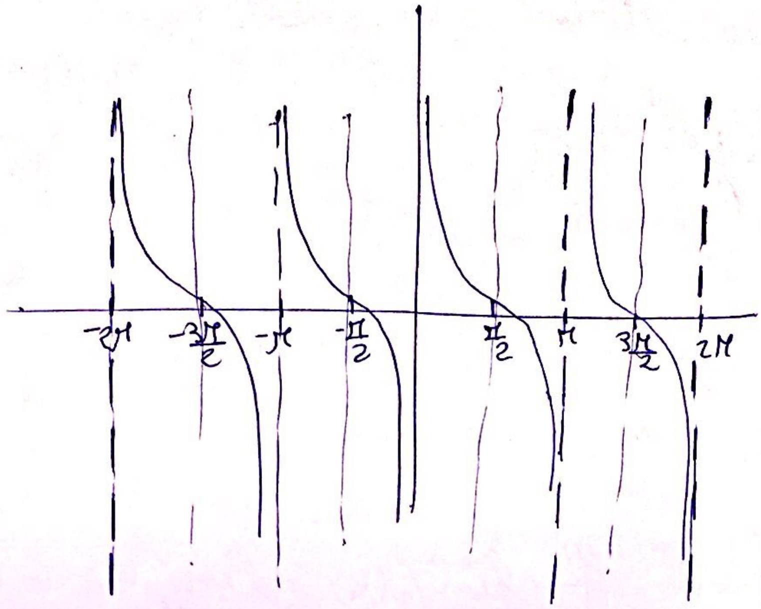
$y = \cot x = \frac{\cos x}{\sin x}$



$D = \mathbb{R} \setminus \{\pi + n2\pi\}$, $n = 0, \pm 1, \dots$

$R = \mathbb{R}$

Trigonometric Identities

1- $\sin^2(x) + \cos^2(x) = 1$

2- $\sin 2x = 2 \sin x \cos x$

3- $\cos 2x = \cos^2 x - \sin^2 x$
$= 1 - \sin^2 x$
$= 2\cos^2 x - 1$

4- $1 + \cot^2 x = \csc^2 x = \frac{1}{\sin^2 x}$

5- $\tan^2 x + 1 = \sec^2 x = \frac{1}{\cos^2 x}$

6- $\sin(A+B) = \sin A \cos B + \sin B \cos A$

7- $\cos(A+B) = \cos A \cos B - \sin A \sin B$

* $\sin x$, $\csc x$, $\tan x$, $\cot x$ $\longrightarrow$ odd
* $\cos x$, $\sec x$ $\longrightarrow$ even

$\tan, \cot x \longrightarrow$ period $\pi \longrightarrow \tan(x+\pi) = \tan x$
$\cot(x+\pi) = \cot x$

$\sin x, \cos x, \csc x, \sec x \longrightarrow$ period $2\pi \longrightarrow \sin(x+2\pi) = \sin x$
$\csc(x+2\pi) = \csc x$
$\cos(x+2\pi) = \cos x$
$\sec(x+2\pi) = \sec x$

* $\sin(x + \frac{\pi}{2}) = \cos x$ ازاحة للشمال
* $\cos(x + \frac{\pi}{2}) = -\sin x$

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Composite "o"

$(f \circ g) = f(g(x))$

$(f \circ g)(x) \neq (g \circ f)(x)$

Chap 2 : limits & continuity

$\lim_{x \to x_0} f(x) = L$ iff $\lim_{x \to x_0^+} f(x) = \lim_{x \to x_0^-} f(x) = L$

- Th:

- The sandwich Theorem: if $g(x) \leq f(x) \leq h(x)$
  if ~~Then~~ $\lim_{x \to c} g(x) = \lim_{x \to c} h(x) = L$
  Then $\lim_{x \to c} f(x) = L$

- $f(x)$ is continious at x_0 iff : $\lim_{x \to x_0} f(x)$ exist and $f(x_0)$ exist and $\lim_{x \to x_0} f(x) = f(x_0)$

- $f(x)$ is ~ ~ $[a, b]$ iff f is cont at every point of $[a, b]$

Exp: - $\sin(x)$ / $\cos(x)$ / $|x|$ / and any polynomials are cont on $\mathbb{R}$
- all Rational function $\frac{f(x)}{g(x)}$ are cont on $\mathbb{R}$ except the zeros of $g(x)$

Discontinuity : when $f(x)$ is Discontinued in a specific point we use a Removable ~~Discontinuity~~ point and we creat an Continious extension : $f(x)$ is Discont at x_0 and $\lim_{x \to x_0} = a$ Then

$$F(x) = \begin{cases} f(x), & x \neq x_0 \\ a, & x = x_0 \end{cases}$$

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The Intermediate Value Theorem IVT

* suppose That $f(x)$ is cont on $[a,b]$ and $f(a) \leq y_0 \leq f(b)$

Then $\exists$ a number $x_0 \in [a,b]$ s.t $f(x_0) = y_0$

Imp: If They ask if $f(x)$ has a Zero at $[a,b]$ we find $f(a)$, $f(b)$ and if $y_0 = 0$ is between

Then There is a zero

To draw a Rational function we use Asymptotes

Asymptotes are 3 Kinds: → H.Asy : line $y = b$

s.t : $\lim_{x \to \infty} f(x) = b$ or $\lim_{x \to -\infty} f(x) = b$

→ V.Asy line : $x = a$

s.t : $\lim_{x \to a^+} f(x) = \pm\infty$ or $\lim_{x \to a^-} f(x) = \pm\infty$

→ oblique.Asy : when $\frac{g(x)}{h(x)}$

$g(x)$ degree $>$ $h(x)$ degree

Notes

$$\lim_{x \to \pm\infty} \frac{\sin x}{x} = 0$$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

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