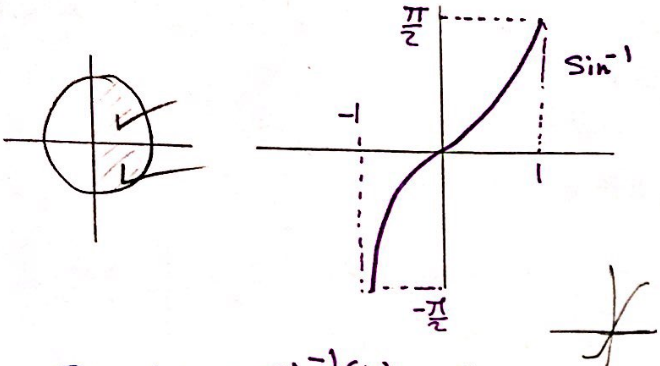
Inverse of Trigonometric functions

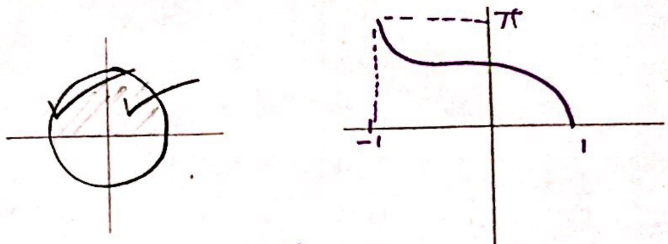
$f(x) = \sin x$ on $[-\frac{\pi}{2}, \frac{\pi}{2}]$ then
$f^{-1}(x) = \sin^{-1} x = \arcsin x$ on $[-1, 1]$



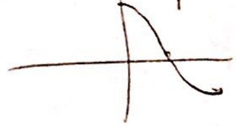
$\sin(x) + \sin^{-1}(x) = 0$

لانه متماثل حول نقطة الاصل

if $f(x) = \cos x$ on $[0, \pi]$ then
$f^{-1}(x) = \cos^{-1} x = \arccos x$ on $[-1, 1]$



$\cos(x) + \cos^{-1} x = \pi$



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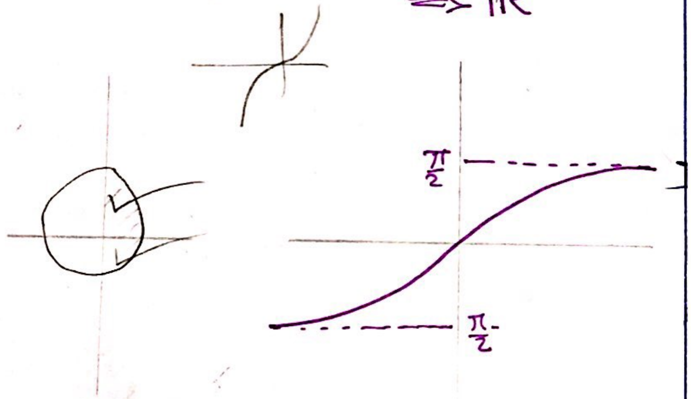
if $f(x) = \tan x$ on $(-\frac{\pi}{2}, \frac{\pi}{2})$ then
$\tan^{-1} x = \arctan x$ on $(-\infty, \infty)$
$\Rightarrow \mathbb{R}$



$\tan x + \tan^{-1} x = 0$

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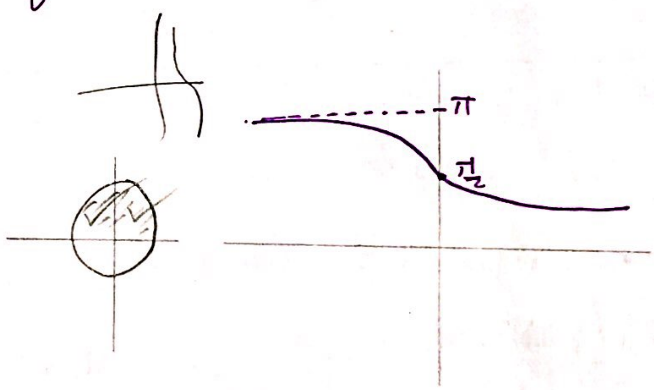
if $f(x) = \cot x$ on $(0, \pi)$ then
$f^{-1}(x) = \cot^{-1} x$ on $\mathbb{R}$



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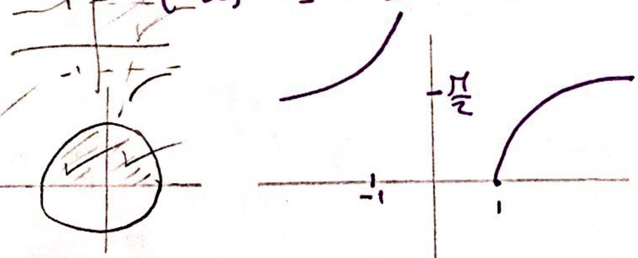
if $f(x) = \sec x$ on $[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$

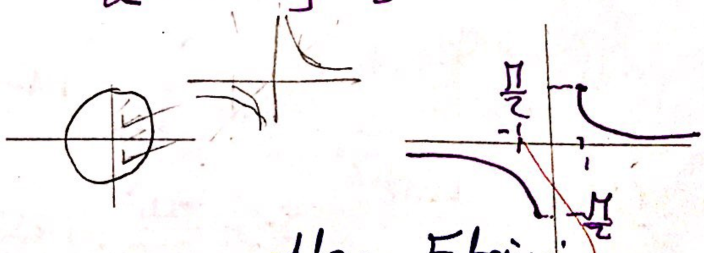
Then

$f^{-1}(x) = \sec^{-1} x = \operatorname{arc\,sec} x$ on
$(-\infty, -1] \cup [1, \infty)$



if $f(x) = \csc x$ on $[-\frac{\pi}{2}, 0) \cup$
$(0, \frac{\pi}{2}]$ then

$f^{-1}(x) = \csc^{-1} x = \operatorname{arc\,csc}$ on
$(-\infty, -1] \cup [-1, \infty)$



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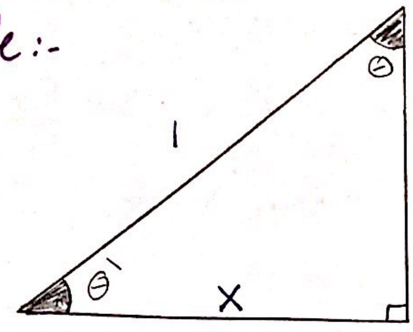
Note :-



$\sin^{-1}(x) = \csc^{-1}\frac{1}{x}$

$\cos^{-1}x = \sec^{-1}\frac{1}{x}$

Note that

المجاور/الوتر $\leftarrow \cos\theta = x \Rightarrow \sec\theta = \frac{1}{x}$ الوتر على المجاور

المقابل/الوتر $\leftarrow \sin\theta = x \Rightarrow \csc\theta = \frac{1}{x}$ الوتر على المقابل

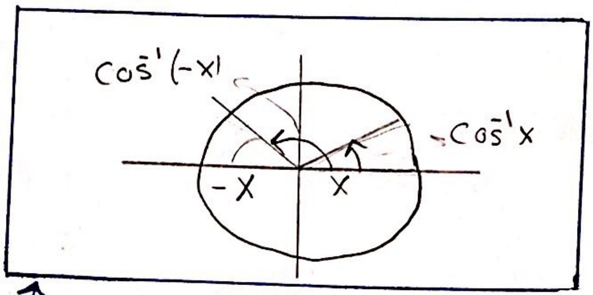
$\sec^{-1}\frac{1}{x} = \cos^{-1}x$

$\csc^{-1}\frac{1}{x} = \sin^{-1}x$

$\cos^{-1}x + \sin^{-1}x = \frac{\pi}{2}$

$\sec^{-1}\frac{1}{x} + \csc^{-1}\frac{1}{x} = \frac{\pi}{2}$

$\cos^{-1}x + \cos^{-1}(-x) = \pi$



* Derivatives for the inverse of Trigonometric functions

- $u(x)$ is diff function of x

1- $\frac{d(\sin^{-1}u(x))}{dx} = \frac{u'}{\sqrt{1-u^2}}$ $|u|<1$

2- $\frac{d(\cos^{-1}u(x))}{dx} = \frac{-u'}{\sqrt{1-u^2}}$ $|u|<1$

3- $\frac{d(\tan^{-1}u(x))}{dx} = \frac{u'}{1+u^2}$

4- $\frac{d(\cot^{-1}u(x))}{dx} = \frac{-u'}{1+u^2}$

5- $\frac{d(\sec^{-1}u(x))}{d(x)} = \frac{u'}{|u|\sqrt{u^2-1}}$ $|u|>1$

6- $\frac{d(\csc^{-1}u(x))}{dx} = \frac{-u'}{|u|\sqrt{u^2-1}}$ $|u|>1$

* Integrals for the inverse of Trigonometric functions

$\rightarrow a \neq 0$

1- $\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a} + C$

2- $\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$

3- $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C$

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② $f(x) = \tan x \Rightarrow f^{-1}(x) = \tan^{-1} x$

$f'(x) = \sec^2 x$

$$\left(f^{-1}(x)\right)' = \frac{1}{f'(f^{-1}(x))} = \frac{1}{f'(\tan^{-1} x)}$$

$$= \frac{1}{\sec^2(\tan^{-1} x)}$$

But

$\tan^2 x = \sec^2 x - 1$

$\sec^2 x = \tan^2 x + 1$

$$= \frac{1}{(\tan(\tan^{-1} x))^2 + 1}$$

$$= \frac{1}{x^2+1}$$

But it's not the same for $\sec^{-1} x$

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③ for $\sec^{-1} x$:-

let $y = \sec^{-1} x$ أدخل sec للطرفين

④ ... $\sec y = \sec(\sec^{-1} x)$ نشتق الطرفين

$\sec y \tan y \, y' = 1$

$$y' = \frac{1}{\sec y \tan y}$$

Back : $\sec y = x$

$$y' = \frac{1}{x \tan y}$$

$\tan y = \sqrt{\sec^2 y - 1}$

$= \sqrt{x^2 - 1}$

$$y' = \frac{1}{x \pm (\sqrt{x^2-1})}$$

$$y' = \frac{1}{|x|\sqrt{x^2-1}}$$ #

• If you get this you will never have a mistake in you exam

(But never say never :P)

- If you forgot The Derivatives of the inverse of Trigonometric functions go back to the Proof

Alaa Etaiwi

① $f(x) = \sin x \Rightarrow f^{-1}(x) = \sin^{-1}(x)$

$f'(x) = \cos x$

$(f^{-1}_{(x)})' = \dfrac{1}{f'(f^{-1}(x))} = \dfrac{1}{f'(\sin^{-1}x)}$

$= \dfrac{1}{\cos(\sin^{-1}x)}$

But :

$\cos^2 x + \sin^2 x = 1$

$\cos x = \sqrt{1-\sin^2 x}$

$= \dfrac{1}{\sqrt{1-\sin^2(\sin^{-1}(x))}}$

$= \dfrac{1}{\sqrt{1-x^2}}$

Same Way for $(\cos^{-1} x)'$