Chap 8: Techniques of integration

8.1:- Integrals by Parts

$$\int u \, dv = uv - \int v \, du$$

$$\int_a^b u \, dv = uv \Big|_a^b - \int_a^b v \, du$$

• How to solve Exercises?

• you need to decide which of the two functions is u and the other will represent dv

• u doesn't have to be the first function

• After that u need to organize the functions let's say we have $\int x \cos x \, dx$

$\rightarrow u = x$ نشتق $du = dx$ — $u \times v$ — $-\int v\,du$

$dv = \cos x \, dx$ نكامل $v = \sin x$

• There is another way to solve these integrals
Note: it works only if one of the functions can be derived to zero.

Ex: $x \underset{\text{نشتق}}{\Rightarrow} 1 \underset{\text{نشتق}}{\Rightarrow} 0$ ✓

if so :-

$\frac{d}{dx}$		$\int$
x	$(+)$	$\cos x$
1	$(-)$	$\sin x$
0		$-\cos x$

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8.2 Trigonometric integrals

⚠ How to integral $\int \cos^m x \sin^n x \, dx$?

Answer :- [1] if m is odd (ضروري)

Then $m = 2K+1$

$$\Rightarrow \cos^m x = \cos^{2K+1} x$$
$$= [\cos^2 x]^K \cos x$$
$$= [1-\sin^2 x]^K \cos x$$

Then let $u = \sin x$

and $du = \cos x \, dx$

[2] if n is odd $\Rightarrow n = 2K+1$

$$\sin^n x = \sin^{2K+1} x$$
$$= [\sin^2 x]^K \sin x$$
$$= [1-\cos^2 x]^K \sin x$$

Then let $u = \cos x$

$du = -\sin x \, dx$

[3] If n and m are both even then:-

we use : $\sin^2 x = \frac{1-\cos 2x}{2}$ → $\cos 2x = 1 - 2\sin^2 x$

$\cos^2 x = \frac{1+\cos 2x}{2}$ → $\cos 2x = 2\cos^2 x - 1$

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Another Case is when n and m are not powers (2)
But Constants to the variable x :-

1- $\int \sin mx \cos nx \, dx$

$= \frac{1}{2} \int [\sin (m-n)x + \sin (m+n)x] \, dx$

2- $\int \sin mx \sin nx \, dx$

$= \frac{1}{2} \int [\cos (m-n) - \cos (m+n)x] \, dx$

Identical But instead of − we use +

3- $\int \cos mx \cos nx \, dx$

$= \frac{1}{2} \int [\cos (m-n)x + \cos (m+n)x] \, dx$

• ملاحظة: دائمًا تكامل sin ناقص تكامل ال cos

Powers of tan and sec :-

① $\int \tan^3 x \, dx$

$= \int \tan^2 x \tan x \, dx$

$= \int (\sec^2 x - 1)(\tan x) \, dx$ — we separate them

$= \int \sec^2 x \tan x \, dx - \int \tan x \, dx$ — let $u = \tan x$, $du = \sec^2 x \, dx$

$= \int u \, du - \int \frac{\sin x}{\cos x} \, dx$

$= \frac{u^2}{2} + \int \frac{(\cos x)'}{\cos x} \, dx$

$= \frac{\tan^2 x}{2} + \ln|\cos x| + C$ ※

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2- $\int \sec^4 x \, dx$

$= \int \sec^2 x \sec^2 x \, dx$

$= \int (1 + \tan^2 x) \sec^2 x \, dx$ let $u = \tan x$

$du = \sec^2 x$

$= \int (1 + u^2) \, du$

$= u + \frac{u^3}{3} + C = \tan x + \frac{\tan^3 x}{3} + C$