

4 Integration of Rational functions using Partial fractions

* A Rational function is : $\frac{f(x)}{g(x)}$

• How to integrate it?

→ The idea of using Partial fraction is to re-write the Rational function $\frac{f(x)}{g(x)}$ as a sum of Partial "simpler" fractions that are easy to integrate

• There are 2 cases :-

1] The degree of $f(x) \geq$ The degree of $g(x)$
→ we use long division

2] The degree of $f(x) <$ The degree of $g(x)$

→ we have the following cases :-

a] if $g(x)$ is a product of linear distinct factors

→ we use Cover Method

b] otherwise we use different approaches

• Remark:- If in case a The products were not distinct : for example :-

$\frac{1}{(x-1)^2}$ we use the cover method but with some changes

$$\frac{1}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$$

Alaa Etawi

Another example:-

$$\frac{1}{(x^2+1)^2} = \frac{Ax+B}{x^2+1} + \frac{Cx+1}{(x^2+1)^2}$$

Explanation of the cover method:-

for example: $\int \frac{(x+5)}{(x-4)(x+1)}$

$$\frac{(x+5)}{(x-4)(x+1)} = \frac{A}{x-4} + \frac{B}{x+1}$$

$$\frac{(x+5)}{(x-4)(x+1)} = \frac{A(x+1) + B(x-4)}{(x-4)(x+1)}$$

بعد توحيد المقامات

$$x+5 = A(x+1) + B(x-4)$$

الآن نختار أي قيمة عشوائية

د x لكي نجد A و B

و نفضل ان تكون القيمة

تصفّر معامل A صفر وتصفّر

معامل B مرة اخرى

وهنا تكون هذه القيمة هي

$$x = -1/4$$

① $-1+5 = 0 + -5B$

$$B = \frac{4}{5}$$

② $4+5 = 5A + 0$

$$A = \frac{5}{5} = \frac{9}{5}$$

or :-

$$\frac{(x+5)}{(x-4)(x+1)} = \frac{A}{x-4} + \frac{B}{x+1}$$

لإيجاد A

① نأخذ قيمة x

التي تصفر المقام

② \rightarrow
نضع $(x-4)$ موضع
وتوض
ال 4
في المقام

③ نضع على قيمة A

$$A = \frac{4+5}{4+1} = \frac{9}{5}$$

④ نجد B بتوض $x = -1/4$

Hana Etaini

Explanation of the long division

Exp $\int \frac{x^3}{x^2 - 2x + 1} dx$



$$\begin{array}{r}
 x+2 \\
 \hline
 x^2-2x+1 \overline{) x^3} \\
 \underline{x^3-2x^2+x} \\
 2x^2-x \\
 \underline{2x^2-4x+2} \\
 3x-2
 \end{array}$$

① ضرب القسوم عليه بـ x

② طرح $x^3 - 2x^2 + x$ من x^3

③ ضرب القسوم عليه بـ 2

④ طرح $2x^2 - 4x + 2$ من $2x^2 - x$

$$\int \frac{x^3}{x^2 - 2x + 1} dx = \int (x+2) + \frac{3x-2}{x^2 - 2x + 1}$$

⑤ نضع الجواب + الباقي