3.2 + 3.1 Tangent and Derivative at a point

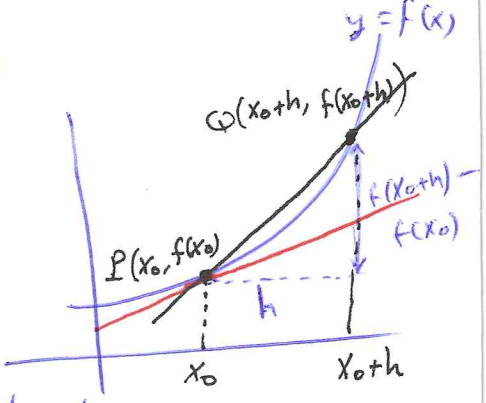
Def The slope of the curve $y = f(x)$ at the point $P(x_0, f(x_0))$ is

$$m = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$$ (provided the limit exists)

→ average rate of change

The tangent line to the curve at P with slope m is

$$y = m(x - x_0) + f(x_0)$$



Def The derivative of a function f at a point x_0 is

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$$ "limit exists"

Example: Find the slope of the function $f(x) = x^2 + 1$ at the point $(\overset{x_0}{2}, \overset{f(x_0)}{5})$ and find an equation for the tangent line

$$m = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0} \frac{(2+h)^2 + 1 - (2^2+1)}{h}$$

$$= \lim_{h \to 0} \frac{4 + 4h + h^2 + 1 - 5}{h} = \lim_{h \to 0} \frac{h(4+h)}{h} = 4 + 0 = 4$$

$$y = m(x - x_0) + f(x_0)$$

$$= 4(x - 2) + 5$$

$$y = 4x - 3 \quad \text{(tangent line)}$$