$\boxed{3.7}$ + $\boxed{3.6}$ The Chain Rule

Theorem If f is differentiable at x and g is differentiable at $f(x)$, then $g \circ f$ is differentiable at x

$$y = (g \circ f)(x) = g(f(x))$$

$$f = \frac{dy}{dx} = (g \circ f)'(x) = g'(f(x))\, f'(x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \qquad \begin{array}{l} y = g(u) \\ u = f(x) \end{array}$$

Example Find the derivatives of

$\boxed{a}$ $x(t) = \cos(t^2+1)$

$$\frac{dx}{dt} = -\sin(t^2+1)\,(2t) = -2t\sin(t^2+1)$$

$\boxed{b}$ $y = \sin(x^2+x)$

$$y' = \cos(x^2+x)\,(2x+1) = (2x+1)\cos(x^2+x)$$

$\boxed{c}$ $g(t) = \tan(5-\sin 2t)$

$$\frac{dg}{dt} = \sec^2(5-\sin 2t)\,\bigl(-\cos 2t\,(2)\bigr)$$

$$= -2\cos 2t\,\sec^2(5-\sin 2t)$$

$\boxed{d}$ $f(x) = (5x^3-x^4)^7$

$$f' = 7(5x^3-x^4)^6\,(15x^2-4x^3)$$

$\boxed{e}$ $y = \sin^5 x \Rightarrow y = [\sin x]^5$

$$y' = 5\sin^4 x\,\cos x$$

$\boxed{f}$ $h(x) = |x| = \sqrt{x^2} = (x^2)^{\frac{1}{2}}$ 67

$h'(x) = \frac{1}{2}(x^2)^{-\frac{1}{2}} \, 2x$

$= \frac{x}{\sqrt{x^2}} \quad , \quad x \neq 0$

$= \frac{x}{|x|} \quad , \quad x \neq 0$

$\boxed{g}$ $y = \frac{1}{(1-2x)^3} = (1-2x)^{-3}$

$y' = -3(1-2x)^{-4}(-2)$

$= \frac{6}{(1-2x)^4}$
