$\boxed{4.3}$ Monotonic Functions and the First Derivative Test

* A function that is increasing or decreasing on an interval is called monotonic on the interval.

Corollary3 Suppose f is continuous on $[a,b]$ and differentiable on (a,b).
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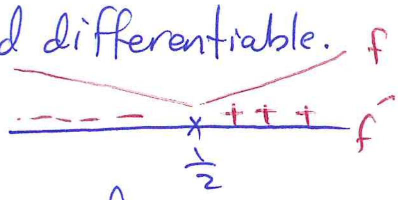
→ If $f'(x) > 0$ for every $x \in (a,b)$, then f is increasing on $[a,b]$

→ If $f'(x) < 0$ for every $x \in (a,b)$, then f is decreasing on $[a,b]$

Example: Find the critical points of $f(x) = x^2 - x$ and identify the intervals on which f is increasing or decreasing.

$f(x)$ is everywhere continuous and differentiable.

$f'(x) = 2x - 1 = 0 \Leftrightarrow x = \frac{1}{2}$



f is increasing on $(\frac{1}{2}, \infty)$ and decreasing on $(-\infty, \frac{1}{2})$

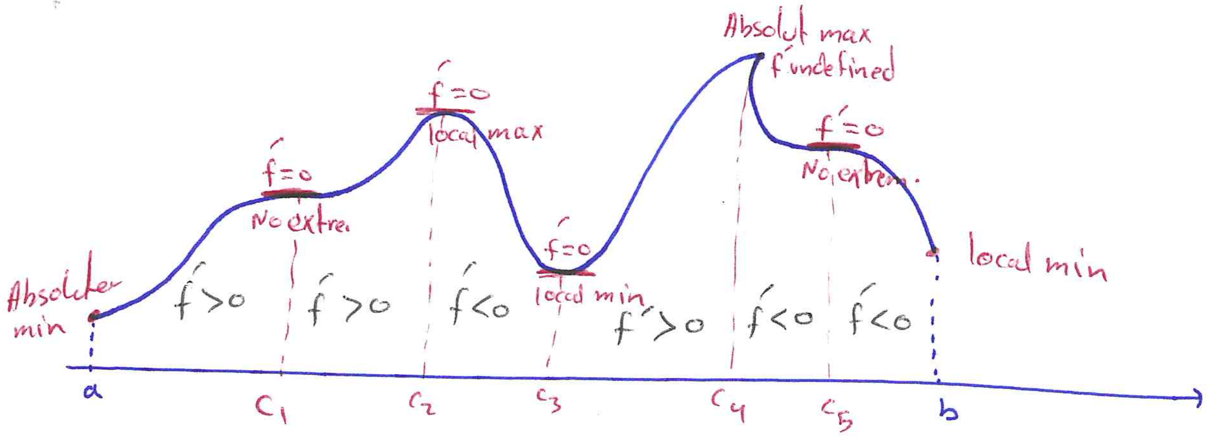
First Derivative Test for Local Extrema:

Suppose c is a critical point of a continuous function f, and f is differentiable on (a,b) except possible at c, where $c \in (a,b)$ then:

$\boxed{1}$ If f' changes from negative to positive at c, then f has a local min at c.

$\boxed{2}$ If f' " " positive to negative at c, " " " " max at c.

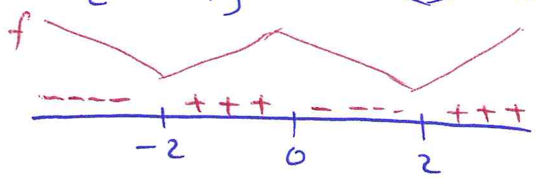
$\boxed{3}$ If f' does not change sign at c, then f has no local extremum at c.



Example: Find the critical points of $f(x) = x^4 - 8x^2 + 16$
Identify the intervals on which f is increasing or decreasing
Find the local max/min and absolute max/min

f is continuous for all $x \Rightarrow f'(x) = 4x^3 - 16x = 0$

$\Leftrightarrow 4x[x^2 - 4] = 0 \Leftrightarrow x = 0, 2, -2$ critical points



- f is increasing on $(-2, 0)$ and on $(2, \infty)$
- f is decreasing on $(-\infty, -2)$ and on $(0, 2)$

- f has local min at $(-2, f(-2)) = (-2, 0)$
  $(2, f(2)) = (2, 0)$
- f has local max at $(0, f(0)) = (0, 16)$
- f has absolute min at 0 when $x = \pm 2$
- f has no absolute max