

4.7

Antiderivatives

(93)

Def A function F is an antiderivative of f on an interval I if $F'(x) = f(x)$ for all $x \in I$.

Example: Find the antiderivative of

$$\textcircled{1} \quad f(x) = 2x, \quad F(x) = x^2$$

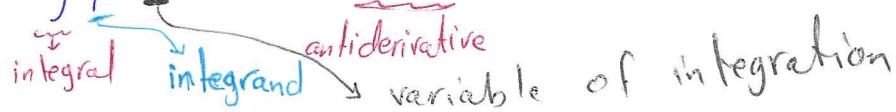
$$\textcircled{2} \quad g(x) = x^2 - 2x + 1, \quad G(x) = \frac{x^3}{3} - x^2 + x$$

$$\textcircled{3} \quad h(x) = \frac{5}{2\sqrt{x}}, \quad H(x) = 5\sqrt{x}$$

$$\textcircled{4} \quad r(x) = -\pi \sin \pi x, \quad R(x) = \cos \pi x$$

Theorem: If F is an antiderivative of f on an interval I , then the most general antiderivative of f on I is $F(x) + C$, where C is an arbitrary constant.

Note that the collection of all antiderivatives of f is called the indefinite integral of f with respect to x and defined by $\int f(x) dx = \underline{F(x)} + C$.


 integral integrand antiderivative variable of integration

Example: Find the most general antiderivatives (or indefinite integral) of

$$\textcircled{1} \quad \int (3x^2 + \frac{x}{2}) dx = x^3 + \frac{x^2}{4} + C$$

$$\textcircled{2} \quad \int -2 \cos t dt = -2 \sin t + C$$

$$\textcircled{3} \quad \int (1 + \tan^2 \theta) d\theta = \int \sec^2 \theta d\theta = \tan \theta + C$$

$$\textcircled{4} \quad \int (2x^3 - 5x + 7) dx = \frac{x^4}{2} - \frac{5x^2}{2} + 7x + C$$

Example: Find an antiderivative of $f(x) = 3x^2$ (94)
that satisfies $F(1) = -1$

The general antiderivative is $F(x) = x^3 + C$

$$F(1) = 1^3 + C = -1 \Leftrightarrow C = -2 \text{ Thus } F(x) = x^3 - 2$$

So we need to find a function $y(x)$ that satisfies

Initial Value Problem $\left\{ \begin{array}{l} \frac{dy}{dx} = f(x) \\ y(x_0) = y_0 \end{array} \right.$

$\frac{dy}{dx} = f(x)$ --- "differential equation"
 "is equation that has derivatives"
 $y(x_0) = y_0$ --- "initial condition"
 To find C

* The general solution $y(x) = F(x) + C$

* When we find C , we find a particular solution $y(x) = x^3 - 2$

Example: Solve the initial value problem

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}} + 2 \sin 2x, \quad y(0) = 0$$

$$\int \frac{dy}{dx} dx = \int \left(\frac{1}{2\sqrt{x}} + 2 \sin 2x \right) dx$$

$$y(x) = \sqrt{x} - \cos 2x + C \quad \text{--- the general solution}$$

$$y(0) = \sqrt{0} - \cos 0 + C = 0$$

$$0 - 1 + C = 0 \Leftrightarrow C = 1$$

The particular solution is
$$y(x) = \sqrt{x} - \cos 2x + 1$$