$\boxed{5.5}$ Indifinite Integrals and Substitution Method

Example: Evaluate the indifinite integrals:

① $\int (x^3+x)^5 (3x^2+1)\,dx$ $\qquad u = x^3 + x$, $\quad du = (3x^2+1)\,dx$

$= \int u^5\,du = \frac{u^6}{6} + C = \frac{1}{6}(x^3+x)^6 + C$

② $\int (3x+2)(3x^2+4x)^4\,dx$ $\qquad u = 3x^2+4x$, $\quad du = (6x+4)\,dx$, $\quad \frac{du}{2} = (3x+2)\,dx$

$\frac{1}{2}\int u^4\,du = \frac{1}{2}\frac{u^5}{5} + C$

$= \frac{1}{10}(3x^2+4x)^5 + C$

③ $\int 7\sqrt{7x-1}\,dx$ $\qquad u = 7x-1$, $\quad du = 7\,dx$

$\int u^{\frac{1}{2}}\,du = \frac{2}{3}u^{\frac{3}{2}} + C = \frac{2}{3}(7x-1)^{\frac{3}{2}} + C$

④ $\int \frac{dx}{\sqrt{x}\,(1+\sqrt{x})^2}$ $\qquad u = 1+\sqrt{x}$, $\quad du = \frac{dx}{2\sqrt{x}}$

$2\int u^{-2}\,du = \frac{-2}{u} + C = \frac{-2}{1+\sqrt{x}} + C$

⑤ $\int \frac{-6\tan^2 x\,\sec^2 x}{(2+\tan^3 x)^3}\,dx$ $\qquad u = 2+\tan^3 x$, $\quad du = 3\tan^2 x\,\sec^2 x\,dx$

$-2\int \frac{du}{u^3} = -2\frac{u^{-2}}{-2} + C = \frac{1}{(2+\tan^3 x)^2} + C$

Example: Solve the IVP $\frac{ds}{dt} = 12t(3t^2-1)^3$, $s(1) = 3$ $\qquad u = 3t^2-1$, $\quad du = 6t\,dt$

$s(t) = \int 12t(3t^2-1)^3\,dt = 2\int u^3\,du$

$s(t) = 2\frac{u^4}{4} + C = \frac{(3t^2-1)^4}{2} + C$ $\qquad s(1) = \frac{2^4}{2} + C = 3 \Leftrightarrow C = -5$

$s(t) = \frac{1}{2}(3t^2-1)^4 - 5$

Example ⓐ $\int \sin^2 x\,dx = \int \frac{1-\cos 2x}{2}\,dx = \frac{x}{2} - \frac{\sin 2x}{4} + C$

ⓑ $\int \cos^2 x\,dx = \int \frac{1+\cos 2x}{2}\,dx = \frac{x}{2} + \frac{\sin 2x}{4} + C$