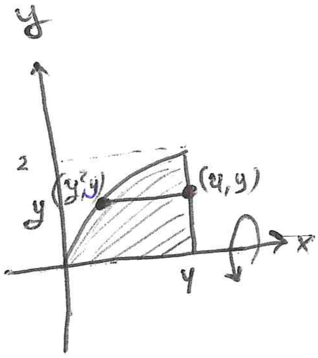
6.2 Volumes Using Shell Method

a The volume of the solid generated by revolving the region about x-axis is

$$V = \int_c^d 2\pi \begin{pmatrix}\text{shell}\\ \text{radius}\end{pmatrix}\begin{pmatrix}\text{shell}\\ \text{length}\end{pmatrix} dy, \quad \text{where}$$

- shell radius: is the distance from the axis of revolution and the shell length.
- shell length: is the segment's length parallel to the axis of revolution.

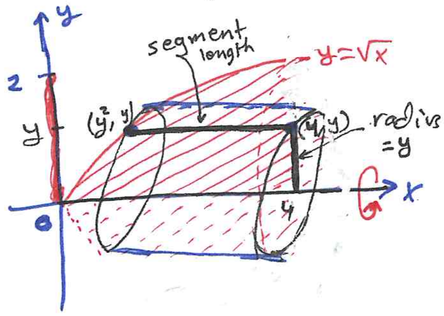
Example: Find the volume of the solid generated by revolving the region bounded by the curve $y=\sqrt{x}$, x-axis and the line $x=4$, about x-axis. Use the shell Method.



$$V = 2\pi \int_0^2 (y)(4-y^2)\, dy$$

$$= 2\pi \int_0^2 (4y - y^3)\, dy$$

$$= 2\pi \left[2y^2 - \frac{y^4}{4}\right]_0^2 = 8\pi$$



The shell thickness variable is y

---

b The volume of the solid generated by revolving the region about y-axis is

$$V = \int_a^b 2\pi \begin{pmatrix}\text{shell}\\ \text{radius}\end{pmatrix}\begin{pmatrix}\text{shell}\\ \text{height}\end{pmatrix} dx, \quad \text{where}$$

- shell radius: is the distance from the axis of revolution and the shell height.
- shell height: is the segmant's height parallel to the axis of revolution.

Example: Use the shell method to find the volume of the solid generated by revolution the region bounded by the curve $y=\sqrt{x}$, the x-axis, and the line $x=4$ about y-axis.

Use Washer Method ⟸

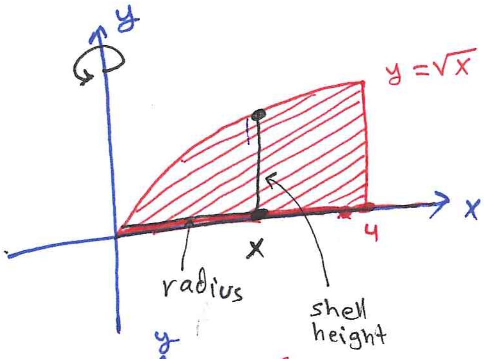
$V=\pi\int_0^2[16-y^4]\,dy$

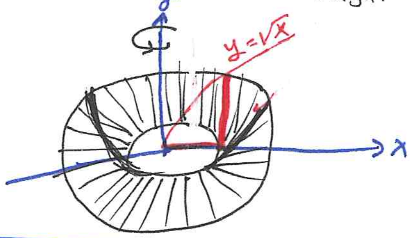
$=\frac{128}{5}\pi$

$$V=2\pi\int_0^4 (x)(\sqrt{x})\,dx$$

$$=2\pi\int_0^4 x^{\frac{3}{2}}\,dx = 2\pi\,\frac{2}{5}\,x^{5/2}\Big|_0^4$$

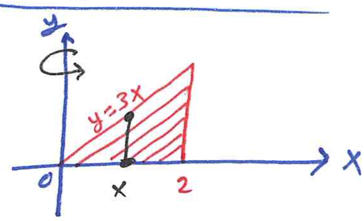
$$=\frac{128}{5}\pi$$





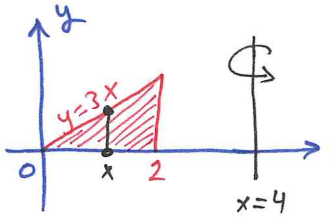
Example Q 23 $\boxed{a}$ $V=2\pi\int_a^b \begin{pmatrix}\text{shell}\\\text{radius}\end{pmatrix}\begin{pmatrix}\text{shell}\\\text{height}\end{pmatrix}dx$

$$=2\pi\int_0^2 (x)(3x)\,dx = 16\pi$$



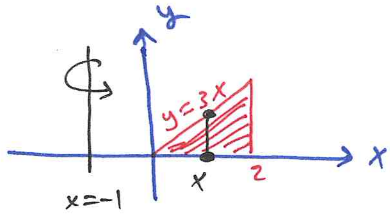
$\boxed{b}$ $V=2\pi\int_a^b \begin{pmatrix}\text{shell}\\\text{radius}\end{pmatrix}\begin{pmatrix}\text{shell}\\\text{height}\end{pmatrix}dx$

$$=2\pi\int_0^2 (4-x)(3x)\,dx = 32\pi$$



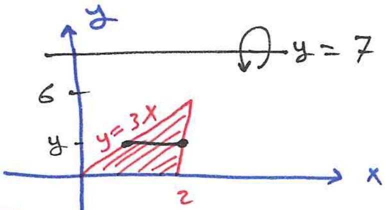
$\boxed{c}$ $V=2\pi\int_a^b \begin{pmatrix}\text{shell}\\\text{radius}\end{pmatrix}\begin{pmatrix}\text{shell}\\\text{height}\end{pmatrix}dx$

$$=2\pi\int_0^2 (x--1)(3x)\,dx = 28\pi$$



$\boxed{e}$ $V=2\pi\int_c^d \begin{pmatrix}\text{shell}\\\text{radius}\end{pmatrix}\begin{pmatrix}\text{shell}\\\text{length}\end{pmatrix}dy$

$$=2\pi\int_0^6 (7-y)\left(2-\frac{y}{3}\right)dy = 60\pi$$

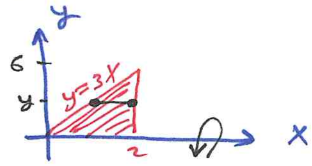


use disk method

$\int_0^2 (3x)^2\pi\,dx$ → $\boxed{d}$ $V=2\pi\int_c^d \begin{pmatrix}\text{shell}\\\text{radius}\end{pmatrix}\begin{pmatrix}\text{shell}\\\text{length}\end{pmatrix}dy$

$=9\pi\int_0^2 x^2\,dx = 24\pi$

$$=2\pi\int_0^6 (y)\left(2-\frac{y}{3}\right)dy = 24\pi$$



$\boxed{f}$ $V=2\pi\int_0^6 (y--2)\left(2-\frac{y}{3}\right)dy = 48\pi.$