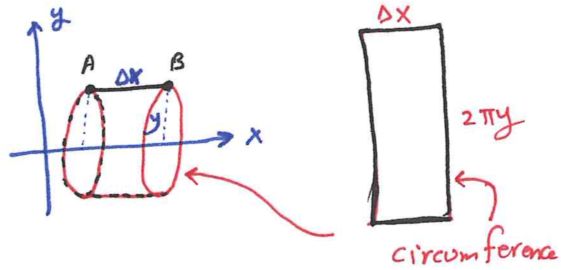
$\boxed{6.4}$ Surfaces Areas of Revolution

* The surface area generated by rotating the horizontal line AB of length Δx about x-axis is $2\pi y\,\Delta x$



Def: If $y=f(x)\geq 0$ is continuously differentiable on $[a,b]$, then the area of surface generating by revolving the graph of $y=f(x)$ about the x-axis is

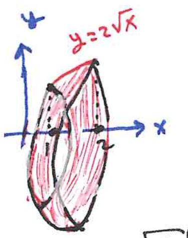
$$S=\int_a^b 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx=\int_a^b 2\pi f(x)\sqrt{1+[f'(x)]^2}\,dx$$

Def: If $x=g(y)\geq 0$ is 〃 〃 〃 $[c,d]$, 〃
〃 〃 〃 〃 〃 〃 〃 〃 〃 〃 $x=g(y)$
〃 〃 y-axis is

$$S=\int_c^d 2\pi x\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy=\int_c^d 2\pi g(y)\sqrt{1+[g'(y)]^2}\,dy$$

Example: Find the area of the surface generated by revolving the curve

$\boxed{1}$ $y=2\sqrt{x}$, $1\leq x\leq 2$ about x-axis



$$S=2\int_1^2 2\pi\sqrt{x}\sqrt{1+\left(\frac{1}{\sqrt{x}}\right)^2}\,dx=4\pi\int_1^2\sqrt{x}\sqrt{\frac{x+1}{x}}\,dx=4\pi\int_1^2\sqrt{x+1}\,dx$$

$u=x+1$

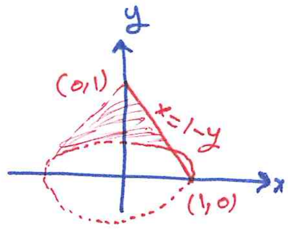
$du=dx$

$$=4\pi\int_2^3 u^{\frac{1}{2}}\,du=\frac{8\pi}{3}u^{\frac{3}{2}}\Big|_2^3=\frac{8\pi}{3}(3\sqrt{3}-2\sqrt{2}).$$

$\boxed{2}$ $x=1-y$, $0\leq y\leq 1$ about y-axis.

$$S=2\pi\int_0^1(1-y)\sqrt{1+(-1)^2}\,dy=2\pi\int_0^1\sqrt{2}(1-y)\,dy$$

$$=2\sqrt{2}\pi\left[y-\frac{y^2}{2}\right]\Big|_0^1=2\pi\sqrt{2}\left(1-\frac{1}{2}\right)=\pi\sqrt{2}$$



Cone

The base not included

"Only the Lateral surface area".

مساحة مثلث = $\frac{1}{2}$ x القاعدة x الارتفاع

$$S=\cancel{\frac{1}{2}}\cancel{2}(1)(\pi)\sqrt{2}=\pi\sqrt{2}$$

$\boxed{3}$ $x = \sqrt{2y-1}$, $\frac{5}{8} \le y \le 1$, y-axis (117)

$$S = \int_c^d 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$\frac{dx}{dy} = \frac{1}{2}(2y-1)^{-\frac{1}{2}}(2) = \frac{1}{\sqrt{2y-1}}$$

$$(x')^2 = \frac{1}{2y-1}$$

$$1 + (x')^2 = \frac{2y-1+1}{2y-1} = \frac{2y}{2y-1}$$

$$= \int_{\frac{5}{8}}^{1} 2\pi \sqrt{2y-1} \sqrt{\frac{2y}{2y-1}}\, dy$$

$$= \int_{\frac{5}{8}}^{1} 2\pi \sqrt{2y}\, dy$$

$$= 2\pi \sqrt{2}\, y^{\frac{3}{2}} \frac{2}{3} \Big|_{\frac{5}{8}}^{1} = \frac{4\sqrt{2}\pi}{3}\left[1 - \sqrt{\left(\frac{5}{8}\right)^3}\right]$$