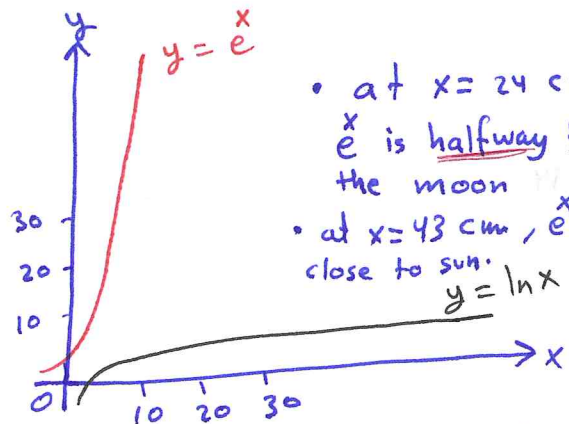
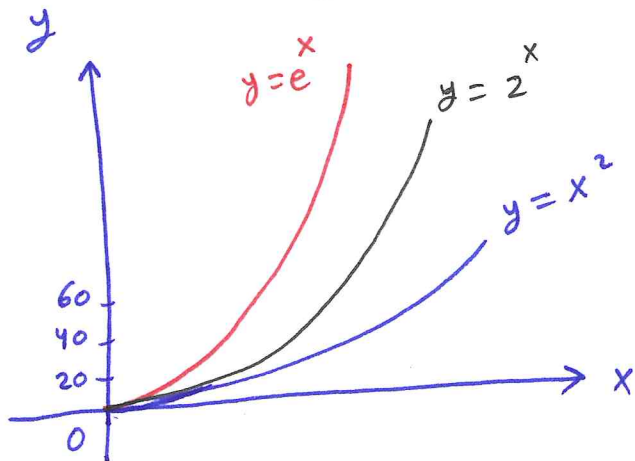


# 7.8 Relative Rates of Growth

(29)



- at  $x = 24$  cm,  $e^x$  is halfway to the moon
- at  $x = 43$  cm,  $e^x$  is close to sun.

we need  $\approx 5$  light-years on  $x$ -axis to find point where  $y = \ln x = 43$  cm

Def: Let  $f(x)$  and  $g(x)$  be positive for large  $x$ :

- If  $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = \infty$ , then  $f$  grows faster than  $g$  as  $x \rightarrow \infty$
- If  $\lim_{x \rightarrow \infty} \frac{g(x)}{f(x)} = 0$ , then  $g$  grows slower than  $f$  as  $x \rightarrow \infty$ .
- If  $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = L$  " $0 < L < \infty$ ", then  $f$  and  $g$  grow at the same rate as  $x \rightarrow \infty$

Exp. ①  $4^x$  grows faster than  $e^x$  as  $x \rightarrow \infty$  since

$$\lim_{x \rightarrow \infty} \frac{4^x}{e^x} = \left(\frac{4}{e}\right)^x = \infty \quad e \approx 2.718$$

②  $\left(\frac{3}{2}\right)^x$  grows slower than  $e^x$  as  $x \rightarrow \infty$  since

$$\lim_{x \rightarrow \infty} \frac{\left(\frac{3}{2}\right)^x}{e^x} = \lim_{x \rightarrow \infty} \frac{\left(\frac{3}{2e}\right)^x}{1} = 0$$

③  $\log_3 x$  grows same as  $\ln x$  as  $x \rightarrow \infty$  since

$$\lim_{x \rightarrow \infty} \frac{\log_3 x}{\ln x} = \lim_{x \rightarrow \infty} \frac{\frac{\ln x}{\ln 3}}{\ln x} = \lim_{x \rightarrow \infty} \frac{1}{\ln 3} = \frac{1}{\ln 3}$$

④  $\ln x$  grows slower than  $e^x$  as  $x \rightarrow \infty$  since

$$\lim_{x \rightarrow \infty} \frac{\ln x}{e^x} = \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{e^x} = \lim_{x \rightarrow \infty} \frac{1}{x e^x} = 0$$