$\boxed{8.1}$ Integration by Parts

① $$\int u\,dv = uv - \int v\,du$$

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

Exp ① $$\int x\cos x\,dx = \int u\,dv$$

$u = x \qquad dv = \cos x\,dx$

$du = dx \Rightarrow v = \sin x$

$$= uv - \int v\,du$$

$$= x\sin x - \int \sin x\,dx$$

$$= x\sin x + \cos x + c$$

② $$\int_0^{\pi} x\cos x\,dx = x\sin x\Big|_0^{\pi} - \int_0^{\pi}\sin x\,dx$$

$$= 0 + \cos x\Big|_0^{\pi} = -1 - 1 = -2$$

③ $$\int_0^{\pi} x\cos x\,dx$$

f(x) and its derivatives		g(x) and its integrals
$f(x) = x$	(+)	$g(x) = \cos x$
1	(−)	$\sin x$
0		$-\cos x$

$$= x\sin x + \cos x + c$$

Exp $$\int \ln x\,dx = uv - \int v\,du$$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x}\,dx \Rightarrow v = x$

$$= x\ln x - \int dx$$

$$= x\ln x - x + c$$

Exp $\int x^2 e^x \, dx$

$f(x)$		$g(x)$
$f(x) = x^2$		$g(x) = e^x$
$2x$	$(+)$	e^x
2	$(-)$	e^x
0	$(+)$	e^x

$$\int x^2 e^x dx = x^2 e^x - 2x e^x + 2e^x + C$$

$\int x^2 e^x \, dx$ $\qquad u = x^2 \quad dv = e^x dx$

$\qquad du = 2x\,dx \quad V = e^x$

$= x^2 e^x - \int 2x e^x dx$ $\qquad$ we need $\int x e^x dx$

$\qquad u = x \quad dv = e^x dx$

$= x^2 e^x - 2\int x e^x dx$ $\qquad du = dx \quad v = e^x$

$= x^2 e^x - 2\left[x e^x - \int e^x dx\right]$

$= x^2 e^x - 2x e^x + 2\int e^x dx = x^2 e^x - 2x e^x + 2e^x + C$

Exp* $\int e^x \cos x \, dx$

$f(x) = e^x$		$g(x) = \cos x$
e^x	$(+)$	$\sin x$
e^x	$(-)$ stop	$-\cos x$

$$\int e^x \cos x \, dx = e^x \sin x + e^x \cos x - \int e^x \cos x \, dx$$

$$2\int e^x \cos x \, dx = e^x \sin x + e^x \cos x$$

$$\int e^x \cos x \, dx = \frac{1}{2} e^x (\sin x + \cos x) + C$$

$\int e^x \cos x \, dx$ $\qquad u = e^x \quad dv = \cos x \, dx$

$\qquad du = e^x dx \quad V = \sin x$

$\int e^x \cos x \, dx = e^x \sin x - \int e^x \sin x$ $\qquad u = e^x \quad dv = \sin x \, dx$

$\qquad du = e^x dx \quad V = -\cos x$

$= e^x \sin x - \left[-e^x \cos x + \int e^x \cos x \, dx\right]$

$$\int e^x \cos x \, dx = e^x \sin x + e^x \cos x - \int e^x \cos x \, dx$$

$$2\int e^x \cos x \, dx = e^x \sin x + e^x \cos x$$

$$\int e^x \cos x \, dx = \frac{1}{2} e^x [\sin x + \cos x] + C$$

<u>Exp</u> $\displaystyle\int_1^2 x \ln x \, dx$

$u = \ln x \qquad dv = x \, dx$

$du = \dfrac{dx}{x} \qquad v = \dfrac{x^2}{2}$

$$\int_1^2 x \ln x \, dx = \frac{x^2}{2} \ln x \Big|_1^2 - \int_1^2 \frac{1}{x} \frac{x^2}{2} \, dx$$

$$= \frac{x^2}{2} \ln x \Big|_1^2 - \frac{1}{2}\int_1^2 x \, dx$$

$$= 2\ln 2 - 0 - \frac{1}{2} \frac{x^2}{2} \Big|_1^2 = 2\ln 2 - 1 + \frac{1}{4}$$

$$= \ln 4 - \frac{3}{4}$$

$$\int x \ln x \, dx = x^2 \ln x - x^2 - \int x \ln x \, dx + \frac{x^2}{2}$$

$$2\int x \ln x \, dx = x^2 \ln x - \frac{x^2}{2}$$

$$\int_1^2 x \ln x \, dx = \left[\frac{x^2}{2} \ln x - \frac{x^2}{4}\right]_1^2$$

$$= [2\ln 2 - 1] - [0 - \tfrac{1}{4}]$$

$$= 2\ln 2 - \frac{3}{4} \checkmark$$

$f(x) = x$		$g(x) = \ln x$
	$(+)$	$x \ln x - x$
1	$(-)$	$\int x \ln x \, dx - \frac{x^2}{2}$
0		