

8.1 Integration by Parts

30

$$\textcircled{1} \int u dv = uv - \int v du$$

$$\int_a^b u dv = uv \Big|_a^b - \int_a^b v du$$

Exp $\textcircled{1} \int x \cos x dx = \int u dv$

$u = x \rightarrow dv = \cos x dx$
 $du = dx \rightarrow v = \sin x$

$$= uv - \int v du$$

$$= x \sin x - \int \sin x dx$$

$$= x \sin x + \cos x + c$$

$$\textcircled{2} \int_0^{\pi} x \cos x dx = x \sin x \Big|_0^{\pi} - \int_0^{\pi} \sin x dx$$

$$= 0 + \cos x \Big|_0^{\pi} = -1 - 1 = -2$$

$$\textcircled{3} \int_0^{\pi} x \cos x dx$$

f(x) and its derivatives g(x) and its integrals

$f(x) = x \xrightarrow{(+)} g(x) = \cos x$

$1 \xrightarrow{(-)} \sin x$

$0 \xrightarrow{(-)} -\cos x$

$$= x \sin x + \cos x + c$$

Exp $\int \ln x dx = uv - \int v du$

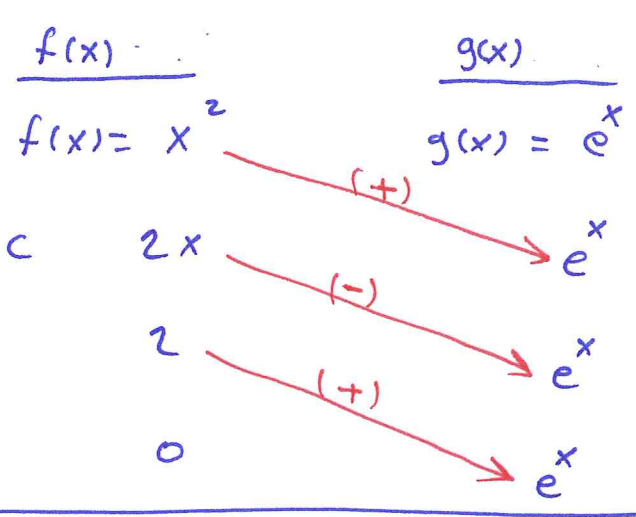
$$= x \ln x - \int dx$$

$$= x \ln x - x + c$$

$u = \ln x \rightarrow dv = dx$
 $du = \frac{1}{x} dx \rightarrow v = x$

Exp $\int x^2 e^x dx$

(31)



$\int x^2 e^x dx = x^2 e^x - 2x e^x + 2e^x + c$

$\int x^2 e^x dx$	$u = x^2$	$dv = e^x dx$
	$du = 2x dx$	$v = e^x$

$= x^2 e^x - \int 2x e^x dx$

we need $\int x e^x dx$

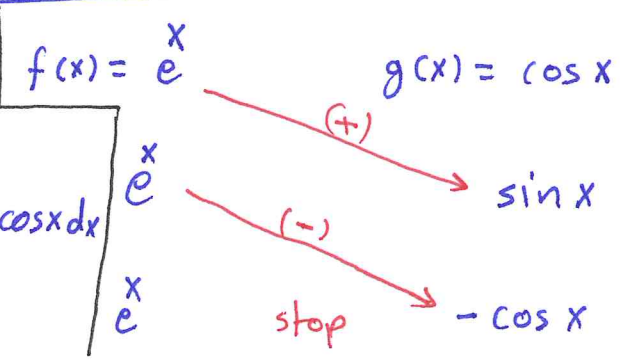
$= x^2 e^x - 2 \int x e^x dx$

$u = x$	$dv = e^x dx$
$du = dx$	$v = e^x$

$= x^2 e^x - 2 [x e^x - \int e^x dx]$

$= x^2 e^x - 2x e^x + 2 \int e^x dx = x^2 e^x - 2x e^x + 2e^x + c$

Exp* $\int e^x \cos x dx$



$\int e^x \cos x dx = e^x \sin x + e^x \cos x - \int e^x \cos x dx$

$2 \int e^x \cos x dx = e^x \sin x + e^x \cos x$

$\int e^x \cos x dx = \frac{1}{2} e^x (\sin x + \cos x) + C$

$\int e^x \cos x dx$	$u = e^x$	$dv = \cos x dx$
	$du = e^x dx$	$v = \sin x$

$\int e^x \cos x dx = e^x \sin x - \int e^x \sin x$

$u = e^x$	$dv = \sin x dx$
$du = e^x dx$	$v = -\cos x$

$= e^x \sin x - [-e^x \cos x + \int e^x \cos x dx]$

$$\int e^x \cos x \, dx = e^x \sin x + e^x \cos x - \int e^x \cos x \, dx$$

$$2 \int e^x \cos x \, dx = e^x \sin x + e^x \cos x$$

$$\int e^x \cos x \, dx = \frac{1}{2} e^x [\sin x + \cos x] + C$$

Exp $\int_1^2 x \ln x \, dx$

$$u = \ln x \quad dv = x \, dx$$
$$du = \frac{dx}{x} \quad v = \frac{x^2}{2}$$

$$\int_1^2 x \ln x \, dx = \left. \frac{x^2}{2} \ln x \right|_1^2 - \int_1^2 \frac{1}{x} \frac{x^2}{2} \, dx$$
$$= \left. \frac{x^2}{2} \ln x \right|_1^2 - \frac{1}{2} \int_1^2 x \, dx$$

$$= 2 \ln 2 - 0 - \frac{1}{2} \left. \frac{x^2}{2} \right|_1^2 = 2 \ln 2 - 1 + \frac{1}{4}$$
$$= \ln 4 - \frac{3}{4}$$

$\int x \ln x \, dx = \left. \frac{x^2}{2} \ln x - \frac{x^2}{2} \int x \ln x \, dx + \frac{x^2}{2} \right _0^1$	$f(x) = x$	$g(x) = \ln x$
	1	(+)
0	(-)	$\rightarrow \int x \ln x \, dx - \frac{x^2}{2}$

$$2 \int x \ln x \, dx = \left. \frac{x^2}{2} \ln x - \frac{x^2}{2} \right|_0^1$$

$$\int_1^2 x \ln x \, dx = \left[\frac{x^2}{2} \ln x - \frac{x^2}{4} \right]_1^2$$

$$= [2 \ln 2 - 1] - [0 - \frac{1}{4}]$$

$$= 2 \ln 2 - \frac{3}{4} \quad \checkmark$$