① $\boxed{1}$

1.1.4 Trigonometry

Angles are measured in either degrees or radians

Complete revolution on a unit circle corresponds to 360° or 2π

$$\frac{\theta \text{ measured in degrees}}{360^\circ} = \frac{\theta \text{ measured in radians}}{2\pi}$$

Example:

$\theta = 23^\circ$. Find θ in radians

$$\theta \text{ measured in radians} = \frac{2\pi}{360} \cdot \theta \text{ measured in degrees}$$

$$= \frac{2\pi}{360^\circ} \times 23^\circ$$

Example:
Compute θ in degrees if $\theta = \frac{\pi}{6}$

$$\theta \text{ measured in degrees} = \left(\cancel{\frac{2\pi}{360}}\right)$$

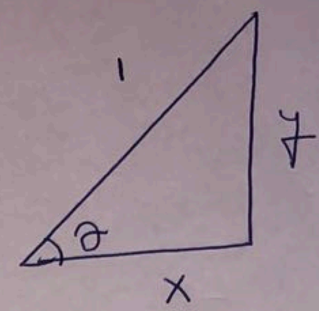
$$= \left(\frac{360}{2\pi}\right) \theta \text{ measured in degrees}$$

$$= \frac{360}{2\pi} \cdot \frac{\pi}{6} = 30^\circ$$

$\sin\theta = \frac{y}{1} = y$ θ∠

$\cos\theta = \frac{x}{1} = x$ θ∠

$\tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$ θ∠



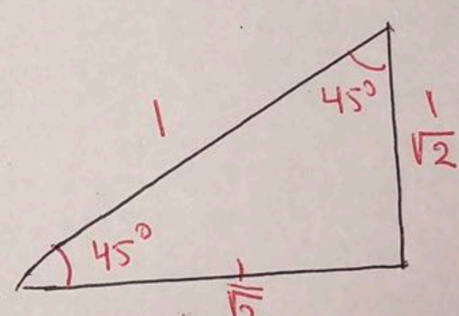
$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{y}$

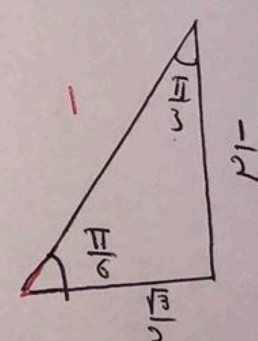
$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{x}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{x}{y}$

Trigonemetric Identities

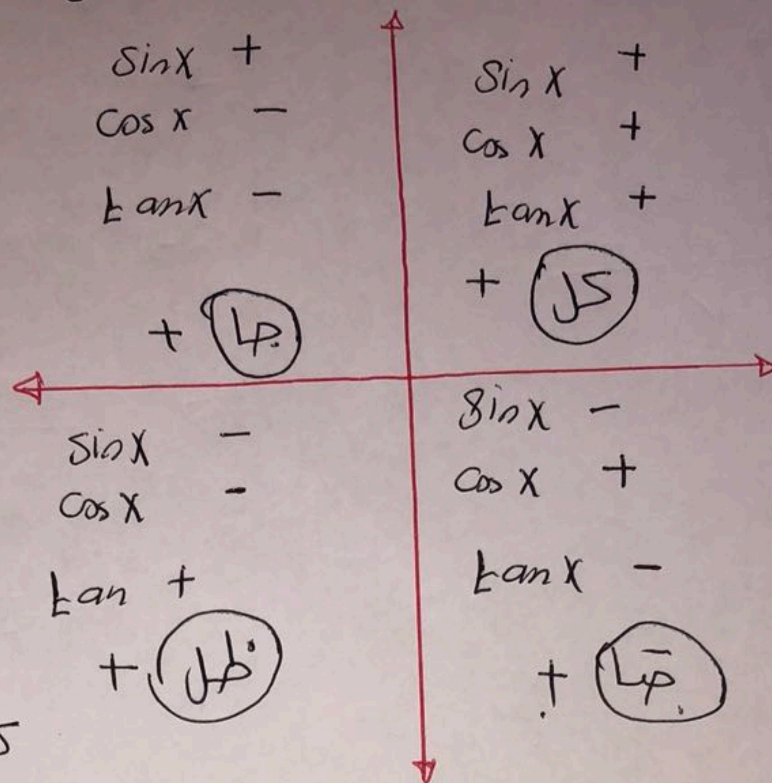
$\sin^2\theta + \cos^2\theta = 1$





θ	0	$\frac{\pi}{6}$ (30°)	$\frac{\pi}{4}$ (45°)	$\frac{\pi}{3}$ (60°)	$\frac{\pi}{2}$ (90°)	π (180°)	
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undefind	0	

Reference angle.



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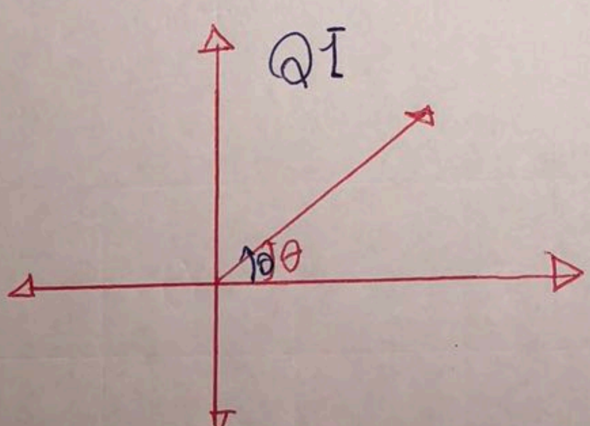
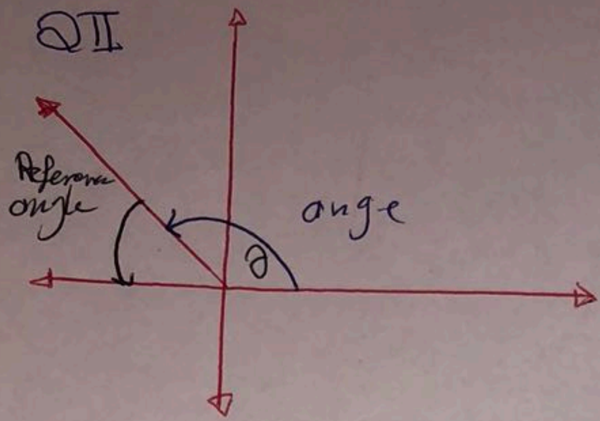
Reference angle for angle θ is the positive acute angle made by the terminal side of angle θ and the x- axis



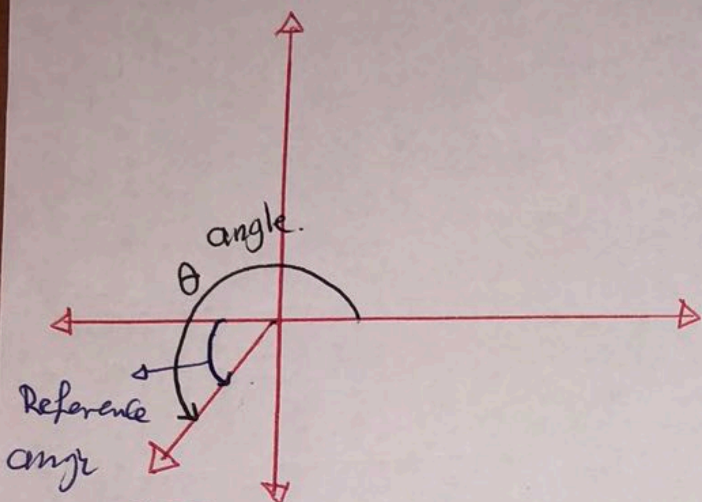
Reference angle = angle.

QII



angle = θ

Reference angle = $180^\circ - \theta$

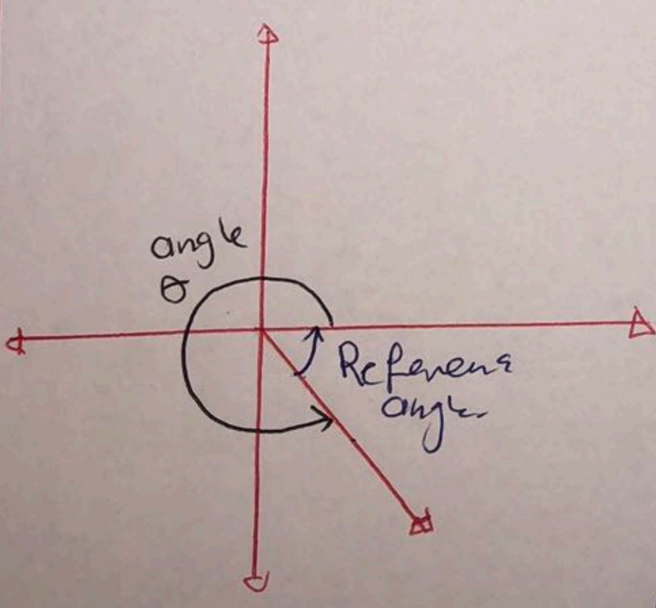


QIII

angle = θ

Reference angle = ~~180~~

= $\theta - 180$



angle = θ

Reference angle = $360^\circ - \theta$

Angles and Reference angles have the same $\boxed{\text{values}}$ for $\sin x$, $\cos x$, $\tan x$, $\sec x$, $\csc x$, $\cot x$.

"مع مراعاة اقترانات الاشارة بناءً على الربع"

Example: Find the reference angle of 150°

Reference angle $= 180^\circ - 150^\circ$

$= 30^\circ$

$\sin 30^\circ = \frac{1}{2}$

$\sin 150^\circ = \frac{1}{2}$

$\cos 30^\circ = \frac{\sqrt{3}}{2}$

$\cos 150^\circ = \frac{-\sqrt{3}}{2}$

$\tan 30 = \frac{1}{\sqrt{3}}$

$\tan 150 = \frac{-1}{\sqrt{3}}$

1.1.5 Exponentials and Logarithms

An exponential is an expression of the form

$$a^r$$

r = exponent

a = Base.

$$a^r a^s = a^{r+s}$$

$$\frac{a^r}{a^s} = a^{r-s}$$

$$a^{-r} = \frac{1}{a^r}$$

$$(ab)^r = a^r b^r$$

$$\left(\frac{a}{b}\right)^r = \frac{a^r}{b^r}$$

$$(a^r)^s = a^{rs}$$

Example:

Evaluate the following exponential expressions:

(a) $3^2 3^{\frac{5}{2}} = 3^{2+\frac{5}{2}} = 3^{\frac{4}{2}+\frac{5}{2}} = 3^{\frac{9}{2}}$

(b) $\frac{2^{-4} \cdot 2^3}{2^2} = \frac{2^{-1}}{2^2} = 2^{-1-2} = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$.

(c) $\frac{a^k a^{3k}}{a^{5k}} = \frac{a^{4k}}{a^{5k}} = a^{-k} = \frac{1}{a^k}$.

Logarithms:

$$2^3 = 8$$
$$\Updownarrow$$
$$\log_2 8 = 3$$

$$\boxed{x = \log_a y \text{ is equivalent to } y = a^x}$$

Example: which real number x satisfies

(a) $\log_3 x = -2$?

$x = 3^{-2} \Longrightarrow x = \frac{1}{3^2} = \frac{1}{9}$

(b) $\log_{\frac{1}{2}} 8 = x$?

$\left(\frac{1}{2}\right)^x = 8$

$\left(2^{-1}\right)^x = 8$

$2^{-x} = 8 = 2^3$

so $x = -3$

Important Properties of logarithms

1. $\log_a(xy) = \log_a x + \log_b y$

2. $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

3. $\log_a x^r = r \log_a x$

Natural logarithm $= \log_e x = \ln x$ لوغاريتم طبيعي

Common logarithm $= \log_{10} x = \log x$ لوغاريتم عادي

Example: Assume that x and y are positive and simplify the following expressions:

(a) $\log_3 9x^2 = \log_3 9 + \log_3 x^2$

$= \log_3 3^2 + \log_3 x^2$

$= 2\log_3 3 + 2\log_3 x$

$= 2 * 1 + 2\log_3 x$

$= 2 + 2\log_3 x$ ▤

(b) $\log_5 \frac{x^2+3}{5x} = \log_5 (x^2+3) - \log_5 (5x)$

$= \log_5 (x^2+3) - [\log_5 5 + \log_5 x]$

$= \log_5 (x^2+3) - \log_5 5 - \log_5 x$

$= \log_5 (x^2+3) - 1 - \log_5 x$ ▤

10 (4)

$$\boxed{\log_2 x^3 = 3\log_2 x}$$

$$\boxed{\log_2 (x+1)^3 = 3\log_2 (x+1)}$$

$$\boxed{\log_2 x^3+1 \neq 3\log_2 x+1}$$

$\boxed{\text{Also}}$

$$\log_a 1 = 0$$

$$\boxed{\log_a a^x = x} \quad \text{and} \quad \boxed{a^{\log_a x} = x}$$

Note that $\log_5 x^2+3$ can't be simplified any further.

(c) $-\ln \frac{1}{2} = -[\ln 1 - \ln 2] = -[0 - \ln 2] = \ln 2$ ∎

(d) $\ln\left(\frac{3x^2}{\sqrt{y}}\right) = \ln 3x^2 - \ln \sqrt{y}$ $\quad\left(\sqrt{y} = y^{\frac{1}{2}}\right)$

$= \ln 3 + \ln x^2 - \frac{1}{2}\ln y$ ∎

Example:

Solve for x

(a) $e^{2x} = 3$

take ln for both sides

$\ln e^{2x} = \ln 3$

$2x = \ln 3$

$x = \frac{\ln 3}{2} = \frac{1}{2}\ln 3 = \ln 3^{\frac{1}{2}} = \ln\sqrt{3}$

(b) $\ln(x+1) = 5$ "logarithmic form"

$e^{5} = x+1$ "Exponential form"

$\boxed{x = e^{5} - 1}$

(c) $5^{2x-1} = 2^{x}$

$\ln 5^{2x-1} = \ln 2^{x}$

$(2x-1)(\ln 5) = x(\ln 2)$

$2\ln 5\, x - \ln 5 = \ln 2 \cdot x$

$2\ln 5\, x - \ln 2\, x = \ln 5 \Rightarrow x[2\ln 5 - \ln 2] = \ln 5$

$x = \ln 5 / 2\ln 5 - \ln 2$

1.1.6 Complex Number

$$\boxed{i^2 = -1}$$

i = imaginary unit

$\sqrt{-17} = \sqrt{-1 * 17} = \sqrt{-1}\ \sqrt{17} = i\sqrt{17}$

$\sqrt{-5} = \sqrt{5}\, i$

$\sqrt{-2} = i\sqrt{2}$

A complex Number is a number of the form

$$z = a + bi$$

where a and b are real numbers,

real number a is the real part of $a+bi$

real number b is the imaginary part

Examples:

$2-5i$: 2 = real part

-5 = imaginary part

$\mathbb{R} \subseteq \mathbb{C}$ $\mathbb{R}$ = real numbers

$\mathbb{C}$ = complex numbers

complex number of the form bi are called pure imaginary numbers

Operation:

① $a+bi = c+di$

$\Longleftrightarrow \quad a=c \text{ and } b=d$

② $(a+bi)+(c+di) = a+c+bi+di$

$= (a+c) + i(b+d)$

③ $(a+bi)-(c+di) = a-c+bi-di$

$= (a-c) + i(b-d)$

④ $(a+bi)(c+di) = ac + adi + bic + \underbrace{bd\,i^2}_{-bd}$

$= [ac-bd] + i[ad+bc]$

$i = \sqrt{-1}$

$i^2 = -1$

Example: Find

ⓐ $(2+3i)-(5-6i)$

$= 2+3i-5+6i = -3+9i$

ⓑ $(5-3i)(1+2i) = 5+10i-3i-\underbrace{6(i)^2}_{6}$

$= 11+7i$

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If $z = a+bi$ is a complex number its Conjugate denoted by $\bar{z} = a-bi$

Conjugate = مرافق

$\bar{\bar{z}} = z$

$\overline{z+w} = \bar{z} + \bar{w}$

$\overline{zw} = \bar{z}\bar{w}$

If $z = a+ib$, $\bar{z} = a-ib$

$$z\bar{z} = (a+ib)(a-ib) = a^2 - i\,ab + i\,ab + b^2$$
$$= a^2 + b^2$$

If $z = a+bi$

$$z\bar{z} = a^2 + b^2$$

Example:

Let $z = 3+2i$

(a) $\bar{z} = 3-2i$

(b) $z\bar{z} = (3+2i)(3-2i) = 9 - 6i + 6i + 4$
$$= 13$$

Or use the Rule $z\bar{z} = 3^2 + 2^2 = 9+4 = 13$

Quadratic Equation

$ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2a}$$

المميز Discriminant

$b^2 - 4ac$

1. $b^2 - 4ac$ (+) 2 different real solutions
2. $b^2 - 4ac = 0 \to 1$ solution
3. $b^2 - 4ac$ (−) 2 complex solution

Example:

solve $x^2 + 4x + 5 = 0$

$$x = \frac{-4 \pm \sqrt{16 - 4 \cdot 1 \cdot 5}}{2}$$

$$= \frac{-4 \pm \sqrt{-4}}{2}$$

$\sqrt{-4} = \sqrt{4} \cdot \sqrt{-1}$

$= 2i$

$$= \frac{-4 \pm 2i}{2}$$

$= -2 \pm i$

solution $= \{-2 + i, -2 - i\}$

Example: without solving $2x^2 - 3x + 7 = 0$ what can you say about the solution?

$b^2 - 4ac = (-3)^2 - 4(2)(7)$

$= 9 - 56 = -47 < 0$

So the equation has two complex solution which are conjugates of each other