4.5 Derivatives of Trigonometric Functions

$f_1(x) = \sin x$	$f_1'(x) = \cos x$
$f_2(x) = \cos x$	$f_2'(x) = -\sin x$
$f_3(x) = \tan x$	$f_3'(x) = \sec^2 x$
$f_4(x) = \cot x$	$f_4'(x) = -\csc^2 x$
$f_5(x) = \sec x$	$f_5'(x) = \sec x \tan x$
$f_6(x) = \csc x$	$f_6'(x) = -\csc x \cot x$.

Example: $y = -4\sin x + \cos\frac{\pi}{6}$

$y' = -4\cos x + 0$

$\boxed{y' = -4\cos x}$

Example: $y = \cos(x^2+1)$

$y' = -\cos(x^2+1) \cdot 2x$

$\boxed{y' = -2x\cos(x^2+1)}$

Example: $y = -x^2 \sin 3x - \cos 5x$

$y' = -x^2(\cos 3x)\cdot 3 + \sin 3x(-2x) + \sin 5x \cdot 5$

$\boxed{y' = -3x^2\cos 3x - 2x\sin 3x + 5\sin 5x}$

Example: $y = \tan x^2$

$y' = \sec x^2 \cdot 2x$

$$\boxed{y' = 2x \sec x^2}$$

Examples $y = \tan^2 x$

$y' = 2 \tan x \sec^2 x$

$$\boxed{y' = 2 \tan x \sec^2 x}$$

Example $y = \sec \sqrt{x^2+1}$

$y = \sec (x^2+1)^{\frac{1}{2}}$

$y' = \frac{1}{2} \sec (x^2+1)^{\frac{1}{2}} \tan (x^2+1)^{\frac{1}{2}} \cdot \frac{1}{2} (x^2+1)^{-\frac{1}{2}} \cdot 2x$

$$= \frac{x \sec \left(\sqrt{x^2+1}\right) \tan \left(\sqrt{x^2+1}\right)}{\sqrt{x^2+1}}$$