4.6 Derivatives of Exponential Functions.

$$f(x) = e^x \qquad f'(x) = e^x$$

$$h(x) = e^{f(x)} \qquad h'(x) = e^{f(x)} f'(x)$$

$$g(x) = a^x \qquad g'(x) = a^x \ln a$$

Example: $f(x) = e^{-x^2/2}$

$$f'(x) = e^{\frac{-x^2}{2}} \cdot \frac{-2x}{2}$$

$$f'(x) = -x\, e^{\frac{-x^2}{2}}$$

Example: $f(x) = 3^{\sqrt{x}}$

$$f(x) = 3^{x^{\frac{1}{2}}}$$

$$f'(x) = (3^{x^{\frac{1}{2}}})\left(\tfrac{1}{2} x^{-\frac{1}{2}}\right)(\ln 3)$$

$$= \frac{\ln 3 \; 3^{\sqrt{x}}}{2\sqrt{x}}$$

Example: $y = e^{\sin(\sqrt{x})}$

$$y' = e^{\sin(\sqrt{x})} \cdot \cos\sqrt{x} \cdot \frac{1}{2\sqrt{x}}$$

$$y' = \frac{\cos\sqrt{x}\; e^{\sin\sqrt{x}}}{2\sqrt{x}}$$

Rule: $\lim_{h\to 0} \frac{a^h - 1}{h} = \ln a$.

Recall that $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$f(x+h) = a^h$

so $f(x) = a^x$

$f(x+h) = a^h$ so $x = 0$

$f(x) = a^x$

$f'(x) = a^x \ln a$.

$f'(0) = a^0 \ln a = \ln a$.

Example: $\lim_{h\to 0} \frac{3^{2h} - 1}{h}$

Let $m = 2h$

$h \to 0$, $m \to 0$

$h = \frac{m}{2}$

$\lim_{m\to 0} \frac{3^m - 1}{\frac{m}{2}} = 2 \lim_{m\to 0} \frac{3^m - 1}{m} = 2 \ln 3$