Note Title

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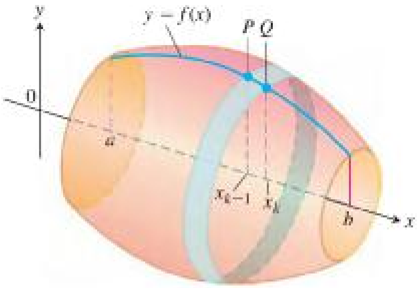
6.4 Area of Surfaces of Revolution



DEFINITION If the function $f(x) \geq 0$ is continuously differentiable on $[a, b]$, the **area of the surface** generated by revolving the graph of $y = f(x)$ about the x-axis is

$$S = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \int_a^b 2\pi f(x)\sqrt{1 + (f'(x))^2}\, dx. \qquad (3)$$

Surface Area for Revolution About the y-Axis
If $x = g(y) \geq 0$ is continuously differentiable on $[c, d]$, the area of the surface generated by revolving the graph of $x = g(y)$ about the y-axis is

$$S = \int_c^d 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy = \int_c^d 2\pi g(y)\sqrt{1 + (g'(y))^2}\, dy. \qquad (4)$$

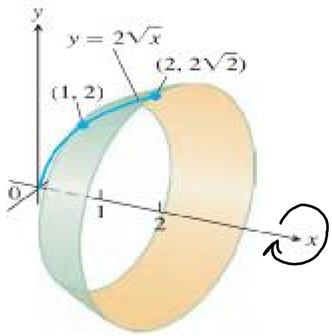
EXAMPLE 1 Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}$, $1 \leq x \leq 2$, about the x-axis

Sol: $f(x) = 2\sqrt{x} \Rightarrow f' = \frac{1}{\sqrt{x}}$ which is cont. on $[1, 2]$. Moreover $f(x) \geq 0 \ \forall \ x \in [1,2]$. So,

$$S = 2\pi \int_1^2 f(x)\sqrt{1+(f')^2}\, dx = 2\pi \int_1^2 2\sqrt{x}\sqrt{1+\frac{1}{x}}\, dx$$

$$= 4\pi \int_1^2 \sqrt{x+1}\, dx = 4\pi \frac{2}{3}(x+1)^{3/2}\Big]_1^2 = \frac{8\pi}{3}(3\sqrt{3} - 2\sqrt{2}).$$

المنحنى $y=2\sqrt{x}$ على الفترة $[1,2]$, الجسم الدوراني الذي أوجدنا مساحة سطحه موضح من الرسمة التالية .



EXAMPLE 2 The line segment $x = 1 - y$, $0 \le y \le 1$, is revolved about the y-axis to generate the cone in Figure 6.35. Find its lateral surface area (which excludes the base area).

in book

sol:

Clearly $x = f(y) = 1 - y \ge 0$ on $[0,1]$. Moreover $\frac{dx}{dy} = -1$ is cont. on $[0,1]$. So the surface area of the cone is

$$S = 2\pi \int_0^1 f(y)\sqrt{1 + f'(y)^2}\, dy = 2\pi \int_0^1 (1-y)\sqrt{1 + (-1)^2}\, dy$$

$$= 2\sqrt{2}\,\pi \left(y - \frac{y^2}{2} \right]_0^1 = \boxed{\sqrt{2}\,\pi}$$

الرسمة التالية توضح الجسم الدوراني cone الذي نتج عن دوران القطعة المستقيمة $x = 1 - y$, $y \in [0,1]$ حول محور y .