Second [redacted]

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MATHEMATICS DEPARTMENT
Math1431 -SECOND HOUR EXAM-
FIRST SEMESTER 2018/2019

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Question One (60 points) Circle the best answer in each of the following parts:

1. The limit $\lim_{h\to 0} \frac{3(a+h)^2-3a^2}{h}$ represents the derivative of $f(x)$ at the point $(a, f(a))$, then

$\lim_{h\to 0} \frac{3(a+h)^2 - 3a^2}{h}$ $\lim \frac{f(a+h) - f(a)}{h}$ 15

   (a) $f(x) = x^6$
   (b) $f(x) = x^2$
   (c) $f(x) = 3x^2$ $\boxed{3x^2}$
   (d) $f(x) = 3(x+2)^2$

(1,3)

$f(x) = y = x^2 + 2x$

2. The tangent line of the curve $y = x^2 + 2x$ at the point $(1,3)$ is

$f'(x) = 2x + 2$
$m = f'(1) = 2 + 2 = 4$
$y - 3 = 4(x-1)$
$y - 3 = 4x - 4$
$\boxed{y = 4x - 1}$

   (a) $y = \frac{-1}{4}x + \frac{13}{4}$
   (b) $y = 4x - 1$
   (c) $y = x - 3$
   (d) $y = 2x + 1$

3. The horizontal tangent lines to the curve $y = x^3 + \frac{15}{2}x^2 + 12x - 2$ occur at

$y' = 3x^2 + 15x + 12 = 0$
$3(x^2 + 5x + 4) = 0$
$3(x^2 + 5x + 4) = 0$
$x^2 + 5x + 4$

$y = x(2x^2 + 15x + 24)$

   (a) $x = 2, x = 5$
   (b) $x = -1, x = -4$
   (c) $x = 0$
   (d) No horizontal tangent lines.

$f(g(x))$ $1 - 5 + 4$
$f'(g(x))\,g'(x)$ $16 - 20 + 4$

4. Suppose that $f(1) = 4$, $f'(2) = 5$, $f'(1) = -1$, $g(1) = 2$, and $g'(1) = 3$, then $(f \circ g)'(1) =$

$5 \cdot 3$
$\boxed{15}$

$f'(g(x))\,g'(x)$
$f'(g(1))\,g'(1) = f'(2)(3)$

   (a) -3
   (b) 10
   (c) 15
   (d) None of the above

$1 - 4$
-3
$4 - 1$
3
$4 - 1 = 3$
$1 - 4 = -3$

5. The value(s) of c in the conclusion of the Mean Value Theorem of $f(x) = x - \frac{4}{x}$ in $[1, 4]$

$1 - 4$

$f'(c) = \frac{f(4) - f(1)}{4 - 1}$

   (a) $-2, 2$
   (b) -2
   (c) 2
   (d) None of the above.

$f'(c) = 0$ $f'(x) = 1 + \frac{4}{x^2}$

6. The derivative of $x^2y = 3xy + 1$ at $x = 1$ is

$y = 3y + 1$
$0 = 3y - y + 1$
$-\frac{1}{2} = 2y$

$-2y' = -\frac{1}{2}$
$y' = \frac{1}{4}$

$-4x^{-1}$
$4x^{-2}$
$\boxed{1 + \frac{4}{x^2}}$

   (a) $-\frac{1}{4}$
   (b) $\frac{1}{4}$
   (c) $-\frac{1}{2}$
   (d) $\frac{1}{2}$

$x^2y = 3xy + 1$
$x^2y' + y\,2x = 3xy' + 3y$
$x^2y' - 3xy' = 3y - 2xy$
$y' - 3y' = 3y - 2y$
$-2y' = \frac{-3}{2} + 1$

$0 = 1 + \frac{4}{c^2}$
$\frac{4}{c^2} = -1$
$-c^2 = 4$
$c^2 = -4$
$c = \sqrt{-4}$
$c = \pm 2$

$3 - \frac{1}{2}$
$-\frac{3}{2}$
$-2 \cdot \frac{1}{2}$
$\frac{2}{2} - \frac{3}{2}$
$\frac{4}{\sqrt{2}}$

$\cancel{2 = x^3 + \frac{15}{2}x^2 + 12x}$
$\cancel{x(x^2 + \frac{15}{2}x + 12)}$
$-1 + \frac{15}{2} - 12$
$13 + \frac{15}{2}$

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$\ln y = \ln e^{\tan x^3}$ $\boxed{y' = 3x^2 e^{\tan x^3} \sec^2 x^3}$

$\ln y = \tan x^3$

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7. The derivative of the function $y = e^{\tan x^3}$ is

   (a) $3x^2 e^{\tan x^3} \sec^2 x$

   (b) $3x^2 \tan x \sec^2 x^2$

   (c) $3x^2 e^{\tan x^3} \sec^2 x^3$

   (d) $3x e^{\tan x} \sec x$

$\frac{y'}{y} = \sec^2 x^3 \; 3x^2 \; y$

$y' = 3x^2 e^{\tan x^3} \sec^2 x^3$

8. Let $f(x) = e^{-x^2} + 2x$, find $\frac{df^{-1}(x)}{dx}$ at $x = 1$ is

   $e^{-x^2}(-2x) + 2$

   (a) 0

   (b) $\frac{1}{2}$

   (c) 1

   (d) 2

$\left.\frac{df^{-1}}{dx}\right|_{x=1}$ $\boxed{b = f(a)}$ $1 = e^{-a^2} + 2a$ $1 = e^{-0^2} + 2(0)$ $1 = \boxed{e^{-0}} + 0$ $\boxed{a = 0}$

$1 = e^{-a^2} + 2a$

$\frac{1}{f'(0)} =$

$\ln y = \ln (\cos x)^x$

$\ln y = x \ln \cos x$

9. Let $y = (\cos x)^x$, then $\frac{dy}{dx} =$

   $\frac{y'}{y} = \frac{x(-\sin x)}{\cos x} + \ln \cos x$

   (a) $-x(\cos x)^{x-1} \sin x$

   (b) $-\sin x (\cos x)^x$

   (c) $-x \tan x + \ln \cos x$

   (d) $(\cos x)^x \ln \cos x - x(\cos x)^x \tan x$

$y' = (-x \tan x + \ln \cos x) \cos x^x$

10. Suppose that the bacterial colony grows in such a way that at time t the population size is $N(t) = N_0 e^{rt}$, $t \geq 0$ where N_0 is a positive constant and r is a real number, then $\frac{dN}{dt} =$

    $N(t) = \boxed{N_0 e^{rt}}\, r$

    (a) N

    (b) $2N$

    (c) $N_0 N$

    (d) rN

$N'(t) = Nr$

$y = \ln(\sec(\ln x))$

$y' = \frac{\sec \tan \frac{1}{x}}{\sec(\ln x)}$

$\sec = \sec \tan$

11. Let $y = \ln(\sec(\ln x))$, then $\frac{dy}{dx} =$

    $\frac{1}{x} \tan(\ln x)$

    (a) $\tan(\ln x)$

    (b) $\frac{\tan(\ln x)}{x}$

    (c) $\sec(\ln x) \tan(\ln x)$

    (d) $\frac{\sec^2(\ln x)}{x}$

$\frac{\tan(\ln x)}{x}$

12. The function $y = 2x^3 - 6x^2 + 4$ on $[1, 3]$ has an absolute maximum at $x =$

    $f'(x) = 6x^2 - 6x = 0$

    $6x(x-1) = 0$

    $x = 0$

    $x = 1$

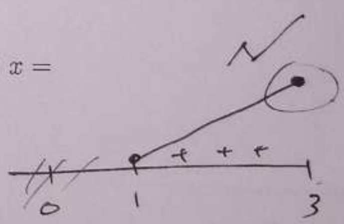
    $x = 3$

    (a) 0

    (b) 1

    (c) 2

    (d) 3



$6(4) - 6(3)$

$24 - 12 =$

13. The height y in feet of a tree as a function of the tree's age x in years is given by $y = 121 e^{-\frac{17}{x}}$, $x > 0$, the limit of the height as $x \to \infty$ is

    (a) 17

    (b) 121

    (c) 0

    (d) does not exist.

$f(x) = 121 e^{\frac{-17}{x}}$

$\lim_{x \to \infty} \boxed{121} e^{\frac{-17}{x}}$

$y = 2$

H. Asy.

14. If the rational function $f(x) = \frac{g(x)}{x-4}$ has a horizontal asymptote $y = 2$, then a possible value for $g(x)$ is

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(a) $g(x) = 2x^2 + 4x$

(b) $g(x) = 2x + 5$

(c) $g(x) = 2$

(d) $g(x) = 0$

$\lim_{x\to\infty} \frac{g(x)}{x-4} = 2$

$2 = \frac{2x+5}{x-4}$

15. Suppose that $f(x)$ twice differentiable function on an interval containing 4 such that $f''(4) = 1$, then $f(x)$ has a relative minimum at $x = 4$

$f''(4) = 1$ min

(a) True.

(b) False

16. Suppose that $f(x) = x^4$, the inflection point(s) of $f(x)$ is(are)

(a) $x = -1$

(b) $x = 0$

(c) $x = 1$

(d) No inflection points.

$f(x) = x^4$

$f'(x) = 4x^3$

$f''(x) = 12x^2 = 0$

$x = 0$

17. If $f(x) = \frac{x}{x^2 - x}$, then the vertical asymptote(s) is (are)

(a) $x = 0$

(b) $x = 1$

(c) $x = 0, x = 1$

(d) $y = 0, y = 1$

$\lim_{x\to 1^+} \frac{1}{\text{small}^+} = \infty$

18. If $y = \log x$, then $\frac{dy}{dx} =$

(a) $\frac{1}{x}$

(b) $\frac{1}{x \ln 10}$

(c) $\frac{\ln 10}{x}$

(d) $\frac{1}{\ln 10}$

$y = \log_{10} x$

$y = \frac{\ln x}{\ln 10}$

$y' = \frac{\ln 10}{x}$

19. $\lim_{x\to 0} \frac{e^x - 1}{\sin x} =$

(a) 0

(b) -1

(c) 1

(d) does not exist.

$\lim_{x\to 0} \frac{e^x - 1}{\sin x}$ $\frac{0}{0}$

$\lim_{x\to 0} \frac{e^x}{\cos x}$ $\frac{1}{1} = 1$

20. $\lim_{x\to\infty} (1 + \frac{3}{x})^x =$ $\lim_{x\to\infty} e^{x \ln(1+\frac{3}{x})}$

(a) 3

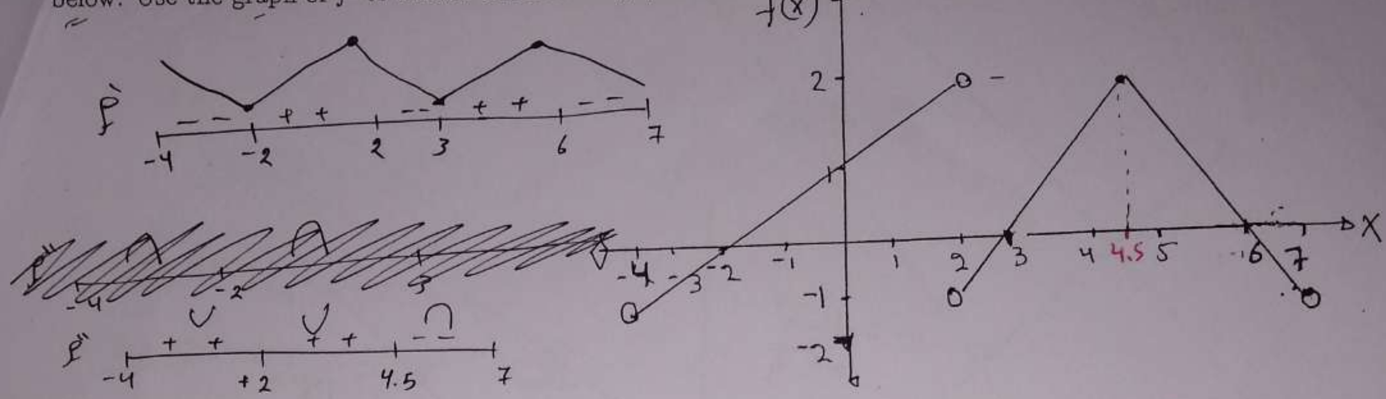
(b) e^2

(c) e^3

(d) does not exist.

$\lim_{x\to\infty} e^{\frac{3}{1+\frac{3}{x}}}$ $e^{\frac{3}{1+0}}$ e^3

Question Two (16 points) Let $f(x)$ be continuous on $[-4, 7]$. The graph of its derivative $f'(x)$ is given below. Use the graph of f' to answer the following questions.



1. The critical value(s) of $f(x)$ is(are)

   critical point (values) ① $\in \mathbb{R}$ [redacted] ② diff. ③ cont. [redacted] there is no critical values

2. $f(x)$ is decreasing on

   $]-4,-2[\cup]2,3[\cup]6,7[$ 3

3. $f(x)$ is increasing on

   $]-2,2[\cup]3,6[$ 2

4. The inflection point(s) is(are)

   $x = 4.5$ 1

5. $f(x)$ is concave up on

   $]-4,2[\cup]2,4.5[$ 1/2

6. $f(x)$ is concave down on

   $]4.5,7[$ 1

7. $f(x)$ has local maximum at

   $f(4.5)$ local max $x = 4.5$

8. $f(x)$ has local minimum at

   no local minimum.

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Question Three (25 points) Let $f(x) = \frac{1-x^2}{x^2-9}$, $f'(x) = \frac{16x}{(x^2-9)^2}$, $f''(x) = \frac{-48(x^2+3)}{(x^2-9)^3}$. Find

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1. Domain of $f(x)$. $\mathbb{R} \setminus \{-3, 3\}$

2. $\lim_{x\to\infty} f(x) =$ $\lim_{x\to\infty} \frac{1-x^2}{x^2-9} = -1$

3. $\lim_{x\to-\infty} f(x) =$ $\lim_{x\to-\infty} \frac{1-x^2}{x^2-9} = -1$

4. $\lim_{x\to3^+} f(x) =$ $\lim_{x\to3^+} \frac{1-x^2}{x^2-9} = \frac{-8}{small^+} = \infty$

5. $\lim_{x\to3^-} f(x) =$ $\lim_{x\to3^-} \frac{1-x^2}{x^2-9} = \frac{-8}{small^-} = -\infty$

6. $\lim_{x\to-3^+} f(x) =$ $\lim_{x\to-3^+} \frac{1-x^2}{x^2-9} = \frac{-8}{small^-} = -\infty$

7. $\lim_{x\to-3^-} f(x) =$ $\lim_{x\to-3^-} \frac{1-x^2}{x^2-9} = \frac{-8}{small^+} = \infty$

8. Horizontal asymptotes (if any) are:
$y = -1$

9. Oblique asymptotes (if any) are:
|

10. Vertical asymptotes (if any) are:
$x = -3$, $x = 3$

11. Interval of increasing.
$[0,3[\cup]3,\infty[$

12. Interval of decreasing.
$]-\infty,-3[\cup]-3,0]$

13. Local maximum point(s).
no local max

14. Local minimum point(s).
$x = 0$ $\left(0, \frac{-1}{9}\right)$

15. When the graph is concave up?
$]-\infty,-3[\cup]-3,3[\cup]3,\infty[$

16. When the graph is concave down?
no concave down

17. Inflection point(s)
no inflection point

18. Graph the function using the above information.

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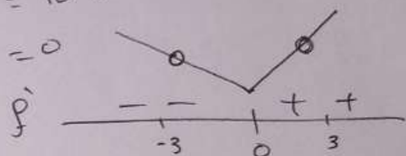
$f'(x) = \frac{16x}{(x^2-9)^2}$

$f'(x) = 0 = 16x$

$x = 0$



f'': + (−3) − + (3) + +

$f''(x) = \frac{-48(x^2+3)}{(x^2-9)^3}$

$f''(x) = 0 = \frac{-48(x^2+3)}{(x^2-9)3}$

$\frac{0}{-48} = \frac{-48(x^2+3)}{-48}$

$0 = x^2 + 3$

$x^2 = -3$

$x = \sqrt{-3}$

Key point

$(0, \frac{-1}{9})$

$(1, 0)$

$(-1, 0)$