First

MATHEMATICS DEPARTMENT
Math1431 -FIRST HOUR EXAM-
FIRST SEMESTER 2018/2019

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-3

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Question One (66 points) Circle the best answer in each of the following parts:

9

1. The solution of the inequality $|1-2x|>7$ is

   (a) $x<3$ or $x>4$
   (b) $x<-3$ or $x>4$
   (c) $x>-3$ or $x<-4$
   (d) $x>3$ or $x<4$

$1-2x>7$, $-2x>6$, $x<\frac{6}{-2}$, $x<-3$ or $1-2x<-7$, $-2x<-8$, $x>\frac{-8}{-2}$, $x>4$

-3 4

2. The equation of the line whose x-intercept 4 and y-intercept 2 is

   (a) $2y+x=4$
   (b) $y+2x=4$
   (c) $y=2x+2$
   (d) $y=-\frac{1}{2}x+4$

$y=0$, $x=4$ $y=2$, $x=0$

3. The solution of the equation

$$\ln(x+\sqrt{3})+\ln(x-\sqrt{3})=0$$

is

   (a) $\{-2,2\}$
   (b) $\{-2\}$
   (c) $\{2\}$
   (d) No Solution.

$\ln(x+\sqrt{3})(x-\sqrt{3})=0$

$\ln(x^2-3)=0$

$\ln(x^2-3)=e$

$x^2-3=1$

$x^2=4 \Rightarrow x=\pm 2$

4. The center and radius of the circle given by the equation

$$x^2+y^2-4x+2y-11=0$$

is respectively given by

   (a) $(2,-1)$, 16
   (b) $(-1,-2)$, 16
   (c) $(-1,-2)$, 4
   (d) $(2,-1)$, 4

$x^2-4x+y^2+2y=11$

$(x-2)^2-4+(y+1)^2=r^2$

$(x-2)^2+(y+1)^2=4+5+11$

16

$(2,-1)$

$r=4$

x^2+4-4x

y^2+1+2y

x^2+4-2x

y^2+1+2y

R

12

5. The period of the function $f(x) = \sin(\frac{\pi x}{4}) + 1$ is

$P = 2\pi$

(a) 2π
(b) 8
(c) π
(d) 4

$\frac{2\pi}{\frac{\pi}{4}}$ $\quad \frac{2\pi}{1} \times \frac{4}{\pi} = 8$

6. The domain of the function $f(x) = \frac{1}{4-\sqrt{x}}$ is

(a) $[0, \infty) - \{16\}$
(b) $\Re - \{16\}$
(c) $\Re - \{4\}$
(d) $(0, \infty) - \{16\}$

$\frac{1}{4-\sqrt{x}}$ $\quad 0 \quad 16 \quad \mathbb{R}$

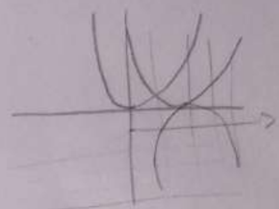
$4 - \sqrt{x} = 0$
$\sqrt{x} = 4$
$x = 16$
$[0, \infty[- \{16\}$

7. The range of the function $f(x) = \frac{1}{1+x^2}$ is

(a) $(-\infty, \infty)$
(b) $[0, 1]$
(c) $(0, 1]$
(d) $(0, 1)$

$\frac{1}{1} \quad \frac{1}{1+0} \quad]0,$

$1 + x^2 \neq 0 \quad \frac{1}{x}$



8. The inverse of the function $f(x) = x^3 - 1$ is

(a) $g(x) = (x+1)^3$
(b) $h(x) = \sqrt[3]{x+1}$
(c) $k(x) = \sqrt{x-1}$
(d) $l(x) = (x-1)^2$

$y = x^3 - 1$
$y + 1 = x^3$
$\sqrt[3]{y+1} = \sqrt[3]{x}$
$x = \sqrt[3]{y+1}$
$y = \sqrt[3]{x+1}$
$= \sqrt[3]{x+1}$

9. $5^{5 \log_{\frac{1}{5}} x} =$

(a) x^5
(b) $-5x$
(c) $5x$
(d) x^{-5}

$5 \log_{\frac{1}{5}} x$

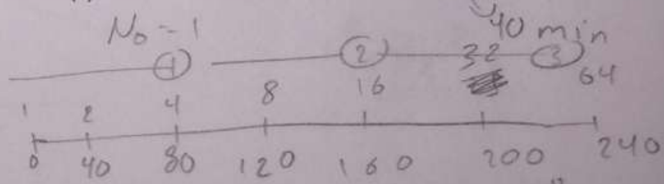
$5^{\log_{\frac{1}{5}} x^5}$

$5^{\log_{\frac{1}{5}} x^{-5}}$

10. Assume $N(t)$ represents the population size at time t. Given that $N_0 = 1$ and the population doubles every 40 minutes. Assume one unit of time is 80 minutes. Then $N(t)$ can be written as

(a) $N(t) = 2^t,\ t = 0, 1, 2, \ldots$
(b) $N(t) = 4^t,\ t = 0, 1, 2, \ldots$
(c) $N(t) = \frac{1}{2}^t,\ t = 0, 1, 2, \ldots$
(d) $N(t) = 2^{-2t},\ t = 0, 1, 2, \ldots$

40 min



$N_0 = 1 \quad 4^0$
$N_1 = 4 \quad 4^1$
$N_2 = 16 \quad 4^2$
$N_3 = 64 \quad 4^3$

$N_t = 4^t$

11. The best expression for a_n whose terms start as $\frac{1}{2}, \frac{1}{8}, \frac{1}{26}, \frac{1}{80}, \ldots$ is

(a) $a_n = \frac{1}{n^2-1},\ n = 1, 2, 3, \ldots$
(b) $a_n = \frac{1}{3^n-1},\ n = 1, 2, 3, \ldots$
(c) $a_n = \frac{1}{-1+3^n},\ n = 1, 2, 3, \ldots$
(d) $a_n = \frac{1}{2(n+1)},\ n = 1, 2, 3, \ldots$

$a_1 = \frac{1}{2}$
$a_2 = \frac{1}{8}$
$a_3 = \frac{1}{26}$
$a_4 = \frac{1}{80}$

18

12. The fixed points of $\{a_n\}$ if $a_{n+1} = \frac{3}{-2+a_n}$ are

(a) $a = -3,\ a = 3$

(b) $a = -3,\ a = -1$

(c) $a = 1,\ a = 3$

(d) $a = -1,\ a = 3$

$a = \frac{3}{-2+a}$

$a(-2+a) = 3$

$-2a + a^2 = 3$

$a^2 - 2a - 3 = 0$

$(a-3)(a+1) = 0$

$a = 3,\ a = -1$

13. $\lim_{n\to\infty} \frac{n+2^{-n}}{n} =$

(a) 1

(b) $\frac{3}{2}$

(c) 3

(d) ∞

1.99 × 2 3.98

14. $\lim_{x\to 2^-} \frac{4}{4-2x} =$ $\frac{4}{small^+} = \infty$

(a) 0

(b) 2

(c) ∞

(d) $-\infty$

15. If

$$f(x) = \begin{cases} \frac{1}{x} & x \geq 1 \\ 2x + c & x < 1 \end{cases}$$

is continuous , then the value of c is

(a) -1

(b) 1

(c) 2

(d) 3

$\frac{1}{x} = 2x + c$

$\frac{1}{1} = 2(1) + c$

$1 = 2 + c$

$1 - 2 = c$

$c = -1$

16. $\lim_{x\to\infty} \frac{(1-2x)^2}{4x^2+5x+2} =$

(a) -1

(b) -0.5

(c) 1

(d) 0

17. $\lim_{x\to 0} \frac{\sec x - 1}{x \sec x} = \lim \frac{\frac{1}{\cos x} - 1}{x \cdot \frac{1}{\cos x}}$

$\lim \frac{\frac{1 - \cos x}{\cos x}}{x \cdot \frac{1}{\cos x}}$

$1^2 + 4/x^2 - 4x$

csc

$\frac{1}{\text{جا}} = $ جا sec x

(a) -1

(b) 0

(c) 1

(d) 0.5

$= \lim_{x\to 0} \frac{1 - \cos x}{x} = 0$

18. $\lim_{x\to 0} \frac{|x-3|-3}{x} =$

(a) 0

(b) −1

(c) 1

(d) Does not exist.

$x-3=0$

$x=3$

$\lim_{x\to 0^+} \frac{x-3-3}{x}$

$\lim_{x\to 0^-} \frac{3-x-3}{x}$

$\lim \frac{-x+3-3}{x}$

$\lim_{x\to 0} -1 = \boxed{-1}$

$\cos x = x$

19. The equation $\cos x = x$ has a solution on the interval

(a) $[\pi, 2\pi]$

(b) $[0, \frac{\pi}{2}]$

(c) $[\frac{\pi}{2}, \pi]$

(d) None of the above.

root $y=0$

$f(x) = \cos x - x$

$f(0) = 1 - 0 = 1$

$f(\frac{\pi}{2}) = 0 - \frac{\pi}{2} < 0$

$x - \cos x = 0$

$\boxed{\cos x} - x = 0$

حل تجريبي

20. The discontinuities of the function $y = \sec x$ occur at the points

(a) $\{(2n+1)\frac{\pi}{2}, n \in Z\}$

(b) $\Re - \{(2n+1)\frac{\pi}{2}, n \in Z\}$

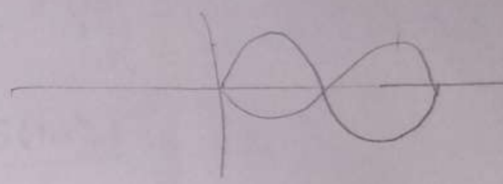
(c) $\{n\pi, n \in Z\}$

(d) $\Re - \{n\pi, n \in Z\}$

$= \frac{1}{\cos x}$

$x \neq \frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm 5\frac{\pi}{2}$

$(2n+1)\frac{\pi}{2}$



21. One of the following functions is an even function

(a) $y = \sin x$

(b) $y = \csc x$

(c) $y = \sec x$

(d) $y = \tan x$

$f(x) = f(-x)$

$\csc = \frac{1}{\sin} \to$ odd

$\sec = \frac{1}{\cos}$ even

$f(x) = \sin x$

$\sin(-x)$

$-\sin(x)$

$f(x) = \cos(x)$ even

22. $\lim_{x\to 1} \frac{1-\sqrt{x}}{1-x} \times \frac{1+\sqrt{x}}{1+\sqrt{x}}$

(a) $\frac{1}{2}$

(b) 1

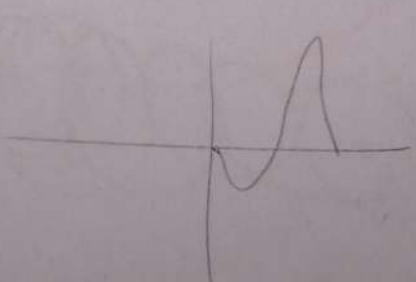
(c) 2

(d) Does not exist.

$= \lim_{x\to 1} \frac{(1-x)}{(1-x)(1+\sqrt{x})}$

$\frac{1}{2}$

$\frac{1}{\cos}$



Question Two (5 points) $f(x) = \frac{x^2+4x+3}{x+1}$

Find a continuous extension $F(x)$ for $f(x)$ (if possible). disc. $x = -1$

$\lim_{x \to a} f(x) = f(a)$

$\lim_{x \to -1} \frac{x^2+4x+3}{x+1} = \cancel{\frac{a^2+4a+3}{a+1}}$

~~( )( ) = f(a)~~

~~f(x) is continuous on IR~~
~~so there is no need to find a~~
continuous extension

$$F(x) = \begin{cases} \frac{x^2+4x+3}{x+1} & x \neq -1 \\ \lim_{x \to -1} f(x) & x = -1 \end{cases}$$

Question Three (10 points) Find the following limits.

a- $\lim_{x \to 5} \frac{x^3-25x}{x^2-8x+15} =$ $\lim_{x \to 5} \frac{x(x^2-25)}{(x-5)(x-3)} = \lim_{x \to 5} \frac{x\cancel{(x-5)}(x+5)}{\cancel{(x-5)}(x-3)}$

$= \lim_{x \to 5} \frac{x(x+5)}{(x-3)} = \frac{5(5+5)}{(5-3)} = \frac{5(10)}{2} = 25$

5

b- $\lim_{x \to 0} x^2 \cos \frac{1}{x} = 0$

$$-1 < \cos \frac{1}{x} < 1$$

$$-x^2 < x^2 \cos \frac{1}{x} < x^2$$

$$\lim_{x \to 0} -x^2 < \lim_{x \to 0} x^2 \cos \frac{1}{x} < \lim_{x \to 0} x^2$$

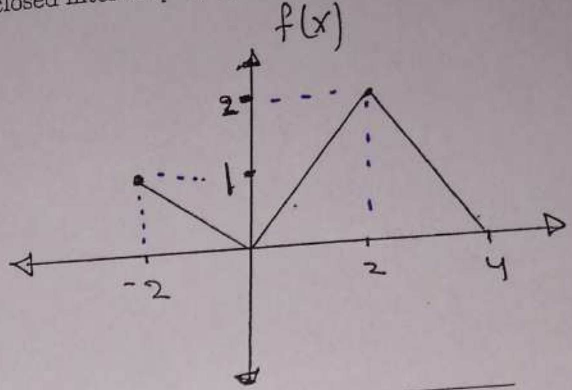
$$0 < \lim_{x \to 0} x^2 \cos \frac{1}{x} < 0$$

Then $\lim_{x \to 0} x^2 \cos \frac{1}{x} = 0$

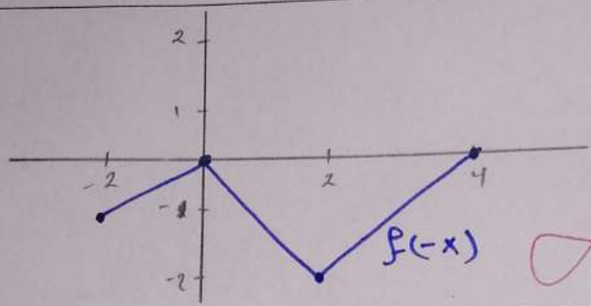
5

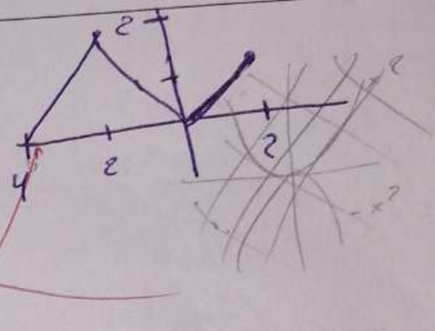
Question Four(20 points) Use the graph of $f(x)$ defined on the closed interval $[-2, 4]$ to sketch the graph of the functions

Graph of $f(x)$

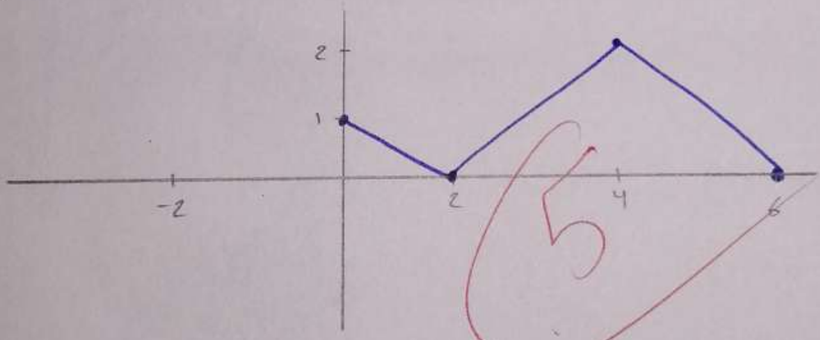


a- $f(-x)$

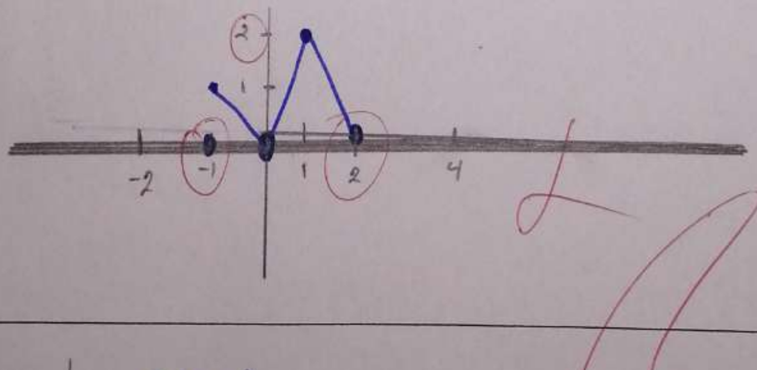


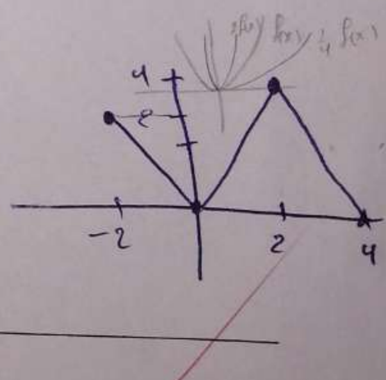


b- $f(x-2)$



c- $2f(x)$





d- $f(x+1)-2$