Birzeit University-Mathematics Department

Math 2311-Calculus III

First Exam — First Semester 2018/2019

Name:..........KEY.......... Number:...............

Instructor:.............................. Section:...............

Question 1.(21 points) Circle the correct answer:

(1) The domain of the function $f(x, y) = \ln(x + y)$ is

(a) $\{(x, y)|y \neq -x\}$

(b) the xy-plane except $(0, 0)$

(c) $\{(x, y)|y \geq -x\}$

(d) $\{(x, y)|y > -x\}$

(2) Consider the function $f(x, y, z) = x^2yze^z + 3xy\cos z$. Then $f_{xyz}(1, 1, 0) =$

(a) 3

(b) 2

(c) -1

(d) 1

(3) $\lim\limits_{(x,y)\to(1,1)} \frac{x^3-y^3}{x-y} =$

(a) 1

(b) 0

(c) 3

(d) Does not exist.

(4) Let $f(x, y) = x^2y + y + x$. The equation of the tangent line to the level curve $f(x, y) = 3$ at $(1, 1)$ is

(a) $3x + 2y = 5$

(b) $2x + 3y = 5$

(c) $2x - y = 1$

(d) $3x + y = 4$

(5) The length of the curve $\mathbf{r}(t) = t\mathbf{i} + \frac{\sqrt{6}}{2}t^2\mathbf{j} + t^3\mathbf{k}$, $-1 \leq t \leq 1$ is

(a) 2

(b) 4.

(c) 6.

(d) 3.

(6) Let $f(x, y)$ be a given function and let $x = e^{st}$, $y = s^2t^2$, $f_x(1, 0) = 2$ and $f_y(1, 0) = -1$. Then f_s at $(s, t) = (0, 1)$ is

(a) 1

(b) 2

(c) -1

(d) 0

(7) The tangent line to the curve $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$ at $t = 1$ has parametric equations

(a) $x = t, y = t, z = t,\ t \in (-\infty, \infty)$

(b) $x = 1 + t, y = 1 + t, z = 1 + t,\ t \in (-\infty, \infty)$

(c) $x = t, y = 2t, z = 3t,\ t \in (-\infty, \infty)$

(d) $x = 1 + t, y = 1 + 2t, z = 1 + 3t,\ t \in (-\infty, \infty)$

(8) Assume that the equation $2x\cos(z - 1) + \ln z = x + y$ defines z as a function of x and y. At the point $(x, y, z) = (0, 0, 1)$, $z_x =$

(a) 1

(b) 2

(c) -2

(d) -1

(9) $\lim_{(x,y)\to(0,0)} \frac{x-y}{\sqrt{x^2+y^2}} =$

(a) 0

(b) 1

(c) $+\infty$

(d) Does not exist.

(10) Let $f(x, y) = \tan^{-1}(xy)$, then $f_{xy}(0, 0) =$

(a) 1
(b) 2
(c) -2
(d) 4

(11) The function $f(x, y) = \begin{cases} \frac{\sin(x^2+y^2)}{x^2+y^2} & , \quad (x, y) \neq (0, 0) \\ 1 & , \quad (x, y) = (0, 0) \end{cases}$

(a) is continuous at $(0, 0)$
(b) is not continuous at $(0, 0)$
(c) $\lim\limits_{(x,y)\to(0,0)}$ does not exist.
(d) None of the above.

(12) The function $f(x, y) = x^2 + \ln(x + y)$ has zero derivative at $(0, 1)$ in the direction

(a) $\frac{\mathbf{i}+\mathbf{j}}{\sqrt{2}}$
(b) $\frac{\mathbf{i}-\mathbf{j}}{\sqrt{2}}$
(c) $\frac{-\mathbf{i}-\mathbf{j}}{\sqrt{2}}$
(d) None of the above

(13) The directional derivative of the function $f(x, y, z) = x^2 + 2y^2 + 3z^2$ at $P_0(1, 1, 1)$ in the direction of $\mathbf{u} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ is

(a) 12
(b) $12\sqrt{3}$
(c) $4\sqrt{3}$
(d) 4.

(14) Let $z = 4e^x + \ln y$, $x = \frac{\sqrt{3}}{2}u - \cos v$, $y = u \sin v$, then z_u at $(u, v) = (1, \pi/6) =$

(a) $2\sqrt{3}$
(b) $1 - 2\sqrt{3}$
(c) $4\sqrt{3}$
(d) $1 + 2\sqrt{3}$

Question 2.(8 points) Let $\mathbf{r}(t) = (\cos t + t\sin t)\mathbf{i} + (\sin t - t\cos t)\mathbf{j}$, $t > 0$.

(a) Find $\mathbf{v}(t)$ (1 point each).

$$\vec{v}(t) = t\cos t\,\hat{i} + t\sin t\,\hat{j}$$

(b) Find $|\mathbf{v}(t)|$

$$|\vec{v}(t)| = t$$

(c) Find the unit tangent vector $\mathbf{T}$

$$\vec{T} = \cos t\,\hat{i} + \sin t\,\hat{j}$$

(d) Find the unit normal vector $\mathbf{N}$

$$\frac{d\vec{T}}{dt} = -\sin t\,\hat{i} + \cos t\,\hat{j}, \quad \left|\frac{d\vec{T}}{dt}\right| = 1$$

$$\vec{N} = -\sin t\,\hat{i} + \cos t\,\hat{j}$$

(e) Calculate the curvature, κ, of the curve.

$$\kappa = \frac{1}{|\vec{v}|}\left|\frac{d\vec{T}}{dt}\right| = \frac{1}{t}$$

(f) Find the arc length parameter $s(t)$. Take $t_0 = 0$.

$$s(t) = \int_0^t \tau\, d\tau = \frac{t^2}{2}$$

(g) Find $\mathbf{a}_T$

$$a_T = \frac{d}{dt}|\vec{v}| = 1$$

(h) Find $\mathbf{a}_N$

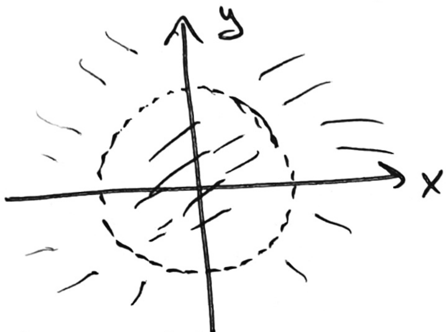
$$\vec{a}_N = \kappa|\vec{v}|^2 = \frac{1}{t}t^2 = t.$$

Question 3.(11 points) Consider the function $f(x,y) = \frac{1}{4-x^2-y^2}$

2pts (a) Find the domain of f.

$$D = \{(x,y) \mid x^2+y^2 \neq 4\}$$

2pts (b) Sketch the domain of f in the xy−plane.



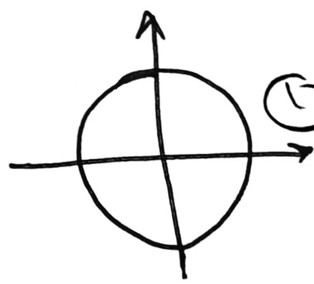
2pts (c) Find the range of f.

$$R = (-\infty, 0) \cup [\tfrac{1}{4}, \infty)$$

(d) Sketch the level curve $f(x,y) = c$.

$$\frac{1}{4-x^2-y^2} = c$$

$\Longleftrightarrow$ (1) $x^2+y^2 = 4-\frac{1}{c}$ Circle



(1)

(e) Find the boundary of the domain.

(1) $\{(x,y) \mid x^2+y^2 = 4\}$

(f) Is the domain open or closed?

(1) open

(g) Is the domain bounded or unbounded?

(1) unbounded.

Question 4.(10 points) Consider the function $f(x, y, z) = \ln(2x + 3y + 6z)$.

(a) Find $\nabla f(x, y, z)$.

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$$\vec{\nabla} f(x,y,z) = \frac{2\hat{i}}{2x+3y+6z} + \frac{3}{2x+3y+6z}\hat{j} + \frac{6}{2x+3y+6z}\hat{k}$$

(b) Calculate the directional derivative of f at $P_0(-1, -1, 1)$ in the direction of $\mathbf{v} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$

1 $\vec{\nabla} f(-1,-1,1) = 2\hat{i} + 3\hat{j} + 6\hat{k}$

1 $\vec{u} = \frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}k$

1 $D_u f(-1,-1,1) = (2\hat{i}+3\hat{j}+6k)\cdot(\frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}k) = \frac{13}{3}$

(c) Find the direction in which f increases most rapidly and find the derivative in that direction.

1 $\vec{u} = \dfrac{\vec{\nabla} f(-1,-1,-1)}{|\nabla f|} = \frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k})$

1 Derivative $= |\vec{\nabla} f| = 7$.

(d) Find the direction in which f decreases most rapidly and find the derivative in that direction.

1 $-\vec{u} = -\frac{1}{7}(2\hat{i} + 3\hat{j} + 6k)$

1 Derivative $= -|\vec{\nabla} f| = -7$.

(e) Show that $xf_x + yf_y + zf_z = 1$

1 $xf_x + yf_y + zf_z = \dfrac{2x+3y+6z}{2x+3y+6z} = 1$.