Birzeit University-Mathematics Department
Math 2311-Calculus III

First Hour Exam | First Semester 2019/2020
Name:.....KEY.....
Number:...............
Instructor:...
Section:...............

Question 1.(28 points) Circle the correct answer:

(1) The domain of the function $f(x,y) = \frac{1}{\ln(1-x^2-y^2)}$ is

(a) $\{(x,y)|x^2+y^2 > 1\}$
(b) $\{(x,y)|x^2+y^2 < 1\}$
(c) $\{(x,y)|0 < x^2+y^2 < 1\}$
(d) $\{(x,y)|x^2+y^2 \neq 1\}$

(2) The level curves of the function $f(x,y) = \sqrt{2x^2+3y^2}$ are

(a) circles.
(b) ellipses
(c) parabolas
(d) lines

(3) $\lim_{(x,y)\to(0,0)} \frac{e^{-x^2-y^2}-1}{x^2+y^2} =$

(a) 1
(b) 0
(c) -1
(d) Does not exist.

(4) Let $z = f(x,y)$ be a function such that $f_x(0,0) = 1$, $f_y(0,0) = -2$, where $x = \sin(s+t)$, $y = \tan(s)$. Then f_s at $(s,t) = (0,0)$ is

(a) -1
(b) 1
(c) 2
(d) 0

(5) Suppose that the equation $zy + x \ln z - x + y = 0$ defines z as a function of x and y. Then $\frac{\partial z}{\partial x} =$

(a) $\frac{z+\ln z}{yz+x}$

(b) $\frac{z+z\ln z}{yz+x}$

(c) $\frac{z-z\ln z}{yz+x}$

(d) $\frac{x-z\ln z}{yz-x}$

(6) If $|\mathbf{v}(t)| = 1$ then one of the following statements is false

(a) $a_T = 0$

(b) $\mathbf{a}(t)$ is parallel to $\mathbf{N}$

(c) $\mathbf{a}(t).\mathbf{v}(t) = 0$

(d) $\mathbf{a}(t) = 0$

(7) The tangent line to the curve $\mathbf{r}(t) = (t+1)\mathbf{i} + e^t\mathbf{j} + t\mathbf{k}$ at $t = 0$ has parametric equations

(a) $x = 1 + t, y = t, z = t,\ t \in (-\infty, \infty)$

(b) $x = 1 + t, y = 1 + t, z = t,\ t \in (-\infty, \infty)$

(c) $x = t, y = t, z = 1 + t,\ t \in (-\infty, \infty)$

(d) $x = 1 + t, y = 1 - t, z = t,\ t \in (-\infty, \infty)$

(8) The region $R = \{(x, y)|1 < x^2 + y^2 < 4\}$ is

(a) open and bounded

(b) closed and unbounded

(c) closed and bounded

(d) open and unbounded

(9) $\lim\limits_{(x,y)\to(0,0)} \frac{y}{x^2 - y} =$

(a) 0

(b) 1

(c) $+\infty$

(d) Does not exist.

(10) Let $f(x,y) = \tan^{-1}\left(\frac{x}{y}\right)$, then $f_{xy}(0,1) =$

(a) 1
(b) 0
(c) 2
(d) -1

(11) The function $f(x,y) = \begin{cases} \frac{\sin(x-y)}{|x|+|y|} & , \quad (x,y) \neq (0,0) \\ a & , \quad (x,y) = (0,0) \end{cases}$

(a) is continuous at $(0,0)$ if $a = 0$
(b) is continuous at $(0,0)$ if $a = 1$
(c) $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist.
(d) None of the above.

(12) One of the following vectors is tangent to the surface $x^2y = 1$ at $(1,1)$

(a) $\mathbf{i} + \mathbf{j}$
(b) $\mathbf{i} - 2\mathbf{j}$
(c) $2\mathbf{i} + \mathbf{j}$
(d) $2\mathbf{i} - \mathbf{j}$

(13) The directional derivative of the function $f(x,y,z) = \sqrt{x^2 + 2y^2} - z$ at $P_0(1,-2,1)$ in the direction of $\mathbf{u} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ is

(a) $-\frac{2}{3}$
(b) -1
(c) $-\frac{1}{3}$
(d) -3.

(14) At the point $(1,1)$, the directional derivative of $f(x,y)$ is zero in the direction $\frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j})$ and the greatest value of the directional derivative of f at $(1,1)$ is 2, then one possible vector for $\nabla f(1,1)$ is

(a) $\sqrt{2}\mathbf{i} + \sqrt{2}\mathbf{j}$
(b) $\sqrt{2}\mathbf{i} - \sqrt{2}\mathbf{j}$
(c) $-\sqrt{2}\mathbf{i} - \sqrt{2}\mathbf{j}$
(d) $\mathbf{i} - \mathbf{j}$

Question 2.(12 points) Let $\mathbf{r}(t) = 2t\mathbf{i} + t^2\mathbf{j} + \frac{1}{3}t^3$, $t > 0$.

(a) Find the velocity $\mathbf{v}(t)$

$\vec{v}(t) = 2\hat{i} + 2t\hat{j} + t^2\hat{k}$

(b) Find the speed $|\mathbf{v}(t)|$

$|\vec{v}| = \sqrt{4+4t^2+t^4} = \sqrt{(2+t^2)^2} = 2+t^2.$

(c) Find the acceleration $\mathbf{a}(t)$

$\vec{a}(t) = 0i + 2j + 2tk$

(d) Calculate $\mathbf{v} \times \mathbf{a}$

2.

$$\vec{v} \times \vec{a} = \begin{vmatrix} i & j & k \\ 2 & 2t & t^2 \\ 0 & 2 & 2t \end{vmatrix} = 2t^2\hat{i} - 4t\hat{j} + 4\hat{k}$$

(e) Calculate $|\mathbf{v} \times \mathbf{a}|$

$|\vec{v} \times \vec{a}| = \sqrt{4t^4+16t^2+16} = 2\sqrt{t^4+4t^2+4} = 2(t^2+2)$

(f) Calculate the curvature, κ

$$\kappa = \frac{|\vec{v}\times\vec{a}|}{|\vec{v}|^3} = \frac{2(t^2+2)}{(2+t^2)^3} = \frac{2}{(2+t^2)^2}$$

(g) Find $\mathbf{a}_T$

$a_T = \frac{d}{dt}|\vec{v}| = 2t$

(h) Find $\mathbf{a}_N$

$$a_N = \kappa|\vec{v}|^2 = \frac{2}{(2+t^2)^2}(2+t^2)^2 = 2.$$

(i) Find the torsion τ

3.

$$\begin{vmatrix} 2 & 2t & t^2 \\ 0 & 2 & 2t \\ 0 & 0 & 2 \end{vmatrix} = 8$$

$$\tau = \frac{8}{4(2+t^2)^2} = \frac{2}{(2+t^2)^2}.$$

Question 3.(12 points) Consider the function $f(x, y) = \ln(x + y + 1)$

(a) Find the domain D of f.

$$D = \{(x,y) \mid x+y+1>0\}$$

(b) Find the range of f.

$$(-\infty, \infty)$$

(c) What is the boundary of D?

$$\{(x,y) \mid x+y+1=0\}$$

(d) Find the equation of the level curve that passes through the point $(0, 0)$

$$\ln(x+y+1) = 0$$

$$y = -x$$

(e) Find the direction in which f increase most rapidly at the point $(1, 1)$ and the value of the directional derivative in that direction.

$$\vec{\nabla} f(x,y) = \frac{1}{x+y+1}\hat{i} + \frac{1}{x+y+1}\hat{j}$$

$$\vec{\nabla} f(1,1) = \frac{1}{3}\hat{i} + \frac{1}{3}\hat{j}$$

$$\vec{u} = \frac{\vec{\nabla} f}{|\vec{\nabla} f|} = \frac{1}{3}(\hat{i}+\hat{j})\left(\frac{\sqrt{2}}{3}\right)^{-1} = \frac{\hat{i}+\hat{j}}{\sqrt{2}}, \quad D_u f(1,1) = \frac{\sqrt{2}}{3}$$

(f) Find the directions of zero change. at $(1,1)$

$$\frac{\hat{i}-\hat{j}}{\sqrt{2}}, \quad \frac{-\hat{i}+\hat{j}}{\sqrt{2}}.$$